Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/15-1.1.1.4-a+b-x-^m-c+d-x-^n-e+f-x-^p-g+h-x-^q

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [159]. This is test number [15].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.37 (158)	0.63 (1)
Mathematica	96.86 (154)	3.14 (5)
Maple	80.50 (128)	19.50 (31)
Fricas	43.40 (69)	56.60 (90)
Mupad	30.82 (49)	69.18 (110)
Giac	28.30 (45)	71.70 (114)
Maxima	24.53 (39)	75.47 (120)
Sympy	18.87 (30)	81.13 (129)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

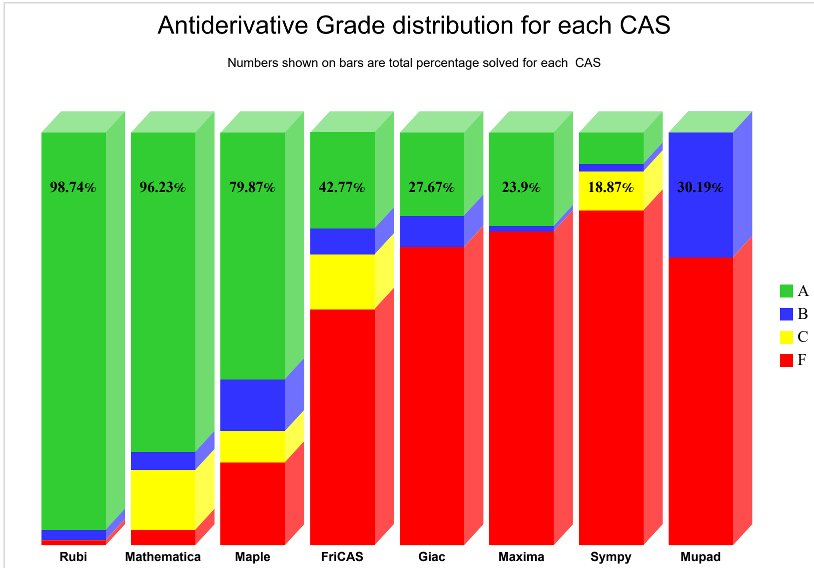
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

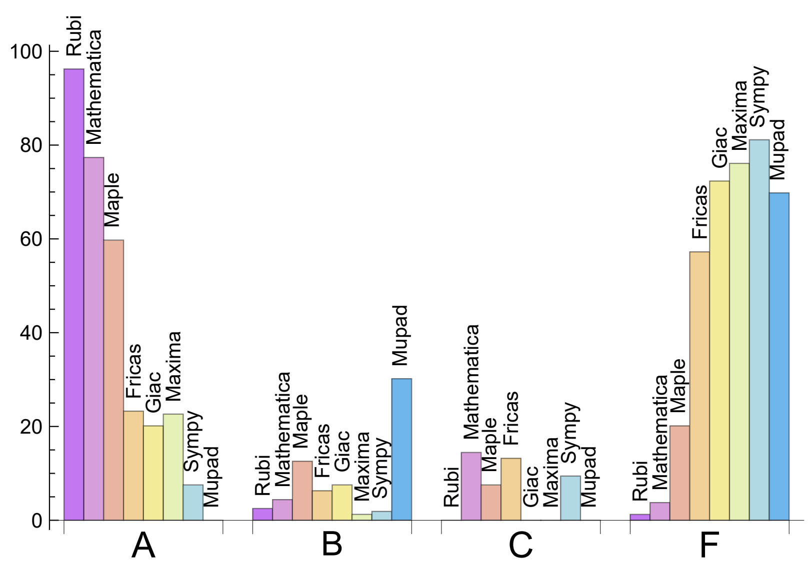
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.226	2.516	0.000	1.258
Mathematica	77.358	4.403	14.465	3.774
Maple	59.748	12.579	7.547	20.126
Fricas	23.270	6.289	13.208	57.233
Maxima	22.642	1.258	0.000	76.101
Giac	20.126	7.547	0.000	72.327
Sympy	7.547	1.887	9.434	81.132
Mupad	0.000	30.189	0.000	69.811

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	5	100.00	0.00	0.00
Maple	31	100.00	0.00	0.00
Fricas	90	84.44	15.56	0.00
Mupad	110	0.00	100.00	0.00
Giac	114	98.25	0.88	0.88
Maxima	120	95.00	0.00	5.00
Sympy	129	58.91	29.46	11.63

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.16
Maxima	0.26
Giac	0.30
Fricas	0.80
Maple	2.07
Mupad	4.37
Mathematica	8.09
Sympy	11.43

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	100.77	1.10	84.00	1.05
Giac	171.29	1.73	109.00	1.32
Rubi	224.26	1.06	191.50	1.00
Mathematica	304.35	1.19	155.00	0.95
Fricas	364.51	2.17	69.00	1.16
Maple	385.31	1.59	178.00	1.20
Sympy	464.57	4.10	197.50	2.41
Mupad	694.12	4.83	244.00	2.38

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

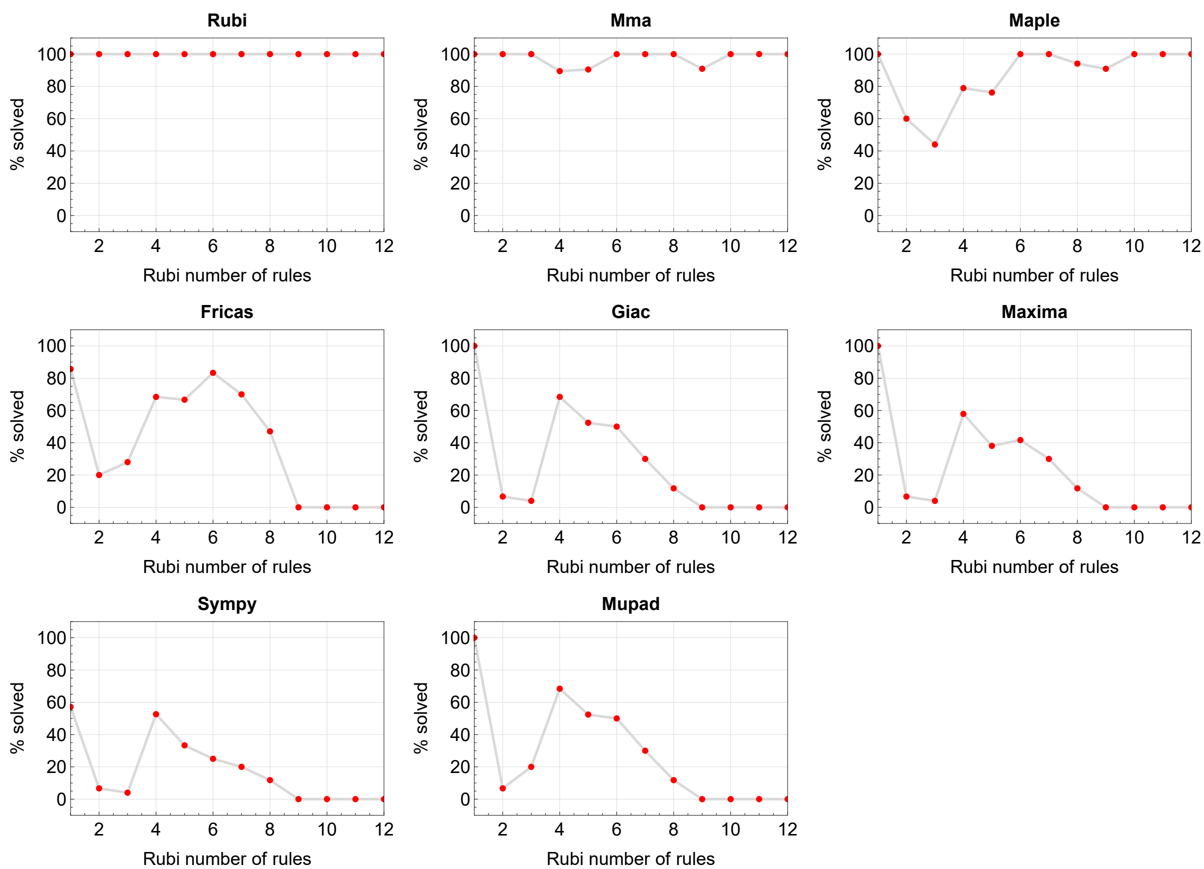


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

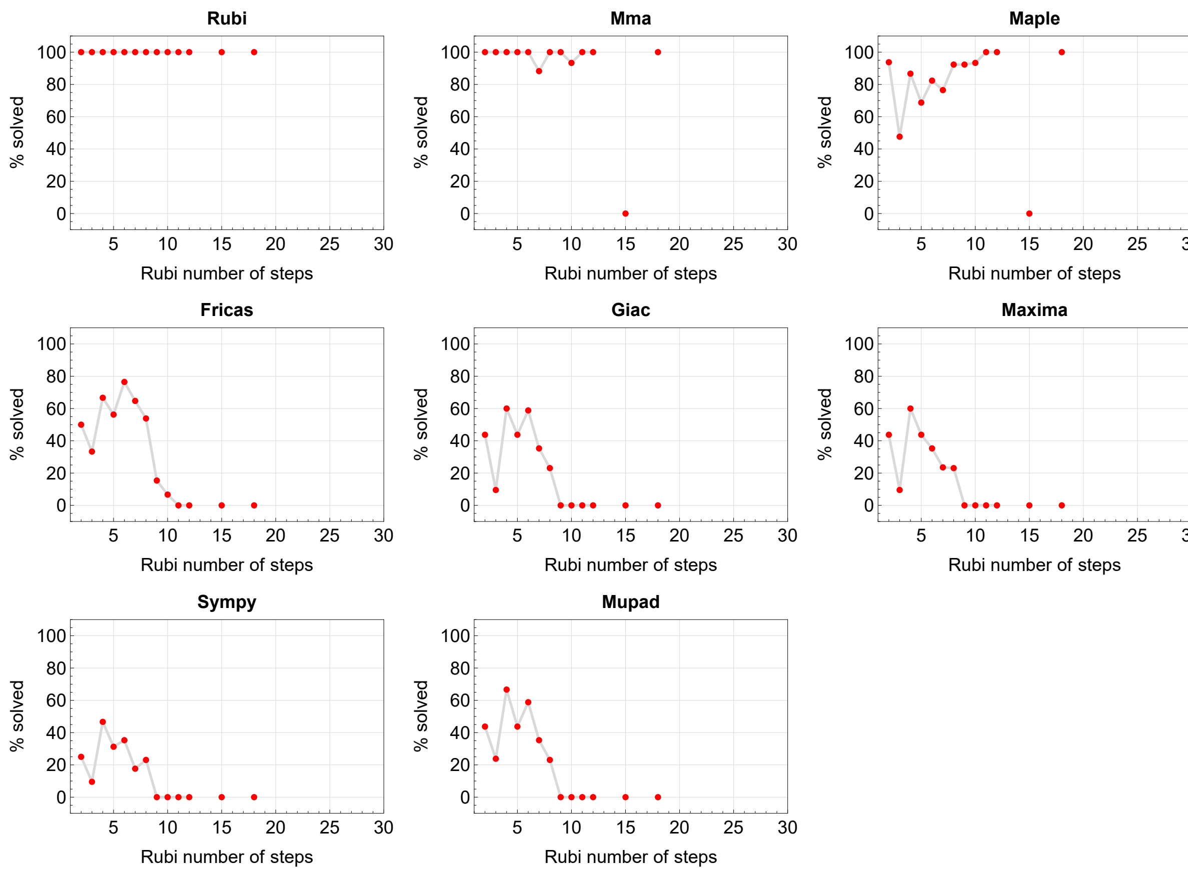


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

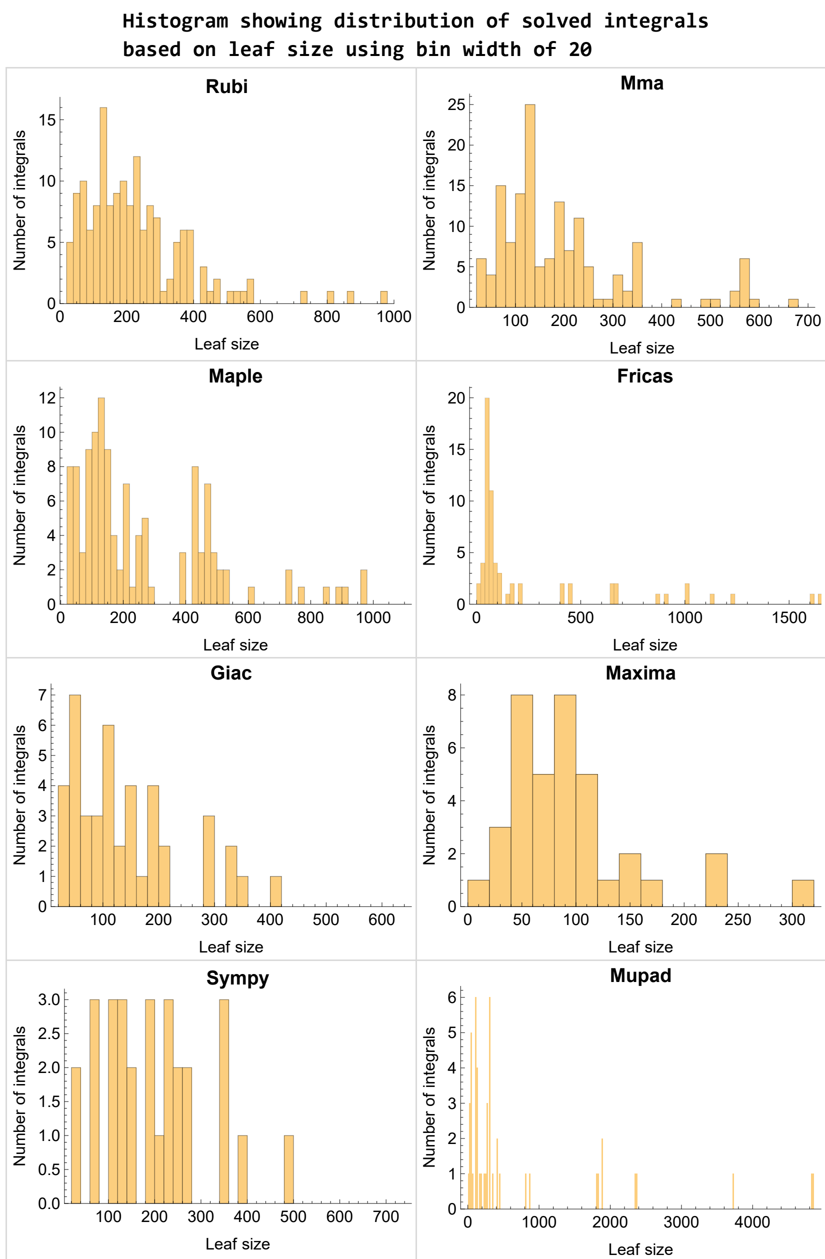


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

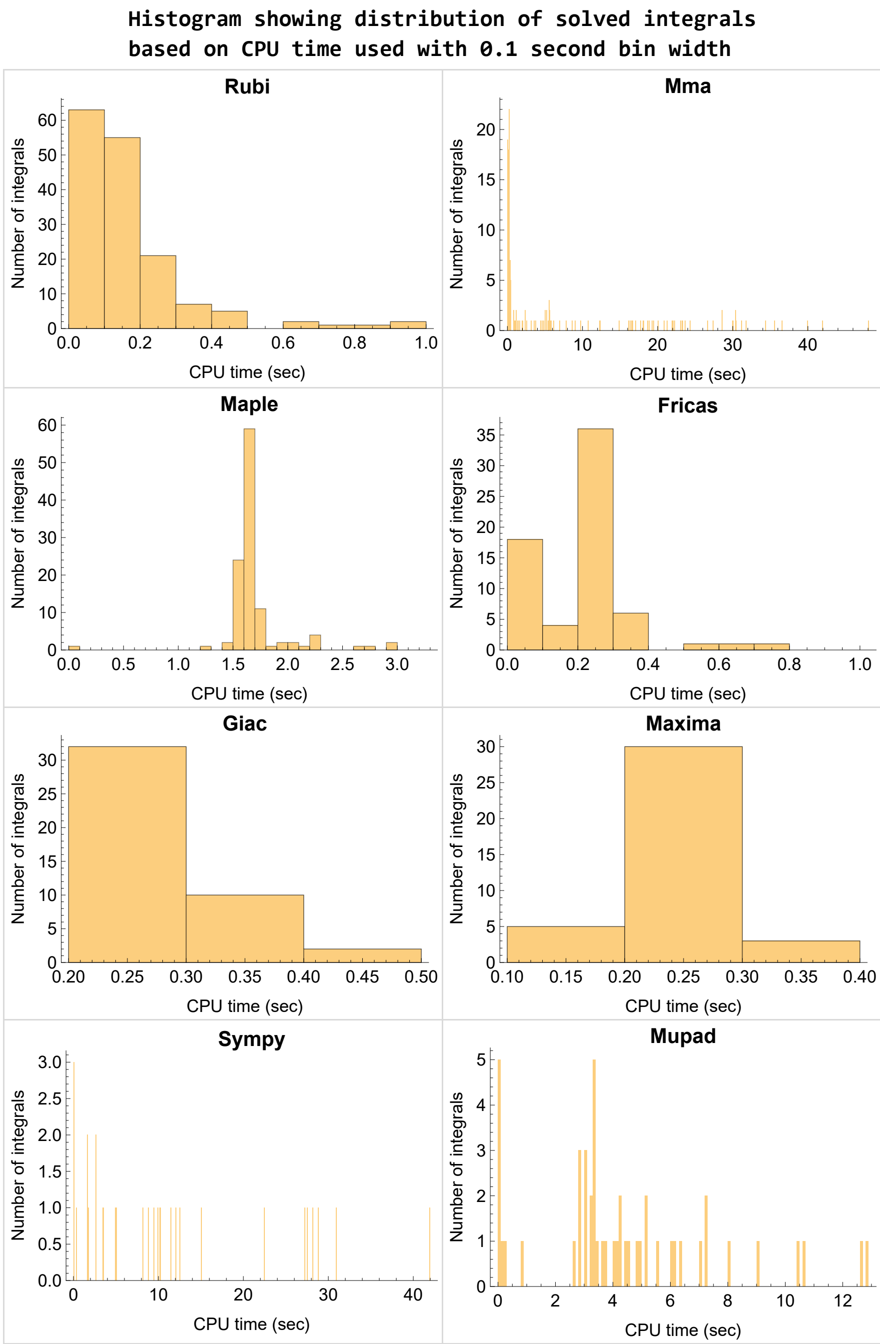


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

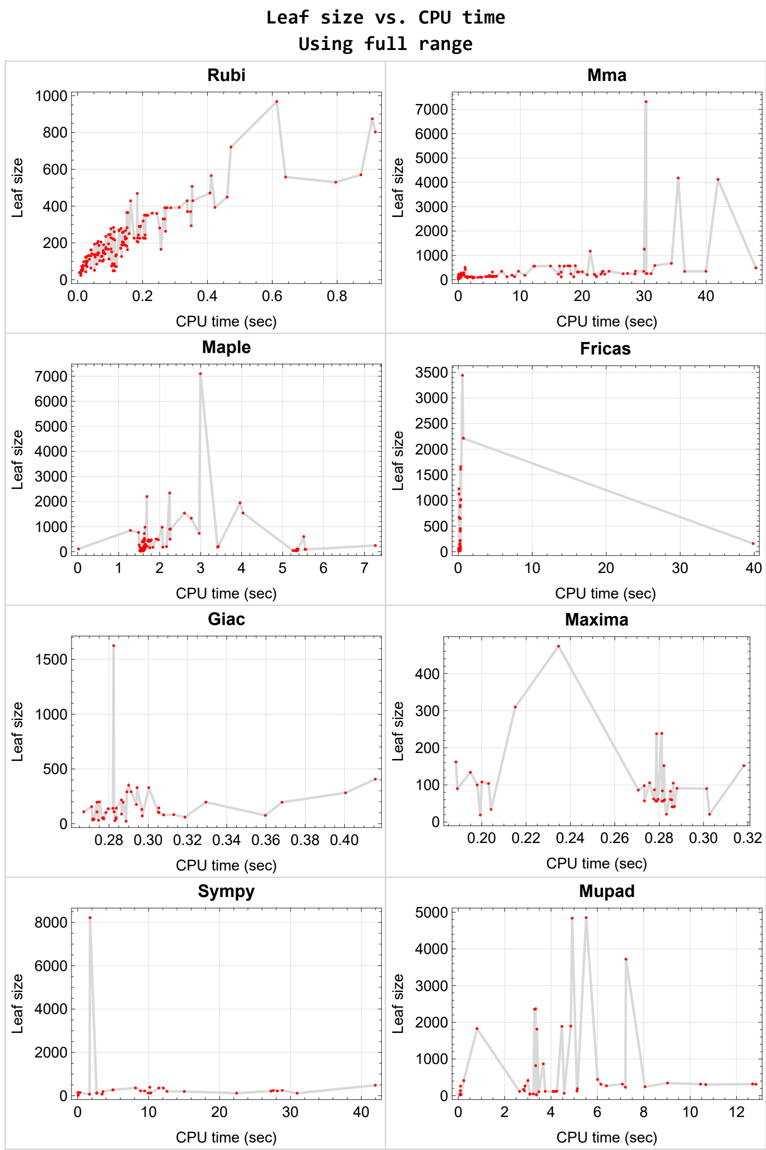


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{143}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {85, 86, 87, 88, 89, 93, 94, 95, 96, 99, 101, 102, 107, 110, 146, 155, 156, 157, 158}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

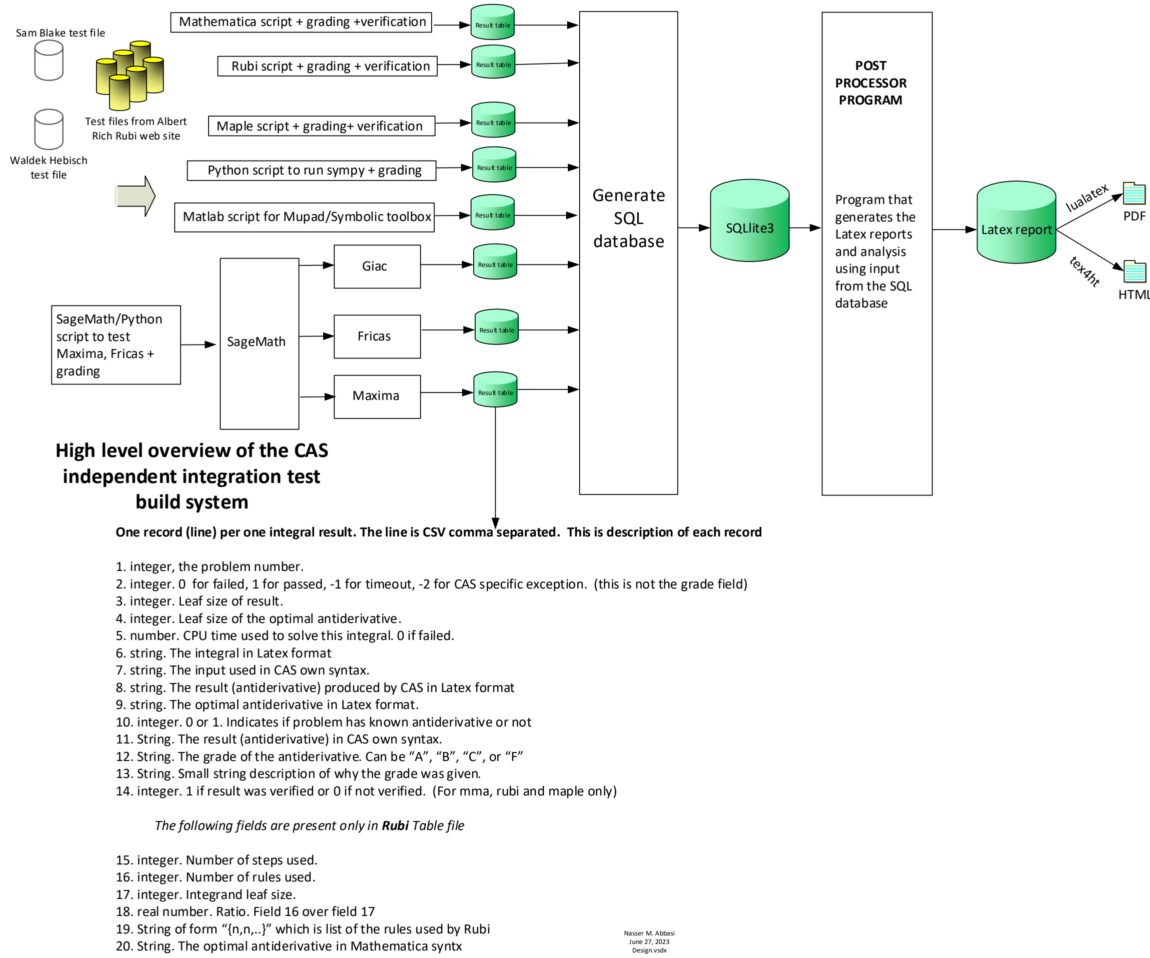
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 158, 159 }

B grade { 97, 155, 156, 157 }

C grade { }

F normal fail { 111 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 60, 61, 62, 63, 64, 66, 67, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 142, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

B grade { 25, 26, 54, 97, 107, 108, 110 }

C grade { 33, 34, 43, 58, 59, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F normal fail { 132, 139, 140, 141, 144 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 101, 102, 104, 106, 107, 109, 154, 157, 158, 159 }

B grade { 33, 34, 71, 73, 74, 75, 76, 89, 99, 100, 105, 108, 110, 111, 119, 130, 131, 134, 135, 155 }

C grade { 22, 23, 24, 25, 97, 103, 149, 150, 151, 152, 153, 156 }

F normal fail { 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

B grade { 13, 14, 20, 21, 26, 119, 130, 131, 134, 135 }

C grade { 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 51, 52, 53, 54, 60, 61, 62, 63, 64, 68, 69 }

F normal fail { 39, 40, 41, 42, 48, 49, 50, 55, 56, 57, 65, 66, 67, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

F(-1) timedout fail { 5, 43, 58, 59, 70, 71, 72, 73, 74, 75, 76, 99, 107, 108 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159 }

B grade { 119, 155 }

C grade { }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

F(-1) timedout fail { }

F(-2) exception fail { 12, 13, 14, 19, 20, 21 }

Giac

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 149, 150, 154, 155, 156, 157 }

B grade { 5, 26, 27, 28, 29, 30, 119, 151, 152, 153, 158, 159 }

C grade { }

F normal fail { 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 144, 145, 146, 147, 148 }

F(-1) timedout fail { 139 }

F(-2) exception fail { 33 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 119, 130, 131, 134, 135, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18 }

B grade { 12, 19, 119 }

C grade { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 151, 152, 156, 157 }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 120, 121 }

F(-1) timedout fail { 3, 4, 5, 13, 14, 20, 21, 77, 78, 83, 84, 85, 91, 92, 93, 101, 106, 118, 123, 132, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 154, 155, 158, 159 }

F(-2) exception fail { 112, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 137, 138 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	109	108	142	148	142	115
N.S.	1	1.00	1.00	0.97	0.96	1.27	1.32	1.27	1.03
time (sec)	N/A	0.096	0.028	0.019	0.200	0.199	0.027	0.284	2.641

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	123	175	162	163	146	200	174
N.S.	1	1.00	0.98	1.39	1.29	1.29	1.16	1.59	1.38
time (sec)	N/A	0.135	0.047	1.519	0.188	0.236	0.310	0.275	2.816

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	85	102	104	117	0	108	105
N.S.	1	1.00	1.01	1.21	1.24	1.39	0.00	1.29	1.25
time (sec)	N/A	0.059	0.037	1.566	0.203	0.261	0.000	0.274	3.471

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	102	108	134	160	0	156	127
N.S.	1	1.00	0.94	1.00	1.24	1.48	0.00	1.44	1.18
time (sec)	N/A	0.080	0.043	1.603	0.195	39.869	0.000	0.271	5.118

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	164	164	310	0	0	351	317
N.S.	1	1.00	1.01	1.01	1.90	0.00	0.00	2.15	1.94
time (sec)	N/A	0.153	0.112	1.759	0.215	0.000	0.000	0.290	7.079

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83
time (sec)	N/A	0.010	0.007	1.536	0.199	0.261	0.074	0.289	0.068

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	30	34	53	32	31	29
N.S.	1	1.00	0.77	0.70	0.79	1.23	0.74	0.72	0.67
time (sec)	N/A	0.025	0.015	1.538	0.204	0.238	0.084	0.275	0.125

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	197	214	239	649	355	330	413
N.S.	1	1.00	0.87	0.94	1.05	2.86	1.56	1.45	1.82
time (sec)	N/A	0.184	0.290	1.663	0.281	0.268	12.080	0.300	0.236

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	131	144	152	405	223	196	263
N.S.	1	1.00	0.90	0.99	1.04	2.77	1.53	1.34	1.80
time (sec)	N/A	0.064	0.166	1.607	0.282	0.259	9.432	0.287	2.875

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	81	83	91	219	122	102	136
N.S.	1	1.00	1.05	1.08	1.18	2.84	1.58	1.32	1.77
time (sec)	N/A	0.024	0.094	1.562	0.288	0.284	10.296	0.278	0.093

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	60	111	70	55	45
N.S.	1	1.00	0.98	0.85	1.11	2.06	1.30	1.02	0.83
time (sec)	N/A	0.011	0.046	5.252	0.286	0.292	1.672	0.277	0.081

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	103	0	449	196	109	2368
N.S.	1	1.00	1.00	1.02	0.00	4.45	1.94	1.08	23.45
time (sec)	N/A	0.076	0.196	5.349	0.000	0.309	15.023	0.267	3.334

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	123	132	0	1018	0	140	1827
N.S.	1	1.00	0.97	1.04	0.00	8.02	0.00	1.10	14.39
time (sec)	N/A	0.073	0.459	1.617	0.000	0.348	0.000	0.282	0.809

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	195	213	0	2216	0	292	4852
N.S.	1	1.00	0.94	1.02	0.00	10.65	0.00	1.40	23.33
time (sec)	N/A	0.183	0.833	1.692	0.000	0.666	0.000	0.291	5.517

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	236	217	238	641	355	330	413
N.S.	1	1.00	1.04	0.96	1.05	2.84	1.57	1.46	1.83
time (sec)	N/A	0.174	0.297	1.619	0.279	0.259	11.437	0.294	3.005

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	157	145	152	403	223	196	263
N.S.	1	1.00	1.08	1.00	1.05	2.78	1.54	1.35	1.81
time (sec)	N/A	0.062	0.203	1.590	0.318	0.261	8.859	0.274	0.094

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	91	86	90	217	122	102	136
N.S.	1	1.00	1.18	1.12	1.17	2.82	1.58	1.32	1.77
time (sec)	N/A	0.016	0.136	1.555	0.301	0.250	9.942	0.305	2.855

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	60	111	70	55	45
N.S.	1	1.00	0.98	0.85	1.11	2.06	1.30	1.02	0.83
time (sec)	N/A	0.016	0.064	5.287	0.282	0.242	1.665	0.284	0.073

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	103	0	450	199	109	2355
N.S.	1	1.00	1.00	1.02	0.00	4.46	1.97	1.08	23.32
time (sec)	N/A	0.077	0.204	5.377	0.000	0.283	12.593	0.283	3.290

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	122	110	0	1008	0	137	1814
N.S.	1	1.00	0.95	0.86	0.00	7.88	0.00	1.07	14.17
time (sec)	N/A	0.075	0.464	1.623	0.000	0.323	0.000	0.280	3.391

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	194	214	0	2211	0	291	4839
N.S.	1	1.00	0.95	1.04	0.00	10.79	0.00	1.42	23.60
time (sec)	N/A	0.186	0.913	1.645	0.000	0.711	0.000	0.289	4.912

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	103	132	105	65	484	46	345
N.S.	1	1.00	0.93	1.19	0.95	0.59	4.36	0.41	3.11
time (sec)	N/A	0.036	0.299	1.584	0.286	0.311	41.972	0.284	9.026

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	111	83	57	393	40	269
N.S.	1	1.00	1.09	1.28	0.95	0.66	4.52	0.46	3.09
time (sec)	N/A	0.027	0.227	1.560	0.285	0.247	10.164	0.272	6.393

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	87	90	61	49	269	34	191
N.S.	1	1.00	1.38	1.43	0.97	0.78	4.27	0.54	3.03
time (sec)	N/A	0.016	0.170	1.541	0.285	0.272	4.939	0.272	5.140

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	75	70	41	43	133	28	118
N.S.	1	1.00	2.03	1.89	1.11	1.16	3.59	0.76	3.19
time (sec)	N/A	0.009	0.114	1.535	0.286	0.247	2.690	0.283	3.751

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	68	38	41	47	71	51	47
N.S.	1	1.00	2.34	1.31	1.41	1.62	2.45	1.76	1.62
time (sec)	N/A	0.010	0.088	1.538	0.286	0.241	3.435	0.277	3.240

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	29	25	42	27	107	88	24
N.S.	1	1.00	0.64	0.56	0.93	0.60	2.38	1.96	0.53
time (sec)	N/A	0.010	0.069	1.525	0.287	0.238	2.657	0.286	3.376

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	37	33	62	35	189	130	32
N.S.	1	1.00	0.51	0.45	0.85	0.48	2.59	1.78	0.44
time (sec)	N/A	0.013	0.077	1.514	0.278	0.251	3.563	0.297	3.366

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	45	41	84	43	274	175	40
N.S.	1	1.00	0.46	0.42	0.87	0.44	2.82	1.80	0.41
time (sec)	N/A	0.018	0.081	1.550	0.281	0.249	5.001	0.294	3.084

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	53	49	106	51	359	217	48
N.S.	1	1.00	0.44	0.40	0.88	0.42	2.97	1.79	0.40
time (sec)	N/A	0.026	0.088	1.560	0.276	0.249	8.141	0.286	3.100

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	44	21	40	117	43	65
N.S.	1	1.00	0.95	1.13	0.54	1.03	3.00	1.10	1.67
time (sec)	N/A	0.005	0.056	1.577	0.283	0.259	22.402	0.272	4.567

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	44	21	40	117	43	444
N.S.	1	1.00	0.95	1.13	0.54	1.03	3.00	1.10	11.38
time (sec)	N/A	0.008	0.002	5.341	0.303	0.255	30.936	0.277	6.010

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	309	604	0	1228	0	0	0
N.S.	1	1.00	2.13	4.17	0.00	8.47	0.00	0.00	0.00
time (sec)	N/A	0.079	16.593	5.516	0.000	0.099	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	312	729	0	1126	0	0	0
N.S.	1	1.00	1.41	3.30	0.00	5.10	0.00	0.00	0.00
time (sec)	N/A	0.147	19.240	2.964	0.000	0.122	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	135	154	0	69	0	0	0
N.S.	1	1.00	0.48	0.55	0.00	0.25	0.00	0.00	0.00
time (sec)	N/A	0.253	5.063	1.760	0.000	0.074	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	130	149	0	64	0	0	0
N.S.	1	1.00	0.53	0.61	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.208	4.888	1.628	0.000	0.079	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	125	144	0	59	0	0	0
N.S.	1	1.00	0.65	0.75	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.052	1.400	1.611	0.000	0.079	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	120	139	0	54	0	0	0
N.S.	1	1.00	0.74	0.86	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.041	1.685	1.635	0.000	0.078	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	139	174	0	0	0	0	0
N.S.	1	1.00	0.76	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	5.293	1.837	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	130	247	0	0	0	0	0
N.S.	1	1.00	0.69	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	5.611	1.637	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	134	273	0	0	0	0	0
N.S.	1	1.00	0.59	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	5.774	1.636	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	139	299	0	0	0	0	0
N.S.	1	1.00	0.53	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	5.895	1.647	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	570	1254	976	0	0	0	0	0
N.S.	1	1.00	2.20	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.873	30.020	1.643	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	130	149	0	64	0	0	0
N.S.	1	1.00	0.53	0.61	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.187	5.510	1.634	0.000	0.079	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	125	144	0	59	0	0	0
N.S.	1	1.00	0.61	0.70	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.138	4.402	1.608	0.000	0.074	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	120	139	0	54	0	0	0
N.S.	1	1.00	0.74	0.86	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.042	2.024	1.627	0.000	0.073	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	115	134	0	49	0	0	0
N.S.	1	1.00	0.88	1.02	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.033	1.193	1.613	0.000	0.073	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	95	67	0	0	0	0	0
N.S.	1	1.00	0.63	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	2.580	1.577	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	130	247	0	0	0	0	0
N.S.	1	1.00	0.69	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	5.581	1.640	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	135	273	0	0	0	0	0
N.S.	1	1.00	0.60	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	5.291	1.638	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	125	144	0	59	0	0	0
N.S.	1	1.00	0.61	0.70	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.139	9.074	1.600	0.000	0.075	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	120	139	0	54	0	0	0
N.S.	1	1.00	0.72	0.83	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.103	7.855	1.624	0.000	0.082	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	115	134	0	49	0	0	0
N.S.	1	1.00	0.88	1.02	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.034	1.993	1.589	0.000	0.080	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	111	33	0	26	0	0	0
N.S.	1	1.00	2.36	0.70	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.012	2.333	1.580	0.000	0.070	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	52	0	0	0	0	0
N.S.	1	1.00	0.68	0.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	2.382	5.368	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	130	247	0	0	0	0	0
N.S.	1	1.00	0.69	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	5.685	1.646	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	142	273	0	0	0	0	0
N.S.	1	1.00	0.63	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	5.470	1.655	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	202	478	0	0	0	0	0
N.S.	1	1.00	0.69	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	20.825	1.966	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	1176	769	0	0	0	0	0
N.S.	1	1.00	2.62	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	21.295	1.485	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	125	144	0	59	0	0	0
N.S.	1	1.00	0.62	0.71	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.141	22.293	1.640	0.000	0.078	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	120	139	0	54	0	0	0
N.S.	1	1.00	0.73	0.84	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.094	18.637	1.612	0.000	0.075	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	115	134	0	49	0	0	0
N.S.	1	1.00	0.89	1.04	0.00	0.38	0.00	0.00	0.00
time (sec)	N/A	0.040	16.622	1.643	0.000	0.078	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	187	51	0	26	0	0	0
N.S.	1	1.00	1.91	0.52	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.025	8.675	1.590	0.000	0.084	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	79	33	0	11	0	0	0
N.S.	1	1.00	1.65	0.69	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.015	1.471	5.330	0.000	0.076	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	109	34	0	0	0	0	0
N.S.	1	1.00	2.14	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	3.573	1.607	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	130	247	0	0	0	0	0
N.S.	1	1.00	0.69	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	4.655	7.263	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	142	273	0	0	0	0	0
N.S.	1	1.00	0.63	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	6.181	1.656	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	180	210	0	664	0	0	0
N.S.	1	1.00	1.31	1.53	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.049	10.741	1.627	0.000	0.125	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	319	498	0	671	0	0	0
N.S.	1	1.00	1.12	1.75	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.110	19.434	1.620	0.000	0.126	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	226	222	0	0	0	0	0
N.S.	1	1.00	1.37	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	16.175	1.621	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	322	976	0	0	0	0	0
N.S.	1	1.00	0.82	2.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	23.216	2.061	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	875	875	4180	1335	0	0	0	0	0
N.S.	1	1.00	4.78	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.909	35.505	2.771	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	203	184	0	0	0	0	0
N.S.	1	1.00	2.74	2.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	22.077	3.414	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	203	181	0	0	0	0	0
N.S.	1	1.00	2.74	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.115	0.053	2.076	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	218	212	0	0	0	0	0
N.S.	1	1.00	2.53	2.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	21.941	3.431	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	218	205	0	0	0	0	0
N.S.	1	1.00	2.53	2.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.047	2.168	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	567	500	0	0	0	0	0
N.S.	1	1.00	1.20	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.408	18.030	2.253	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	565	473	0	0	0	0	0
N.S.	1	1.00	1.32	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	16.344	1.797	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	560	446	0	0	0	0	0
N.S.	1	1.00	1.43	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	12.202	1.780	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	554	421	0	0	0	0	0
N.S.	1	1.00	1.58	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	12.320	1.762	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	564	435	0	0	0	0	0
N.S.	1	1.00	1.62	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	14.881	1.608	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	559	464	0	0	0	0	0
N.S.	1	1.00	1.43	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	17.087	1.610	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	574	493	0	0	0	0	0
N.S.	1	1.00	1.74	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	18.888	1.631	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	569	522	0	0	0	0	0
N.S.	1	1.00	1.54	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	17.714	1.628	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	345	473	0	0	0	0	0
N.S.	1	1.00	0.80	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	39.977	1.730	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	340	446	0	0	0	0	0
N.S.	1	1.00	0.87	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	36.533	1.715	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	347	421	0	0	0	0	0
N.S.	1	1.00	0.99	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	28.583	1.718	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	318	397	0	0	0	0	0
N.S.	1	1.00	0.87	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	5.506	1.607	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	326	435	0	0	0	0	0
N.S.	1	1.00	1.17	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.134	20.014	1.598	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	246	464	0	0	0	0	0
N.S.	1	1.00	0.85	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	26.606	1.606	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	251	493	0	0	0	0	0
N.S.	1	1.00	0.76	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	30.349	1.620	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	258	522	0	0	0	0	0
N.S.	1	1.00	0.70	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	27.319	1.629	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	340	446	0	0	0	0	0
N.S.	1	1.00	0.87	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	29.932	1.731	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	349	421	0	0	0	0	0
N.S.	1	1.00	0.99	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	23.377	1.717	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	347	397	0	0	0	0	0
N.S.	1	1.00	0.95	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	6.990	1.572	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	170	134	0	0	0	0	0
N.S.	1	1.00	1.68	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	5.027	1.609	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	195	237	435	0	0	0	0	0
N.S.	1	3.25	3.95	7.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	28.522	1.624	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	248	464	0	0	0	0	0
N.S.	1	1.00	0.86	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	30.435	1.609	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	721	721	484	1544	0	0	0	0	0
N.S.	1	1.00	0.67	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	48.039	4.034	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	208	223	1948	0	0	0	0	0
N.S.	1	1.29	1.39	12.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	23.633	3.961	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	347	421	0	0	0	0	0
N.S.	1	1.00	0.99	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	24.323	1.733	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	347	397	0	0	0	0	0
N.S.	1	1.00	0.74	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	9.715	1.599	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	162	0	0	0	0	0
N.S.	1	1.00	0.95	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	3.775	1.614	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	90	133	0	0	0	0	0
N.S.	1	1.00	1.27	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	3.177	1.632	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	270	237	435	0	0	0	0	0
N.S.	1	1.38	1.22	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	18.162	1.625	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	246	464	0	0	0	0	0
N.S.	1	1.00	0.85	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	31.122	1.628	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	968	968	7319	1541	0	0	0	0	0
N.S.	1	1.00	7.56	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.614	30.309	2.605	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	583	848	0	0	0	0	0
N.S.	1	1.00	2.56	3.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	31.713	1.288	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	198	227	270	0	0	0	0	0
N.S.	1	1.23	1.41	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	23.053	1.499	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	4121	2200	0	0	0	0	0
N.S.	1	1.00	9.61	5.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	41.909	1.688	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	786	0	670	7103	0	0	0	0	0
N.S.	1	0.00	0.85	9.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	34.398	2.995	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	285	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.879	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.452	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	153	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	170	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	222	177	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.309	0.000	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	149	726	474	877	8221	1626	819
N.S.	1	1.00	0.89	4.35	2.84	5.25	49.23	9.74	4.90
time (sec)	N/A	0.086	0.211	1.625	0.235	0.260	1.744	0.282	3.338

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	136	120	0	0	0	0	0	0
N.S.	1	1.01	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.225	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	115	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	0.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	193	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	104	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	266	195	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	195	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.265	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	189	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	0.205	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	221	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	205	198	0	0	0	0	0	0
N.S.	1	1.01	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	246	237	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.290	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	220	894	0	1659	0	0	1895
N.S.	1	1.00	0.61	2.47	0.00	4.58	0.00	0.00	5.23
time (sec)	N/A	0.230	0.361	2.246	0.000	0.339	0.000	0.000	4.852

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	279	2343	0	3441	0	0	3720
N.S.	1	1.00	0.55	4.62	0.00	6.79	0.00	0.00	7.34
time (sec)	N/A	0.353	0.488	2.244	0.000	0.573	0.000	0.000	7.233

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	815	803	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.918	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	572	566	422	0	0	0	0	0	0
N.S.	1	0.99	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.412	1.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	360	227	906	0	1608	0	0	1890
N.S.	1	0.99	0.63	2.50	0.00	4.43	0.00	0.00	5.21
time (sec)	N/A	0.244	0.370	2.263	0.000	0.310	0.000	0.000	4.468

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	186	181	509	0	905	0	0	869
N.S.	1	0.99	0.96	2.71	0.00	4.81	0.00	0.00	4.62
time (sec)	N/A	0.065	0.125	1.917	0.000	0.274	0.000	0.000	3.668

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	190	199	0	0	0	0	0	0
N.S.	1	1.07	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	233	174	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.166	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	184	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.308	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	530	530	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.796	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	393	393	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	31	0	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.07
time (sec)	N/A	0.005	0.753	0.118	0.258	0.269	0.000	0.281	2.983

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	283	283	208	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.309	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	215	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.345	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	261	223	0	0	0	0	0	0
N.S.	1	0.99	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.248	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	558	558	508	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.641	1.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	139	87	78	0	76	244
N.S.	1	1.00	0.95	1.76	1.10	0.99	0.00	0.96	3.09
time (sec)	N/A	0.112	0.297	1.631	0.278	0.241	0.000	0.360	8.052

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	117	57	67	0	60	232
N.S.	1	1.00	1.02	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.041	0.243	1.610	0.273	0.245	0.000	0.319	7.207

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	73	96	57	81	245	196	122
N.S.	1	1.00	1.52	2.00	1.19	1.69	5.10	4.08	2.54
time (sec)	N/A	0.114	0.198	1.607	0.282	0.243	28.852	0.368	4.255

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	73	97	57	84	221	282	114
N.S.	1	1.00	1.52	2.02	1.19	1.75	4.60	5.88	2.38
time (sec)	N/A	0.109	0.211	1.600	0.278	0.241	28.140	0.401	4.084

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	108	98	65	0	407	312
N.S.	1	1.00	0.99	1.52	1.38	0.92	0.00	5.73	4.39
time (sec)	N/A	0.119	0.214	1.614	0.273	0.235	0.000	0.416	6.150

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	151	74	108	100	73	0	105	318
N.S.	1	1.74	0.85	1.24	1.15	0.84	0.00	1.21	3.66
time (sec)	N/A	0.105	0.174	1.619	0.198	0.234	0.000	0.305	12.695

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	52	135	63	96	90	61	0	80	312
N.S.	1	2.60	1.21	1.85	1.73	1.17	0.00	1.54	6.00
time (sec)	N/A	0.062	0.131	5.547	0.189	0.241	0.000	0.308	12.839

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	A	A	C	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	55	135	69	95	56	73	240	71	118
N.S.	1	2.45	1.25	1.73	1.02	1.33	4.36	1.29	2.15
time (sec)	N/A	0.119	0.141	1.605	0.279	0.252	27.588	0.297	4.215

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	C	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	55	135	69	95	56	82	216	83	118
N.S.	1	2.45	1.25	1.73	1.02	1.49	3.93	1.51	2.15
time (sec)	N/A	0.119	0.142	1.620	0.281	0.264	27.240	0.313	4.124

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	83	129	60	76	61	69	0	145	316
N.S.	1	1.55	0.72	0.92	0.73	0.83	0.00	1.75	3.81
time (sec)	N/A	0.117	0.127	1.628	0.279	0.238	0.000	0.305	10.447

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	171	71	89	86	90	0	197	304
N.S.	1	1.47	0.61	0.77	0.74	0.78	0.00	1.70	2.62
time (sec)	N/A	0.132	0.175	5.563	0.271	0.241	0.000	0.329	10.676

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [72] had the largest ratio of [.3428999999999999983]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	21	0.048
2	A	2	1	1.00	23	0.043
3	A	2	1	1.00	25	0.040
4	A	2	1	1.00	27	0.037
5	A	2	1	1.00	29	0.034
6	A	2	1	1.00	17	0.059
7	A	3	2	1.00	22	0.091
8	A	6	5	1.00	25	0.200
9	A	5	5	1.00	25	0.200
10	A	4	4	1.00	23	0.174
11	A	4	4	1.00	18	0.222
12	A	6	4	1.00	25	0.160
13	A	6	4	1.00	25	0.160
14	A	7	5	1.00	25	0.200
15	A	6	5	1.00	25	0.200
16	A	5	5	1.00	25	0.200
17	A	4	4	1.00	23	0.174
18	A	4	4	1.00	18	0.222
19	A	6	5	1.00	25	0.200
20	A	6	5	1.00	25	0.200
21	A	7	6	1.00	25	0.240
22	A	8	6	1.00	26	0.231
23	A	7	6	1.00	26	0.231
24	A	6	6	1.00	24	0.250

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	4	1.00	23	0.174
26	A	5	5	1.00	26	0.192
27	A	3	3	1.00	26	0.115
28	A	4	4	1.00	26	0.154
29	A	5	4	1.00	26	0.154
30	A	6	4	1.00	26	0.154
31	A	4	4	1.00	24	0.167
32	A	5	5	1.00	36	0.139
33	A	3	3	1.00	45	0.067
34	A	5	5	1.00	39	0.128
35	A	10	8	1.00	35	0.229
36	A	9	8	1.00	35	0.229
37	A	8	6	1.00	33	0.182
38	A	7	7	1.00	28	0.250
39	A	10	10	1.00	35	0.286
40	A	10	10	1.00	35	0.286
41	A	11	11	1.00	35	0.314
42	A	12	11	1.00	35	0.314
43	A	12	10	1.00	35	0.286
44	A	9	8	1.00	35	0.229
45	A	8	8	1.00	35	0.229
46	A	7	6	1.00	33	0.182
47	A	6	6	1.00	28	0.214
48	A	9	9	1.00	35	0.257
49	A	10	10	1.00	35	0.286
50	A	11	11	1.00	35	0.314
51	A	8	8	1.00	35	0.229
52	A	7	7	1.00	35	0.200
53	A	6	6	1.00	33	0.182
54	A	2	2	1.00	28	0.071
55	A	6	6	1.00	35	0.171
56	A	10	10	1.00	35	0.286
57	A	11	11	1.00	35	0.314
58	A	8	6	1.00	35	0.171
59	A	11	8	1.00	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	8	8	1.00	35	0.229
61	A	7	7	1.00	35	0.200
62	A	7	7	1.00	35	0.200
63	A	5	5	1.00	33	0.152
64	A	2	2	1.00	28	0.071
65	A	3	3	1.00	35	0.086
66	A	10	10	1.00	35	0.286
67	A	11	11	1.00	35	0.314
68	A	3	3	1.00	36	0.083
69	A	6	5	1.00	33	0.152
70	A	4	3	1.00	35	0.086
71	A	10	8	1.00	35	0.229
72	A	18	12	1.00	35	0.343
73	A	3	3	1.00	36	0.083
74	A	4	4	1.00	31	0.129
75	A	3	3	1.00	40	0.075
76	A	4	4	1.00	31	0.129
77	A	12	10	1.00	37	0.270
78	A	11	10	1.00	37	0.270
79	A	10	10	1.00	37	0.270
80	A	9	9	1.00	37	0.243
81	A	9	9	1.00	37	0.243
82	A	10	10	1.00	37	0.270
83	A	9	9	1.00	37	0.243
84	A	10	9	1.00	37	0.243
85	A	11	10	1.00	37	0.270
86	A	10	10	1.00	37	0.270
87	A	9	9	1.00	37	0.243
88	A	9	8	1.00	37	0.216
89	A	7	7	1.00	37	0.189
90	A	8	8	1.00	37	0.216
91	A	9	9	1.00	37	0.243
92	A	10	9	1.00	37	0.243
93	A	10	10	1.00	37	0.270
94	A	9	9	1.00	37	0.243

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	9	8	1.00	37	0.216
96	A	2	2	1.00	37	0.054
97	B	5	5	3.25	37	0.135
98	A	8	8	1.00	37	0.216
99	A	7	7	1.00	37	0.189
100	A	2	2	1.29	37	0.054
101	A	9	9	1.00	37	0.243
102	A	12	10	1.00	37	0.270
103	A	2	2	1.00	37	0.054
104	A	2	2	1.00	37	0.054
105	A	8	7	1.38	37	0.189
106	A	8	8	1.00	37	0.216
107	A	10	8	1.00	37	0.216
108	A	2	2	1.00	37	0.054
109	A	2	2	1.23	37	0.054
110	A	5	5	1.00	37	0.135
111	F	0	0	N/A	0.000	N/A
112	A	8	3	1.00	25	0.120
113	A	6	3	1.00	25	0.120
114	A	4	2	1.00	25	0.080
115	A	3	2	1.00	23	0.087
116	A	3	2	1.00	22	0.091
117	A	5	3	1.00	25	0.120
118	A	6	3	1.00	25	0.120
119	A	2	1	1.00	23	0.043
120	A	2	2	1.01	25	0.080
121	A	3	2	1.00	27	0.074
122	A	5	2	1.00	29	0.069
123	A	6	3	1.00	25	0.120
124	A	3	3	1.00	25	0.120
125	A	3	3	1.00	29	0.103
126	A	3	3	1.00	27	0.111
127	A	3	3	1.00	29	0.103
128	A	3	3	1.01	29	0.103
129	A	3	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	3	1.00	29	0.103
131	A	4	3	1.00	29	0.103
132	A	10	9	0.99	31	0.290
133	A	9	8	0.99	31	0.258
134	A	3	3	0.99	29	0.103
135	A	3	3	0.99	24	0.125
136	A	5	5	1.07	27	0.185
137	A	5	5	1.00	29	0.172
138	A	7	4	1.00	29	0.138
139	A	31	5	1.00	29	0.172
140	A	15	5	1.00	29	0.172
141	A	7	4	1.00	27	0.148
142	A	3	3	1.00	22	0.136
143	N/A	0	0	1.00	29	0.000
144	A	7	4	1.00	33	0.121
145	A	7	4	1.00	34	0.118
146	A	5	5	1.00	34	0.147
147	A	3	3	0.99	34	0.088
148	A	4	3	1.00	34	0.088
149	A	4	4	1.00	31	0.129
150	A	4	4	1.00	30	0.133
151	A	7	7	1.00	33	0.212
152	A	7	7	1.00	33	0.212
153	A	6	6	1.00	33	0.182
154	A	5	5	1.74	30	0.167
155	B	5	5	2.60	29	0.172
156	B	8	8	2.45	32	0.250
157	B	8	8	2.45	32	0.250
158	A	6	6	1.55	32	0.188
159	A	7	7	1.47	32	0.219

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = acegx + \frac{1}{2}(bceg+a(deg+cfg+ceh))x^2 + \frac{1}{3}(b(deg+cfg+ceh)+a(dfg+deh+cfh))x^3 + \frac{1}{4}(adfh+b(dfg+deh+cfh))x^4 + \frac{1}{5}bdfhx^5$$

```
[Out] a*c*e*g*x+1/2*(b*c*e*g+a*(c*e*h+c*f*g+d*e*g))*x^2+1/3*(b*(c*e*h+c*f*g+d*e*g
)+a*(c*f*h+d*e*h+d*f*g))*x^3+1/4*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*x^4+1/5*b*
d*f*h*x^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {147}

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{1}{4}x^4(adfh+b(cfh+deh+dfg)) + \frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg)) + \frac{1}{2}x^2(a(ceh+cfg+deg)+bceg) + acegx + \frac{1}{5}bdfhx^5$$

```
[In] Int[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*(d*e*g + c*f*g + c*e*h))*x^2)/2 + ((b*(d*e*g + c*
f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))*x^3)/3 + ((a*d*f*h + b*(d*f*g + d
*e*h + c*f*h))*x^4)/4 + (b*d*f*h*x^5)/5
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(aceg+(bceg+a(deg+cfg+ceh))x+(b(deg+cfg+ceh)+a(dfg+deh+cfh))x^2 \right. \\
&\qquad \left. +(adfh+b(dfg+deh+cfh))x^3+bdfhx^4\right)\,dx \\
&= acegx+\frac{1}{2}(bceg+a(deg+cfg+ceh))x^2 \\
&\quad +\frac{1}{3}(b(deg+cfg+ceh)+a(dfg+deh+cfh))x^3 \\
&\quad +\frac{1}{4}(adfh+b(dfg+deh+cfh))x^4+\frac{1}{5}bdfhx^5
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx &= acegx+\frac{1}{2}(bceg+adeg+acfg+aceh)x^2 \\
&\quad +\frac{1}{3}(bdeg+bcfg+adfg+bceh+adeh+acfh)x^3 \\
&\quad +\frac{1}{4}(bdfg+bdeh+bcfh+adfh)x^4+\frac{1}{5}bdfhx^5
\end{aligned}$$

```
[In] Integrate[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*d*e*g + a*c*f*g + a*c*e*h)*x^2)/2 + ((b*d*e*g + b
*c*f*g + a*d*f*g + b*c*e*h + a*d*e*h + a*c*f*h)*x^3)/3 + ((b*d*f*g + b*d*e*
h + b*c*f*h + a*d*f*h)*x^4)/4 + (b*d*f*h*x^5)/5
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

method	result
default	$\frac{bdfh\,x^5}{5}+\frac{(((ad+bc)f+bde)h+bdfg)x^4}{4}+\frac{((acf+(ad+bc)e)h+((ad+bc)f+bde)g)x^3}{3}+\frac{(aceh+(acf+(ad+bc)e)g)x^2}{2}+ac$
norman	$\frac{bdfh\,x^5}{5}+\left(\frac{1}{4}adfh+\frac{1}{4}bcfh+\frac{1}{4}bdeh+\frac{1}{4}bdfg\right)x^4+\left(\frac{1}{3}acfh+\frac{1}{3}adeh+\frac{1}{3}adfg+\frac{1}{3}bceh+\frac{1}{3}bcfg\right.$
gosper	$\frac{1}{5}bdfh\,x^5+\frac{1}{4}x^4adfh+\frac{1}{4}x^4bcfh+\frac{1}{4}x^4bdeh+\frac{1}{4}x^4bdfg+\frac{1}{3}x^3acfh+\frac{1}{3}x^3adeh+\frac{1}{3}x^3adfg+\frac{1}{3}x$
risch	$\frac{1}{5}bdfh\,x^5+\frac{1}{4}x^4adfh+\frac{1}{4}x^4bcfh+\frac{1}{4}x^4bdeh+\frac{1}{4}x^4bdfg+\frac{1}{3}x^3acfh+\frac{1}{3}x^3adeh+\frac{1}{3}x^3adfg+\frac{1}{3}x$
parallelrisch	$\frac{1}{5}bdfh\,x^5+\frac{1}{4}x^4adfh+\frac{1}{4}x^4bcfh+\frac{1}{4}x^4bdeh+\frac{1}{4}x^4bdfg+\frac{1}{3}x^3acfh+\frac{1}{3}x^3adeh+\frac{1}{3}x^3adfg+\frac{1}{3}x$

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b*d*f*h*x^5+1/4*(((a*d+b*c)*f+b*d*e)*h+b*d*f*g)*x^4+1/3*((a*c*f+(a*d+b*
c)*e)*h+((a*d+b*c)*f+b*d*e)*g)*x^3+1/2*(a*c*e*h+(a*c*f+(a*d+b*c)*e)*g)*x^2+
a*c*e*g*x
```

Fricas [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx=\frac{1}{5}x^5hfdb+\frac{1}{4}x^4gfdb+\frac{1}{4}x^4hedb+\frac{1}{4}x^4hfcb+\frac{1}{4}x^4hfda+\frac{1}{3}x^3gedb+\frac{1}{3}x^3gfcb+\frac{1}{3}x^3hecb+\frac{1}{3}x^3gfda+\frac{1}{3}x^3heda+\frac{1}{3}x^3hfca+\frac{1}{2}x^2gecb+\frac{1}{2}x^2geda+\frac{1}{2}x^2gfca+\frac{1}{2}x^2heca+xgeca$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*h*f*d*b + 1/4*x^4*g*f*d*b + 1/4*x^4*h*e*d*b + 1/4*x^4*h*f*c*b + 1/4
*x^4*h*f*d*a + 1/3*x^3*g*e*d*b + 1/3*x^3*g*f*c*b + 1/3*x^3*h*e*c*b + 1/3*x^
3*g*f*d*a + 1/3*x^3*h*e*d*a + 1/3*x^3*h*f*c*a + 1/2*x^2*g*e*c*b + 1/2*x^2*g
*e*d*a + 1/2*x^2*g*f*c*a + 1/2*x^2*h*e*c*a + x*g*e*c*a
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = acegx + \frac{bdfhx^5}{5} + x^4\left(\frac{adfh}{4} + \frac{bcfh}{4} + \frac{bdeh}{4} + \frac{bdfg}{4}\right) + x^3\left(\frac{acfh}{3} + \frac{adeh}{3} + \frac{adfg}{3} + \frac{bceh}{3} + \frac{bcfg}{3} + \frac{bdeg}{3}\right) + x^2\left(\frac{aceh}{2} + \frac{acfg}{2} + \frac{adeg}{2} + \frac{bceg}{2}\right)$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] a*c*e*g*x + b*d*f*h*x**5/5 + x**4*(a*d*f*h/4 + b*c*f*h/4 + b*d*e*h/4 + b*d*
f*g/4) + x**3*(a*c*f*h/3 + a*d*e*h/3 + a*d*f*g/3 + b*c*e*h/3 + b*c*f*g/3 +
b*d*e*g/3) + x**2*(a*c*e*h/2 + a*c*f*g/2 + a*d*e*g/2 + b*c*e*g/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{1}{5}\,bdfhx^5 + acegx + \frac{1}{4}\,(bdfg + (bde + (bc+ad)f)h)x^4 + \frac{1}{3}\,((bde + (bc+ad)f)g + (acf + (bc+ad)e)h)x^3 + \frac{1}{2}\,(aceh + (acf + (bc+ad)e)g)x^2$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*f*h*x^5 + a*c*e*g*x + 1/4*(b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^4
 + 1/3*((b*d*e + (b*c + a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^3 + 1/2*(a
*c*e*h + (a*c*f + (b*c + a*d)*e)*g)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{1}{5}\,bdfhx^5 + \frac{1}{4}\,bdfgx^4 + \frac{1}{4}\,bdehx^4 + \frac{1}{4}\,bcfhx^4 + \frac{1}{4}\,adfhx^4 + \frac{1}{3}\,bdegx^3 + \frac{1}{3}\,bcfgx^3 + \frac{1}{3}\,adfgx^3 + \frac{1}{3}\,bcehx^3 + \frac{1}{3}\,adehx^3 + \frac{1}{3}\,acfhx^3 + \frac{1}{2}\,bcegx^2 + \frac{1}{2}\,adegx^2 + \frac{1}{2}\,acfgx^2 + \frac{1}{2}\,acehx^2 + acegx$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] 1/5*b*d*f*h*x^5 + 1/4*b*d*f*g*x^4 + 1/4*b*d*e*h*x^4 + 1/4*b*c*f*h*x^4 + 1/4
*a*d*f*h*x^4 + 1/3*b*d*e*g*x^3 + 1/3*b*c*f*g*x^3 + 1/3*a*d*f*g*x^3 + 1/3*b*
c*e*h*x^3 + 1/3*a*d*e*h*x^3 + 1/3*a*c*f*h*x^3 + 1/2*b*c*e*g*x^2 + 1/2*a*d*e
*g*x^2 + 1/2*a*c*f*g*x^2 + 1/2*a*c*e*h*x^2 + a*c*e*g*x
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{b\,d\,f\,h\,x^5}{5} + \left(\frac{a\,d\,f\,h}{4} + \frac{b\,c\,f\,h}{4} + \frac{b\,d\,e\,h}{4} + \frac{b\,d\,f\,g}{4}\right)\,x^4 + \left(\frac{a\,c\,f\,h}{3} + \frac{a\,d\,e\,h}{3} + \frac{a\,d\,f\,g}{3} + \frac{b\,c\,e\,h}{3} + \frac{b\,c\,f\,g}{3} + \frac{b\,d\,e\,g}{3}\right)\,x^3 + \left(\frac{a\,c\,e\,h}{2} + \frac{a\,c\,f\,g}{2} + \frac{a\,d\,e\,g}{2} + \frac{b\,c\,e\,g}{2}\right)\,x^2 + a\,c\,e\,g\,x$$

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x),x)
```

```
[Out] x^3*((a*c*f*h)/3 + (a*d*e*h)/3 + (a*d*f*g)/3 + (b*c*e*h)/3 + (b*c*f*g)/3 +
(b*d*e*g)/3) + x^2*((a*c*e*h)/2 + (a*c*f*g)/2 + (a*d*e*g)/2 + (b*c*e*g)/2)
+ x^4*((a*d*f*h)/4 + (b*c*f*h)/4 + (b*d*e*h)/4 + (b*d*f*g)/4) + a*c*e*g*x +
 (b*d*f*h*x^5)/5
```

3.2 $\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = \frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3} + \frac{(adfh-b(dfg-deh-cfh))x^2}{2h^2} + \frac{bdfx^3}{3h} - \frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}$$

```
[Out] (b*(-c*h+d*g)*(-e*h+f*g)-a*h*(-c*f*h-d*e*h+d*f*g))*x/h^3+1/2*(a*d*f*h-b*(-c
*f*h-d*e*h+d*f*g))*x^2/h^2+1/3*b*d*f*x^3/h-(-a*h+b*g)*(-c*h+d*g)*(-e*h+f*g)
*ln(h*x+g)/h^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {147}

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = -\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4} + \frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3} + \frac{x^2(adfh-b(-cfh-deh+dfg))}{2h^2} + \frac{bdfx^3}{3h}$$

```
[In] Int[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((b*(d*g - c*h)*(f*g - e*h) - a*h*(d*f*g - d*e*h - c*f*h))*x)/h^3 + ((a*d*f
*h - b*(d*f*g - d*e*h - c*f*h))*x^2)/(2*h^2) + (b*d*f*x^3)/(3*h) - ((b*g -
a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/h^4
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\text{integral} = \int \left(\frac{b(dg-ch)(fg-eh)-ah(dfg-deh-cfh)}{h^3} + \frac{(adfh-b(dfg-deh-cfh))x}{h^2} + \frac{bdfx^2}{h} + \frac{(-bg+ah)(-dg+ch)(-fg+eh)}{h^3(g+hx)} \right) dx$$

$$= \frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3} + \frac{(adfh-b(dfg-deh-cfh))x^2}{2h^2} + \frac{bdfx^3}{3h} - \frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = \frac{hx(3ah(2cfh+d(-2fg+2eh+fhx))+b(3deh(-2g+hx)+3ch(-2fg+2eh+fhx)+df(6g^2-3ghx}{6h^4}$$

```
[In] Integrate[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] (h*x*(3*a*h*(2*c*f*h + d*(-2*f*g + 2*e*h + f*h*x)) + b*(3*d*e*h*(-2*g + h*x
) + 3*c*h*(-2*f*g + 2*e*h + f*h*x) + d*f*(6*g^2 - 3*g*h*x + 2*h^2*x^2))) -
6*(b*g - a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/(6*h^4)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.39

method	result
norman	$\frac{(acf\,h^2+ade\,h^2-adfgh+bce\,h^2-bcfgh-bdegh+bdf\,g^2)x}{h^3}+\frac{(adfh+bcfh+bdeh-bdfg)x^2}{2h^2}+\frac{bdf\,x^3}{3h}+\frac{(ace\,h^3-acfg\,h^2-a}{}$
default	$\frac{\frac{1}{3}bdf\,x^3h^2+\frac{1}{2}adf\,h^2x^2+\frac{1}{2}bcf\,h^2x^2+\frac{1}{2}bde\,h^2x^2-\frac{1}{2}bdfgh\,x^2+acf\,h^2x+ade\,h^2x-adfghx+bce\,h^2x-bcfghx-bdeghx+bdf\,g^2x}{h^3}$
risch	$\frac{bdf\,x^3}{3h}+\frac{adf\,x^2}{2h}+\frac{bcf\,x^2}{2h}+\frac{bde\,x^2}{2h}-\frac{bdfg\,x^2}{2h^2}+\frac{acfx}{h}+\frac{adex}{h}-\frac{adfgx}{h^2}+\frac{bcex}{h}-\frac{bcfgx}{h^2}-\frac{bdegx}{h^2}+\frac{bdf\,g^2x}{h^3}+$
parallelrisch	$\frac{2bdf\,x^3h^3+3x^2adf\,h^3+3x^2bcf\,h^3+3x^2bde\,h^3-3x^2bdfg\,h^2+6\ln(hx+g)ace\,h^3-6\ln(hx+g)acfg\,h^2-6\ln(hx+g)adeg\,h^2+6\ln(h}{}$

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] (a*c*f*h^2+a*d*e*h^2-a*d*f*g*h+b*c*e*h^2-b*c*f*g*h-b*d*e*g*h+b*d*f*g^2)/h^3
*x+1/2/h^2*(a*d*f*h+b*c*f*h+b*d*e*h-b*d*f*g)*x^2+1/3*b*d*f*x^3/h+(a*c*e*h^3
-a*c*f*g*h^2-a*d*e*g*h^2+a*d*f*g^2*h-b*c*e*g*h^2+b*c*f*g^2*h+b*d*e*g^2*h-b*
d*f*g^3)/h^4*ln(h*x+g)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$$
$$=\frac{2\,bdfh^3x^3-3\,(bdfgh^2-(bde+(bc+ad)f)h^3)x^2+6\,(bdfg^2h-(bde+(bc+ad)f)gh^2+(acf+(bc+ad}{6\,h^4}$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d*f*h^3*x^3 - 3*(b*d*f*g*h^2 - (b*d*e + (b*c + a*d)*f)*h^3)*x^2 +
6*(b*d*f*g^2*h - (b*d*e + (b*c + a*d)*f)*g*h^2 + (a*c*f + (b*c + a*d)*e)*h^
3)*x - 6*(b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f +
(b*c + a*d)*e)*g*h^2)*log(h*x + g))/h^4
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = \frac{bdfx^3}{3h} + x^2\left(\frac{adf}{2h} + \frac{bcf}{2h} + \frac{bde}{2h} - \frac{bdfg}{2h^2}\right) + x\left(\frac{acf}{h} + \frac{ade}{h} - \frac{adfg}{h^2} + \frac{bce}{h} - \frac{bcfg}{h^2} - \frac{bdeg}{h^2} + \frac{bdfg^2}{h^3}\right) + \frac{(ah-bg)\,(ch-dg)\,(eh-fg)\log{(g+hx)}}{h^4}$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*f*x**3/(3*h) + x**2*(a*d*f/(2*h) + b*c*f/(2*h) + b*d*e/(2*h) - b*d*f*g/
(2*h**2)) + x*(a*c*f/h + a*d*e/h - a*d*f*g/h**2 + b*c*e/h - b*c*f*g/h**2 -
b*d*e*g/h**2 + b*d*f*g**2/h**3) + (a*h - b*g)*(c*h - d*g)*(e*h - f*g)*log(g
 + h*x)/h**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = \frac{2\,bdfh^2x^3 - 3\,(bdfgh - (bde + (bc+ad)f)h^2)x^2 + 6\,(bdfg^2 - (bde + (bc+ad)f)gh + (acf + (bc+ad)e)h^2}{6\,h^3} - \frac{(bdfg^3 - aceh^3 - (bde + (bc+ad)f)g^2h + (acf + (bc+ad)e)gh^2)\log{(hx+g)}}{h^4}$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*(b*d*f*g*h - (b*d*e + (b*c + a*d)*f)*h^2)*x^2 + 6*
(b*d*f*g^2 - (b*d*e + (b*c + a*d)*f)*g*h + (a*c*f + (b*c + a*d)*e)*h^2)*x)/
h^3 - (b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*
c + a*d)*e)*g*h^2)*log(h*x + g)/h^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$$
$$= \frac{2\,bdfh^2x^3 - 3\,bdfghx^2 + 3\,bdeh^2x^2 + 3\,bcfh^2x^2 + 3\,adfh^2x^2 + 6\,bdfg^2x - 6\,bdeghx - 6\,bcfghx - 6\,adfgh}{6\,h^3}$$
$$- \frac{(bdfg^3 - bdeg^2h - bcfg^2h - adfg^2h + bcegh^2 + adegh^2 + acfgh^2 - aceh^3)\log(|hx+g|)}{h^4}$$

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*b*d*f*g*h*x^2 + 3*b*d*e*h^2*x^2 + 3*b*c*f*h^2*x^2
+ 3*a*d*f*h^2*x^2 + 6*b*d*f*g^2*x - 6*b*d*e*g*h*x - 6*b*c*f*g*h*x - 6*a*d*f
*g*h*x + 6*b*c*e*h^2*x + 6*a*d*e*h^2*x + 6*a*c*f*h^2*x)/h^3 - (b*d*f*g^3 -
b*d*e*g^2*h - b*c*f*g^2*h - a*d*f*g^2*h + b*c*e*g*h^2 + a*d*e*g*h^2 + a*c*f
*g*h^2 - a*c*e*h^3)*log(abs(h*x + g))/h^4
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$$
$$= x\left(\frac{a\,c\,f + a\,d\,e + b\,c\,e}{h} - \frac{g\left(\frac{a\,d\,f + b\,c\,f + b\,d\,e}{h} - \frac{b\,d\,f\,g}{h^2}\right)}{h}\right) + x^2\left(\frac{a\,d\,f + b\,c\,f + b\,d\,e}{2\,h} - \frac{b\,d\,f\,g}{2\,h^2}\right)$$
$$+ \frac{\ln(g + h\,x)\,\left(a\,c\,e\,h^3 - b\,d\,f\,g^3 - a\,c\,f\,g\,h^2 - a\,d\,e\,g\,h^2 - b\,c\,e\,g\,h^2 + a\,d\,f\,g^2\,h + b\,c\,f\,g^2\,h + b\,d\,e\,g^2\,h\right)}{h^4}$$
$$+ \frac{b\,d\,f\,x^3}{3\,h}$$

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x))/(g + h*x),x)
```

```
[Out] x*((a*c*f + a*d*e + b*c*e)/h - (g*((a*d*f + b*c*f + b*d*e)/h - (b*d*f*g)/h^
2))/h) + x^2*((a*d*f + b*c*f + b*d*e)/(2*h) - (b*d*f*g)/(2*h^2)) + (log(g +
 h*x)*(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e*g*h^2 - b*c*e*g*h^2 + a*
d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h))/h^4 + (b*d*f*x^3)/(3*h)
```

3.3 $\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx = \frac{bdx}{fh} + \frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)} - \frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}$$

```
[Out] b*d*x/f/h+(-a*f+b*e)*(-c*f+d*e)*ln(f*x+e)/f^2/(-e*h+f*g)-(-a*h+b*g)*(-c*h+d
*g)*ln(h*x+g)/h^2/(-e*h+f*g)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {147}

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx = \frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)} - \frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)} + \frac{bdx}{fh}$$

```
[In] Int[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] (b*d*x)/(f*h) + ((b*e - a*f)*(d*e - c*f)*Log[e + f*x])/(f^2*(f*g - e*h)) -
((b*g - a*h)*(d*g - c*h)*Log[g + h*x])/(h^2*(f*g - e*h))
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
```

```
(IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{bd}{fh}+\frac{(-be+af)(-de+cf)}{f(fg-eh)(e+fx)}+\frac{(-bg+ah)(-dg+ch)}{h(-fg+eh)(g+hx)}\right)dx \\
&= \frac{bdx}{fh}+\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$
\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx
$$
$$
= \frac{(be-af)(de-cf)h^2\log(e+fx)+f(bdh(fg-eh)x-f(bg-ah)(dg-ch)\log(g+hx))}{f^2h^2(fg-eh)}
$$

```
[In] Integrate[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*h^2*Log[e + f*x] + f*(b*d*h*(f*g - e*h)*x - f*(b*g
 - a*h)*(d*g - c*h)*Log[g + h*x]))/(f^2*h^2*(f*g - e*h))
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

method	result
default	$\frac{bdx}{fh}+\frac{(ac\,h^2-adgh-bcgh+bd\,g^2)\ln(hx+g)}{h^2(eh-fg)}+\frac{(-ac\,f^2+adef+bcef-bd\,e^2)\ln(fx+e)}{f^2(eh-fg)}$
norman	$\frac{bdx}{fh}+\frac{(ac\,h^2-adgh-bcgh+bd\,g^2)\ln(hx+g)}{h^2(eh-fg)}-\frac{(ac\,f^2-adef-bcef+bd\,e^2)\ln(fx+e)}{(eh-fg)f^2}$
parallelrisch	$-\frac{\ln(fx+e)ac\,f^2h^2-\ln(fx+e)adef\,h^2-\ln(fx+e)bcef\,h^2+\ln(fx+e)bd\,e^2h^2-\ln(hx+g)ac\,f^2h^2+\ln(hx+g)ad\,f^2gh+\ln(hx+g)}{f^2h^2(eh-fg)}$
risch	$\frac{bdx}{fh}-\frac{\ln(fx+e)ac}{eh-fg}+\frac{\ln(fx+e)ade}{(eh-fg)f}+\frac{\ln(fx+e)bce}{(eh-fg)f}-\frac{\ln(fx+e)bd\,e^2}{(eh-fg)f^2}+\frac{\ln(-hx-g)ac}{eh-fg}-\frac{\ln(-hx-g)adg}{h(eh-fg)}-$

```
[In] int((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*x/f/h+1/h^2*(a*c*h^2-a*d*g*h-b*c*g*h+b*d*g^2)/(e*h-f*g)*ln(h*x+g)+(-a*c
*f^2+a*d*e*f+b*c*e*f-b*d*e^2)/f^2/(e*h-f*g)*ln(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{(bde^2+acf^2-(bc+ad)ef)h^2\log(fx+e)+(bdf^2gh-bdefh^2)x-(bdf^2g^2+acf^2h^2-(bc+ad)f^2gh)\log(hx+g)}{f^3gh^2-ef^2h^3}$$

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] ((b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*h^2*log(f*x + e) + (b*d*f^2*g*h - b*
d*e*f*h^2)*x - (b*d*f^2*g^2 + a*c*f^2*h^2 - (b*c + a*d)*f^2*g*h)*log(h*x +
g))/(f^3*g*h^2 - e*f^2*h^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{bdx}{fh}+\frac{(bde^2+acf^2-(bc+ad)ef)\log(fx+e)}{f^3g-ef^2h} - \frac{(bdg^2+ach^2-(bc+ad)gh)\log(hx+g)}{fgh^2-eh^3}$$

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b*d*x/(f*h) + (b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*log(f*x + e)/(f^3*g - e
*f^2*h) - (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(h*x + g)/(f*g*h^2 - e*h
^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{bdx}{fh} + \frac{(bde^2 - bcef - adef + acf^2)\log(|fx+e|)}{f^3g - ef^2h} - \frac{(bdg^2 - bcgh - adgh + ach^2)\log(|hx+g|)}{fgh^2 - eh^3}$$

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] b*d*x/(f*h) + (b*d*e^2 - b*c*e*f - a*d*e*f + a*c*f^2)*log(abs(f*x + e))/(f^
3*g - e*f^2*h) - (b*d*g^2 - b*c*g*h - a*d*g*h + a*c*h^2)*log(abs(h*x + g))/
(f*g*h^2 - e*h^3)
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{\ln(e+f\,x)\,\left(a\,c\,f^2 - f\,(a\,d\,e + b\,c\,e) + b\,d\,e^2\right)}{f^3\,g - e\,f^2\,h} + \frac{\ln(g+h\,x)\,\left(a\,c\,h^2 - h\,(a\,d\,g + b\,c\,g) + b\,d\,g^2\right)}{e\,h^3 - f\,g\,h^2} + \frac{b\,d\,x}{f\,h}$$

```
[In] int(((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x)
```

```
[Out] (log(e + f*x)*(a*c*f^2 - f*(a*d*e + b*c*e) + b*d*e^2))/(f^3*g - e*f^2*h) +
(log(g + h*x)*(a*c*h^2 - h*(a*d*g + b*c*g) + b*d*g^2))/(e*h^3 - f*g*h^2) +
(b*d*x)/(f*h)
```

3.4 $\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)} -\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

```
[Out] -(-a*d+b*c)*ln(d*x+c)/(-c*f+d*e)/(-c*h+d*g)+(-a*f+b*e)*ln(f*x+e)/(-c*f+d*e)
/(-e*h+f*g)-(-a*h+b*g)*ln(h*x+g)/(-c*h+d*g)/(-e*h+f*g)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {153}

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)} -\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

```
[In] Int[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*x])/((d*e - c*f)*(d*g - c*h))) + ((b*e - a*f)*Log[
e + f*x])/((d*e - c*f)*(f*g - e*h)) - ((b*g - a*h)*Log[g + h*x])/((d*g - c*
h)*(f*g - e*h))
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
```

```
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{d(-bc+ad)}{(de-cf)(dg-ch)(c+dx)} + \frac{f(-be+af)}{(de-cf)(-fg+eh)(e+fx)} + \frac{h(-bg+ah)}{(dg-ch)(fg-eh)(g+hx)}\right) dx \\ &= -\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)} + \frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)} - \frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)} dx = \frac{(bc-ad)(fg-eh)\log(c+dx) - (be-af)(dg-ch)\log(e+fx) + (de-cf)(bg-ah)\log(g+hx)}{(de-cf)(dg-ch)(-fg+eh)}$$

```
[In] Integrate[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
[Out] ((b*c - a*d)*(f*g - e*h)*Log[c + d*x] - (b*e - a*f)*(d*g - c*h)*Log[e + f*x
] + (d*e - c*f)*(b*g - a*h)*Log[g + h*x])/((d*e - c*f)*(d*g - c*h)*(-(f*g)
+ e*h))
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

method	result
default	$\frac{(ad-bc)\ln(dx+c)}{(cf-de)(ch-dg)} + \frac{(ah-bg)\ln(hx+g)}{(ch-dg)(eh-fg)} - \frac{(af-be)\ln(fx+e)}{(cf-de)(eh-fg)}$
norman	$\frac{(ah-bg)\ln(hx+g)}{ce\,h^2-cfgh-degh+df\,g^2} + \frac{(ad-bc)\ln(dx+c)}{(cf-de)(ch-dg)} - \frac{(af-be)\ln(fx+e)}{(cf-de)(eh-fg)}$
parallelrisch	$\frac{\ln(dx+c)adeh-\ln(dx+c)adfg-\ln(dx+c)bceh+\ln(dx+c)bcfg-\ln(fx+e)acfh+\ln(fx+e)adfg+\ln(fx+e)bceh-\ln(fx+e)bdeg+}{(ce\,h^2-cfgh-degh+df\,g^2)(cf-de)}$
risch	$\frac{\ln(dx+c)ad}{c^2fh-cdeh-cdfg+d^2eg} - \frac{\ln(dx+c)bc}{c^2fh-cdeh-cdfg+d^2eg} - \frac{\ln(-fx-e)af}{cefh-c\,f^2g-d\,e^2h+defg} + \frac{\ln(-fx-e)be}{cefh-c\,f^2g-d\,e^2h+defg} + \frac{\ln}{ce\,h^2-}$

```
[In] int((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
[Out] (a*d-b*c)/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)+(a*h-b*g)/(c*h-d*g)/(e*h-f*g)*ln(h*
x+g)-(a*f-b*e)/(c*f-d*e)/(e*h-f*g)*ln(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 39.87 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.48

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{((bc-ad)fg-(bc-ad)eh)\log(dx+c)-((bde-adf)g-(bce-acf)h)\log(fx+e)+((bde-bcf)g}{(d^2ef-cdf^2)g^2-(d^2e^2-c^2f^2)gh+(cde^2-c^2ef)h^2}$$

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] -(((b*c - a*d)*f*g - (b*c - a*d)*e*h)*log(d*x + c) - ((b*d*e - a*d*f)*g - (
b*c*e - a*c*f)*h)*log(f*x + e) + ((b*d*e - b*c*f)*g - (a*d*e - a*c*f)*h)*lo
g(h*x + g))/((d^2*e*f - c*d*f^2)*g^2 - (d^2*e^2 - c^2*f^2)*g*h + (c*d*e^2 -
 c^2*e*f)*h^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.24

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{(bc-ad)\log(dx+c)}{(d^2e-cdf)g-(cde-c^2f)h} + \frac{(be-af)\log(fx+e)}{(def-cf^2)g-(de^2-cef)h} - \frac{(bg-ah)\log(hx+g)}{dfg^2+ceh^2-(de+cf)gh}$$

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*log(d*x + c)/((d^2*e - c*d*f)*g - (c*d*e - c^2*f)*h) + (b*e -
a*f)*log(f*x + e)/((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h) - (b*g - a*h)*log
(h*x + g)/(d*f*g^2 + c*e*h^2 - (d*e + c*f)*g*h)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.44

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{(bcd-ad^2)\log(|dx+c|)}{d^3eg-cd^2fg-cd^2eh+c^2dfh} + \frac{(bef-af^2)\log(|fx+e|)}{def^2g-cf^3g-de^2fh+cef^2h} - \frac{(bgh-ah^2)\log(|hx+g|)}{dfg^2h-degh^2-cfgh^2+ceh^3}$$

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] -(b*c*d - a*d^2)*log(abs(d*x + c))/(d^3*e*g - c*d^2*f*g - c*d^2*e*h + c^2*d
*f*h) + (b*e*f - a*f^2)*log(abs(f*x + e))/(d*e*f^2*g - c*f^3*g - d*e^2*f*h
+ c*e*f^2*h) - (b*g*h - a*h^2)*log(abs(h*x + g))/(d*f*g^2*h - d*e*g*h^2 - c
*f*g*h^2 + c*e*h^3)
```

Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.18

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = \frac{\ln(e+f\,x)\,(a\,f-b\,e)}{c\,f^2\,g+d\,e^2\,h-c\,e\,f\,h-d\,e\,f\,g} + \frac{\ln(g+h\,x)\,(a\,h-b\,g)}{c\,e\,h^2+d\,f\,g^2-c\,f\,g\,h-d\,e\,g\,h} + \frac{\ln(c+d\,x)\,(a\,d-b\,c)}{d^2\,e\,g+c^2\,f\,h-c\,d\,e\,h-c\,d\,f\,g}$$

```
[In] int((a + b*x)/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

```
[Out] (log(e + f*x)*(a*f - b*e))/(c*f^2*g + d*e^2*h - c*e*f*h - d*e*f*g) + (log(g
 + h*x)*(a*h - b*g))/(c*e*h^2 + d*f*g^2 - c*f*g*h - d*e*g*h) + (log(c + d*x
)*(a*d - b*c))/(d^2*e*g + c^2*f*h - c*d*e*h - c*d*f*g)
```

3.5 $\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)} - \frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

```
[Out] b^2*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)-d^2*ln(d*x+c)/(-a*d+b*c)/(-c
*f+d*e)/(-c*h+d*g)+f^2*ln(f*x+e)/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)-h^2*ln(h*
x+g)/(-a*h+b*g)/(-c*h+d*g)/(-e*h+f*g)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {186}

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)} - \frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

```
[In] Int[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)) + (f^2*Log[e + f*x])/((b*e - a*f)*
(d*e - c*f)*(f*g - e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)*(f*g
 - e*h))
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{b^3}{(bc-ad)(be-af)(bg-ah)(a+bx)} - \frac{d^3}{(bc-ad)(-de+cf)(-dg+ch)(c+dx)} - \frac{f^3}{(be-af)(de-cf)(-fg+eh)(e+fx)} - \frac{h^3}{(bg-ah)(dg-ch)(fg-eh)(g+hx)}\right) dx \\ &= \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)} - \frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2\log(c+dx)}{(bc-ad)(-de+cf)(-dg+ch)} - \frac{f^2\log(e+fx)}{(be-af)(de-cf)(-fg+eh)} - \frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) - (f^2*Log[e + f*x])/((b*e -
 a*f)*(d*e - c*f)*(-(f*g) + e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g -
c*h)*(f*g - e*h))
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

method	result
default	$\frac{d^2\ln(dx+c)}{(ad-bc)(cf-de)(ch-dg)}-\frac{b^2\ln(bx+a)}{(ad-bc)(af-be)(ah-bg)}+\frac{h^2\ln(hx+g)}{(ah-bg)(ch-dg)(eh-fg)}-\frac{f^2\ln(fx+e)}{(af-be)(cf-de)(eh-fg)}$
norman	$\frac{h^2\ln(hx+g)}{ace\,h^3-acfg\,h^2-adeg\,h^2+adf\,g^2h-bceg\,h^2+bcf\,g^2h+bde\,g^2h-bdf\,g^3}+\frac{d^2\ln(dx+c)}{(ad-bc)(cf-de)(ch-dg)}-\frac{f^2\ln(fx+e}{(ac\,f^2-adef-bcef+bd}$
risch	$\frac{d^2\ln(-dx-c)}{a\,c^2dfh-ac\,d^2eh-ac\,d^2fg+a\,d^3eg-b\,c^3fh+b\,c^2deh+b\,c^2dfg-bc\,d^2eg}+\frac{h^2\ln(hx+g)}{ace\,h^3-acfg\,h^2-adeg\,h^2+adf\,g^2h-bceg\,h^2+bcf\,g^2h-}$
parallelrisch	$-\frac{\ln(bx+a)b^2c^2ef\,h^2-\ln(bx+a)b^2c^2f^2gh-\ln(bx+a)b^2cd\,e^2h^2+\ln(bx+a)b^2cd\,f^2g^2+\ln(bx+a)b^2d^2e^2gh-\ln(bx+a)b^2d^2ef\,g^2-\ln}{}$

```
[In] int(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(a*d-b*c)/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)-b^2/(a*d-b*c)/(a*f-b*e)/(a*h-b*
g)*ln(b*x+a)+h^2/(a*h-b*g)/(c*h-d*g)/(e*h-f*g)*ln(h*x+g)-f^2/(a*f-b*e)/(c*f
-d*e)/(e*h-f*g)*ln(f*x+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.90

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^2\log(bx+a)}{((b^3c-ab^2d)e-(ab^2c-a^2bd)f)g-((ab^2c-a^2bd)e-(a^2bc-a^3d)f)h} - \frac{d^2\log(dx+c)}{((bcd^2-ad^3)e-(bc^2d-acd^2)f)g-((bc^2d-acd^2)e-(bc^3-ac^2d)f)h} + \frac{f^2\log(fx+e)}{(bde^2f+acf^3-(bc+ad)ef^2)g-(bde^3+acef^2-(bc+ad)e^2f)h} - \frac{h^2\log(hx+g)}{bdfg^3-aceh^3-(bde+(bc+ad)f)g^2h+(acf+(bc+ad)e)gh^2}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(((b^3*c - a*b^2*d)*e - (a*b^2*c - a^2*b*d)*f)*g - ((a*b^2
*c - a^2*b*d)*e - (a^2*b*c - a^3*d)*f)*h) - d^2*log(d*x + c)/(((b*c*d^2 - a
*d^3)*e - (b*c^2*d - a*c*d^2)*f)*g - ((b*c^2*d - a*c*d^2)*e - (b*c^3 - a*c^
2*d)*f)*h) + f^2*log(f*x + e)/((b*d*e^2*f + a*c*f^3 - (b*c + a*d)*e*f^2)*g
- (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h) - h^2*log(h*x + g)/(b*d*f*g^
3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*c + a*d)*e)*g*h
^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(163) = 326.

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^3\log(|bx+a|)}{b^4ceg-ab^3deg-ab^3cfg+a^2b^2dfg-ab^3ceh+a^2b^2deh+a^2b^2cfh-a^3bdfh} - \frac{d^3\log(|dx+c|)}{bcd^3eg-ad^4eg-bc^2d^2fg+acd^3fg-bc^2d^2eh+acd^3eh+bc^3dfh-ac^2d^2fh} + \frac{f^3\log(|fx+e|)}{bde^2f^2g-bcef^3g-adef^3g+acf^4g-bde^3fh+bce^2f^2h+ade^2f^2h-acef^3h} - \frac{h^3\log(|hx+g|)}{bdfg^3h-bdeg^2h^2-bcfg^2h^2-adfg^2h^2+bcegh^3+adegh^3+acfgh^3-aceh^4}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/(b^4*c*e*g - a*b^3*d*e*g - a*b^3*c*f*g + a^2*b^2*d*f*
g - a*b^3*c*e*h + a^2*b^2*d*e*h + a^2*b^2*c*f*h - a^3*b*d*f*h) - d^3*log(ab
s(d*x + c))/(b*c*d^3*e*g - a*d^4*e*g - b*c^2*d^2*f*g + a*c*d^3*f*g - b*c^2*
d^2*e*h + a*c*d^3*e*h + b*c^3*d*f*h - a*c^2*d^2*f*h) + f^3*log(abs(f*x + e)
)/(b*d*e^2*f^2*g - b*c*e*f^3*g - a*d*e*f^3*g + a*c*f^4*g - b*d*e^3*f*h + b*
c*e^2*f^2*h + a*d*e^2*f^2*h - a*c*e*f^3*h) - h^3*log(abs(h*x + g))/(b*d*f*g
^3*h - b*d*e*g^2*h^2 - b*c*f*g^2*h^2 - a*d*f*g^2*h^2 + b*c*e*g*h^3 + a*d*e*
g*h^3 + a*c*f*g*h^3 - a*c*e*h^4)
```

Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \frac{b^2\,\ln(a+b\,x)}{b^3\,c\,e\,g-a^3\,d\,f\,h-a\,b^2\,c\,e\,h-a\,b^2\,c\,f\,g-a\,b^2\,d\,e\,g+a^2\,b\,c\,f\,h+a^2\,b\,d\,e\,h+a^2\,b\,d\,f\,g}$$
$$+\frac{d^2\,\ln(c+d\,x)}{a\,d^3\,e\,g-b\,c^3\,f\,h-a\,c\,d^2\,e\,h-a\,c\,d^2\,f\,g-b\,c\,d^2\,e\,g+a\,c^2\,d\,f\,h+b\,c^2\,d\,e\,h+b\,c^2\,d\,f\,g}$$
$$+\frac{f^2\,\ln(e+f\,x)}{a\,c\,f^3\,g-b\,d\,e^3\,h-a\,c\,e\,f^2\,h-a\,d\,e\,f^2\,g-b\,c\,e\,f^2\,g+a\,d\,e^2\,f\,h+b\,c\,e^2\,f\,h+b\,d\,e^2\,f\,g}$$
$$+\frac{h^2\,\ln(g+h\,x)}{a\,c\,e\,h^3-b\,d\,f\,g^3-a\,c\,f\,g\,h^2-a\,d\,e\,g\,h^2-b\,c\,e\,g\,h^2+a\,d\,f\,g^2\,h+b\,c\,f\,g^2\,h+b\,d\,e\,g^2\,h}$$

```
[In] int(1/((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x)),x)
```

```
[Out] (b^2*log(a + b*x))/(b^3*c*e*g - a^3*d*f*h - a*b^2*c*e*h - a*b^2*c*f*g - a*b
^2*d*e*g + a^2*b*c*f*h + a^2*b*d*e*h + a^2*b*d*f*g) + (d^2*log(c + d*x))/(a
*d^3*e*g - b*c^3*f*h - a*c*d^2*e*h - a*c*d^2*f*g - b*c*d^2*e*g + a*c^2*d*f*
h + b*c^2*d*e*h + b*c^2*d*f*g) + (f^2*log(e + f*x))/(a*c*f^3*g - b*d*e^3*h
- a*c*e*f^2*h - a*d*e*f^2*g - b*c*e*f^2*g + a*d*e^2*f*h + b*c*e^2*f*h + b*d
*e^2*f*g) + (h^2*log(g + h*x))/(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e
*g*h^2 - b*c*e*g*h^2 + a*d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h)
```

3.6 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{(1+x)(2+x)(3+x)} \, dx = -\frac{1}{2}\log(1+x) + 2\log(2+x) - \frac{3}{2}\log(3+x)$$

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {153}

$$\int \frac{x}{(1+x)(2+x)(3+x)} \, dx = -\frac{1}{2}\log(x+1) + 2\log(x+2) - \frac{3}{2}\log(x+3)$$

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{1}{2(1+x)} + \frac{2}{2+x} - \frac{3}{2(3+x)}\right) dx \\ &= -\frac{1}{2}\log(1+x) + 2\log(2+x) - \frac{3}{2}\log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{1}{2}\log(1+x) + 2\log(2+x) - \frac{3}{2}\log(3+x)$$

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\ln(1+x)}{2} + 2\ln(2+x) - \frac{3\ln(3+x)}{2}$	20
norman	$-\frac{\ln(1+x)}{2} + 2\ln(2+x) - \frac{3\ln(3+x)}{2}$	20
risch	$-\frac{\ln(1+x)}{2} + 2\ln(2+x) - \frac{3\ln(3+x)}{2}$	20
parallelrisch	$-\frac{\ln(1+x)}{2} + 2\ln(2+x) - \frac{3\ln(3+x)}{2}$	20

```
[In] int(x/(1+x)/(2+x)/(3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\log(x+3) + 2\log(x+2) - \frac{1}{2}\log(x+1)$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{\log(x+1)}{2} + 2\log(x+2) - \frac{3\log(x+3)}{2}$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\log(x+3) + 2\log(x+2) - \frac{1}{2}\log(x+1)$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\log(|x+3|) + 2\log(|x+2|) - \frac{1}{2}\log(|x+1|)$$

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = 2\ln(x+2) - \frac{\ln(x+1)}{2} - \frac{3\ln(x+3)}{2}$$

```
[In] int(x/((x + 1)*(x + 2)*(x + 3)),x)
[Out] 2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

3.7 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx = -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}$$

```
[Out] -12/1375/(3+5*x)^2+201/15125/(3+5*x)+20/3993*ln(6-x)+1493/499125*ln(3+5*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1607, 153}

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx = \frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3}\,dx \\
&= \int \left(\frac{20}{3993(-6+x)}+\frac{24}{275(3+5x)^3}-\frac{201}{3025(3+5x)^2}+\frac{1493}{99825(3+5x)}\right)dx \\
&= -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{\frac{99(157+335x)}{(3+5x)^2}+2500\log(-6+x)+1493\log(3+5x)}{499125}$$

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{201x}{3025}+\frac{471}{15125}}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$
norman	$\frac{-\frac{113}{3025}x-\frac{157}{1815}x^2}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$
default	$-\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{1493\ln(3+5x)}{499125}+\frac{20\ln(-6+x)}{3993}$
parallelrisch	$\frac{187500\ln(-6+x)x^2+111975\ln\left(x+\frac{3}{5}\right)x^2+225000\ln(-6+x)x+134370\ln\left(x+\frac{3}{5}\right)x-129525x^2+67500\ln(-6+x)+40311\ln\left(x+\frac{3}{5}\right.}{1497375(3+5x)^2}$

```
[In] int((x^3-x^2)/(-6+x)/(3+5*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 25*(201/75625*x+471/378125)/(3+5*x)^2+20/3993*ln(-6+x)+1493/499125*ln(3+5*x
)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{1493\,(25\,x^2+30\,x+9)\log(5\,x+3)+2500\,(25\,x^2+30\,x+9)\log(x-6)+33165\,x+15543}{499125\,(25\,x^2+30\,x+9)}$$

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{1005x+471}{378125x^2+453750x+136125}+\frac{20\log(x-6)}{3993}+\frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{3\,(335\,x+157)}{15125\,(25\,x^2+30\,x+9)}+\frac{1493}{499125}\,\log(5\,x+3)+\frac{20}{3993}\,\log(x-6)$$

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{3\,(335\,x+157)}{15125\,(5\,x+3)^2} + \frac{1493}{499125}\,\log\left(|5\,x+3|\right) + \frac{20}{3993}\,\log\left(|x-6|\right)$$

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{20\,\ln\left(x-6\right)}{3993} + \frac{1493\,\ln\left(x+\frac{3}{5}\right)}{499125} + \frac{\frac{201\,x}{75625}+\frac{471}{378125}}{x^2+\frac{6\,x}{5}+\frac{9}{25}}$$

```
[In] int(-(x^2 - x^3)/((5*x + 3)^3*(x - 6)),x)
```

```
[Out] (20*log(x - 6))/3993 + (1493*log(x + 3/5))/499125 + ((201*x)/75625 + 471/37
8125)/((6*x)/5 + x^2 + 9/25)
```

3.8 $\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x} dx$

Optimal result

Integrand size = 25, antiderivative size = 227

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x} dx$$
$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$
$$+ \frac{2(c+dx)^{3/2}\left(2(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf))+3bd(21abd^2\cdots\right.}{315d^4}$$
$$- 2a^3\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] 2/21*(2*a*d*f-2*b*c*f+3*b*d*e)*(b*x+a)^2*(d*x+c)^(3/2)/d^2+2/9*f*(b*x+a)^3*
(d*x+c)^(3/2)/d+2/315*(d*x+c)^(3/2)*(40*a^3*d^3*f+6*a^2*b*d^2*(-16*c*f+45*d
*e)-18*a*b^2*c*d*(-4*c*f+7*d*e)+8*b^3*c^2*(-2*c*f+3*d*e)+3*b*d*(21*a*b*d^2*
e-4*(-a*d+b*c)*(2*a*d*f-2*b*c*f+3*b*d*e))*x)/d^4-2*a^3*e*arctanh((d*x+c)^(1
/2)/c^(1/2))*c^(1/2)+2*a^3*e*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {158, 152, 52, 65, 214}

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x} dx = -2a^3\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + 2a^3e\sqrt{c+dx}$$
$$+ \frac{2(c+dx)^{3/2}\left(2(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf))+3bdx(21abd\cdots\right.}{315d^4}$$
$$+ \frac{2(a+bx)^2(c+dx)^{3/2}(2adf-2bcf+3bde)}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

```
[In] Int[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a^3*e*Sqrt[c + d*x] + (2*(3*b*d*e - 2*b*c*f + 2*a*d*f)*(a + b*x)^2*(c + d
*x)^(3/2))/(21*d^2) + (2*f*(a + b*x)^3*(c + d*x)^(3/2))/(9*d) + (2*(c + d*x
)^(3/2)*(2*(20*a^3*d^3*f + 3*a^2*b*d^2*(45*d*e - 16*c*f) - 9*a*b^2*c*d*(7*d
*e - 4*c*f) + 4*b^3*c^2*(3*d*e - 2*c*f)) + 3*b*d*(21*a*b*d^2*e - 4*(b*c - a
*d)*(3*b*d*e - 2*b*c*f + 2*a*d*f))*x))/(315*d^4) - 2*a^3*Sqrt[c]*e*ArcTanh[
Sqrt[c + d*x]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2\int \frac{(a+bx)^2\sqrt{c+dx}\left(\frac{9ade}{2}+\frac{3}{2}(3bde-2bcf+2adf)x\right)}{x}\,dx}{9d}$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{4\int \frac{(a+bx)\sqrt{c+dx}\left(\frac{63}{4}a^2d^2e+\frac{3}{4}(21abd^2e-4(bc-ad)(3bde-2bcf+2adf))x\right)}{x}\,dx}{63d^2}$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2(c+dx)^{3/2}\left(2(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf))+3bd\right.}{315d^4} + \left(a^3e\right)\int \frac{\sqrt{c+dx}}{x}\,dx$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2(c+dx)^{3/2}\left(2(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf))+3bd\right.}{315d^4} + \left(a^3ce\right)\int \frac{1}{x\sqrt{c+dx}}\,dx$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2(c+dx)^{3/2}\left(2(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf))+3bd\right.}{315d^4} + \frac{\left(2a^3ce\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2(c+dx)^{3/2}\left(2(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf))+3b\right.}{315d^4} - 2a^3\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\sqrt{c+dx}(105a^3d^3(3de+cf+dfx)+63a^2bd^2(c+dx)(5de-2cf+3dfx)+9ab^2d(c+dx)\left(8c^2f+3d^2x(\right.}{31} - 2a^3\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[In] Integrate[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(105*a^3*d^3*(3*d*e + c*f + d*f*x) + 63*a^2*b*d^2*(c + d*x
)*(5*d*e - 2*c*f + 3*d*f*x) + 9*a*b^2*d*(c + d*x)*(8*c^2*f + 3*d^2*x*(7*e +
 5*f*x) - 2*c*d*(7*e + 6*f*x)) - b^3*(c + d*x)*(16*c^3*f - 24*c^2*d*(e + f*
x) + 6*c*d^2*x*(6*e + 5*f*x) - 5*d^3*x^2*(9*e + 7*f*x))))/(315*d^4) - 2*a^3
*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-2a^3\sqrt{c}\,d^4e\,\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\sqrt{dx+c}\left(3\left(\frac{\left(\frac{7fx}{9}+e\right)x^3b^3}{7}+\frac{3\left(\frac{5fx}{7}+e\right)x^2ab^2}{5}+x\left(\frac{3fx}{5}+e\right)a^2b+\left(\frac{fx}{3}+e\right)a^3\right)d^4+\left(\frac{3x^2\left(\frac{5fx}{9}\right.}{35}\right.\right.}{}$
derivativedivides	$\frac{\frac{2fb^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6ab^2df(dx+c)^{\frac{7}{2}}}{7}-\frac{6b^3cf(dx+c)^{\frac{7}{2}}}{7}+\frac{2b^3de(dx+c)^{\frac{7}{2}}}{7}+\frac{6a^2bd^2f(dx+c)^{\frac{5}{2}}}{5}-\frac{12ab^2cdf(dx+c)^{\frac{5}{2}}}{5}+\frac{6ab^2d^2e(dx+c)^{\frac{5}{2}}}{5}}{}$
default	$\frac{\frac{2fb^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6ab^2df(dx+c)^{\frac{7}{2}}}{7}-\frac{6b^3cf(dx+c)^{\frac{7}{2}}}{7}+\frac{2b^3de(dx+c)^{\frac{7}{2}}}{7}+\frac{6a^2bd^2f(dx+c)^{\frac{5}{2}}}{5}-\frac{12ab^2cdf(dx+c)^{\frac{5}{2}}}{5}+\frac{6ab^2d^2e(dx+c)^{\frac{5}{2}}}{5}}{}$

```
[In] int((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-3*a^3*c^(1/2)*d^4*e*arctanh((d*x+c)^(1/2)/c^(1/2))+(d*x+c)^(1/2)*(3*(
1/7*(7/9*f*x+e)*x^3*b^3+3/5*(5/7*f*x+e)*x^2*a*b^2+x*(3/5*f*x+e)*a^2*b+(1/3*
f*x+e)*a^3)*d^4+(3/35*x^2*(5/9*f*x+e)*b^3+3/5*(3/7*f*x+e)*x*a*b^2+3*(1/5*f*
x+e)*a^2*b+f*a^3)*c*d^3-6/5*b*(2/21*(1/2*f*x+e)*x*b^2+a*(2/7*f*x+e)*b+a^2*f
)*c^2*d^2+24/35*(1/3*(1/3*f*x+e)*b+a*f)*b^2*c^3*d-16/105*b^3*c^4*f))/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.86

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = \left[\frac{315\,a^3\sqrt{c}d^4e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,(35\,b^3d^4fx^4+5\,(9\,b^3d^4e+(b^3cd^3+27\,ab^2d^4)f)x^3+3\,(3\,(b^3cd^3+2\right.$$

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*a^3*sqrt(c)*d^4*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(35*b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3
+ 3*(3*(b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a
^2*b*d^4)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 10
5*a^3*d^4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c
*d^3)*f - (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^
3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))
/d^4, 2/315*(315*a^3*sqrt(-c)*d^4*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (35*
b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3 + 3*(3*(
b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a^2*b*d^4
)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 105*a^3*d^
4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c*d^3)*f
- (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^3*d - 36
*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))/d^4]
```

Sympy [A] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = \begin{cases} \frac{2a^3ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^3e\sqrt{c+dx}+\frac{2b^3f(c+dx)^{\frac{9}{2}}}{9d^4}+\frac{2(c+dx)^{\frac{7}{2}}\cdot(3ab^2df-3b^3cf+b^3de)}{7d^4}+\frac{2(c+dx)^{\frac{5}{2}}\cdot(3a^2bd^2f-6ab^2cdf+3ab^2d^2e}{5d^4} \\ \sqrt{c}\left(a^3e\log{(x)}+a^3fx+3a^2bex+\frac{b^3fx^4}{4}+\frac{x^3\cdot(3ab^2f+b^3e)}{3}+\frac{x^2\cdot(3a^2bf+3ab^2e)}{2}\right) \end{cases}$$

```
[In] integrate((b*x+a)**3*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Piecewise((2*a**3*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**3*e*sqrt
(c + d*x) + 2*b**3*f*(c + d*x)**(9/2)/(9*d**4) + 2*(c + d*x)**(7/2)*(3*a*b*
*2*d*f - 3*b**3*c*f + b**3*d*e)/(7*d**4) + 2*(c + d*x)**(5/2)*(3*a**2*b*d**
2*f - 6*a*b**2*c*d*f + 3*a*b**2*d**2*e + 3*b**3*c**2*f - 2*b**3*c*d*e)/(5*d
**4) + 2*(c + d*x)**(3/2)*(a**3*d**3*f - 3*a**2*b*c*d**2*f + 3*a**2*b*d**3*
e + 3*a*b**2*c**2*d*f - 3*a*b**2*c*d**2*e - b**3*c**3*f + b**3*c**2*d*e)/(3
*d**4), Ne(d, 0)), (sqrt(c)*(a**3*e*log(x) + a**3*f*x + 3*a**2*b*e*x + b**3
*f*x**4/4 + x**3*(3*a*b**2*f + b**3*e)/3 + x**2*(3*a**2*b*f + 3*a*b**2*e)/2
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = a^3\sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + \frac{2\left(315\sqrt{dx+c}a^3d^4e+35\,(dx+c)^{\frac{9}{2}}b^3f+45\,(b^3de-3\,(b^3c-ab^2d)f)(dx+c)^{\frac{7}{2}}-63\,((2\,b^3cd-3\,ab^2d^2)\right.}{}$$

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^3*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
315*(315*sqrt(d*x + c)*a^3*d^4*e + 35*(d*x + c)^(9/2)*b^3*f + 45*(b^3*d*e -
 3*(b^3*c - a*b^2*d)*f)*(d*x + c)^(7/2) - 63*((2*b^3*c*d - 3*a*b^2*d^2)*e -
 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f)*(d*x + c)^(5/2) + 105*((b^3*c^2*d
 - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*e - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*f)*(d*x + c)^(3/2))/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,a^3ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\left(45\,(dx+c)^{\frac{7}{2}}b^3d^{33}e-126\,(dx+c)^{\frac{5}{2}}b^3cd^{33}e+105\,(dx+c)^{\frac{3}{2}}b^3c^2d^{33}e+189\,(dx+c)^{\frac{5}{2}}ab^2d^{34}e-315\,(d\right.}{}$$

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/315*(45*(d*x + c)^(7/
2)*b^3*d^33*e - 126*(d*x + c)^(5/2)*b^3*c*d^33*e + 105*(d*x + c)^(3/2)*b^3*
c^2*d^33*e + 189*(d*x + c)^(5/2)*a*b^2*d^34*e - 315*(d*x + c)^(3/2)*a*b^2*c
*d^34*e + 315*(d*x + c)^(3/2)*a^2*b*d^35*e + 315*sqrt(d*x + c)*a^3*d^36*e +
 35*(d*x + c)^(9/2)*b^3*d^32*f - 135*(d*x + c)^(7/2)*b^3*c*d^32*f + 189*(d*
x + c)^(5/2)*b^3*c^2*d^32*f - 105*(d*x + c)^(3/2)*b^3*c^3*d^32*f + 135*(d*x
 + c)^(7/2)*a*b^2*d^33*f - 378*(d*x + c)^(5/2)*a*b^2*c*d^33*f + 315*(d*x +
c)^(3/2)*a*b^2*c^2*d^33*f + 189*(d*x + c)^(5/2)*a^2*b*d^34*f - 315*(d*x + c
)^(3/2)*a^2*b*c*d^34*f + 105*(d*x + c)^(3/2)*a^3*d^35*f)/d^36
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \left(c\left(c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b(ad-bc)(adf-2bcf+bde)}{d^4}\right)+\frac{2(ad-bc)^2(adf-4bcf+3bde)}{d^4}\right)-\frac{2(ad-bc)^3(cf-de)}{d^4}\right)\sqrt{c+dx}$$
$$+\left(\frac{c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b(ad-bc)(adf-2bcf+bde)}{d^4}\right)}{3}+\frac{2(ad-bc)^2(adf-4bcf+3bde)}{3d^4}\right)(c+dx)^{3/2}$$
$$+\left(\frac{2b^3de-8b^3cf+6ab^2df}{7d^4}+\frac{2b^3cf}{7d^4}\right)(c+dx)^{7/2}$$
$$+\left(\frac{c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)}{5}+\frac{6b(ad-bc)(adf-2bcf+bde)}{5d^4}\right)(c+dx)^{5/2}$$
$$+\frac{2b^3f(c+dx)^{9/2}}{9d^4}+a^3\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+dx}\,1\mathrm{i}}{\sqrt{c}}\right)2\mathrm{i}$$

```
[In] int(((e + f*x)*(a + b*x)^3*(c + d*x)^(1/2))/x,x)
```

```
[Out] (c*(c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) + (6
*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4) + (2*(a*d - b*c)^2*(a*d*f -
4*b*c*f + 3*b*d*e))/d^4) - (2*(a*d - b*c)^3*(c*f - d*e))/d^4)*(c + d*x)^(1/
2) + ((c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) +
 (6*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4))/3 + (2*(a*d - b*c)^2*(a*
d*f - 4*b*c*f + 3*b*d*e))/(3*d^4))*(c + d*x)^(3/2) + ((2*b^3*d*e - 8*b^3*c*
```

```
f + 6*a*b^2*d*f)/(7*d^4) + (2*b^3*c*f)/(7*d^4))*(c + d*x)^(7/2) + ((c*((2*b
^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4))/5 + (6*b*(a*d - b
*c)*(a*d*f - 2*b*c*f + b*d*e))/(5*d^4))*(c + d*x)^(5/2) + a^3*c^(1/2)*e*ata
n(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*b^3*f*(c + d*x)^(9/2))/(9*d^4)
```

3.9 $\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = 2a^2e\sqrt{c+dx}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} +\frac{2(c+dx)^{3/2}\left(2(10a^2d^2f-b^2c(7de-4cf)+7abd(5de-2cf))+3bd(7bde-4bcf+4adf)x\right)}{105d^3} -2a^2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] 2/7*f*(b*x+a)^2*(d*x+c)^(3/2)/d+2/105*(d*x+c)^(3/2)*(20*a^2*d^2*f-2*b^2*c*(
-4*c*f+7*d*e)+14*a*b*d*(-2*c*f+5*d*e)+3*b*d*(4*a*d*f-4*b*c*f+7*b*d*e)*x)/d^
3-2*a^2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*a^2*e*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {158, 152, 52, 65, 214}

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = -2a^2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) +\frac{2(c+dx)^{3/2}\left(2(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf))+3bdx(4adf-4bcf+7bde)\right)}{105d^3} +2a^2e\sqrt{c+dx}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a^2*e*Sqrt[c + d*x] + (2*f*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + (2*(c + d
*x)^(3/2)*(2*(10*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) + 7*a*b*d*(5*d*e - 2*c*f
)) + 3*b*d*(7*b*d*e - 4*b*c*f + 4*a*d*f)*x))/(105*d^3) - 2*a^2*Sqrt[c]*e*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2\int \frac{(a+bx)\sqrt{c+dx}\left(\frac{7ade}{2}+\frac{1}{2}(7bde-4bcf+4adf)x\right)}{x}\,dx}{7d}$$

$$= \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f - b^2c(7de-4cf) + 7abd(5de-2cf)) + 3bd(7bde-4bcf+4adf)x\right)}{105d^3} + \left(a^2e\right)\int \frac{\sqrt{c+dx}}{x}\,dx$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f - b^2c(7de-4cf) + 7abd(5de-2cf)) + 3bd(7bde-4bcf+4adf)x\right)}{105d^3} + \left(a^2ce\right)\int \frac{1}{x\sqrt{c+dx}}\,dx$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f - b^2c(7de-4cf) + 7abd(5de-2cf)) + 3bd(7bde-4bcf+4adf)x\right)}{105d^3} + \frac{(2a^2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f - b^2c(7de-4cf) + 7abd(5de-2cf)) + 3bd(7bde-4bcf+4adf)x\right)}{105d^3} - 2a^2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \frac{2\sqrt{c+dx}(35a^2d^2(3de+cf+dfx)+14abd(c+dx)(5de-2cf+3dfx)+b^2(c+dx)\,(8c^2f+3d^2x(7e+5}{105d^3}$$
$$-2a^2\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(35*a^2*d^2*(3*d*e + c*f + d*f*x) + 14*a*b*d*(c + d*x)*(5*
d*e - 2*c*f + 3*d*f*x) + b^2*(c + d*x)*(8*c^2*f + 3*d^2*x*(7*e + 5*f*x) - 2
*c*d*(7*e + 6*f*x))))/(105*d^3) - 2*a^2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqr
t[c]]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-2a^2\sqrt{c}\,d^3e\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+\frac{2\sqrt{dx+c}\,(((\frac{3(\frac{5fx}{7}+e)x^2b^2}{5}+2x(\frac{3fx}{5}+e)ab+3(\frac{fx}{3}+e)a^2)d^3+c(\frac{(\frac{3fx}{7}+e)x\,b^2}{5}+2(\frac{fx}{5}+e)a}{3}}{d^3}$
derivativedivides	$\frac{\frac{2b^2f(dx+c)^{\frac{7}{2}}}{7}+\frac{4abdf(dx+c)^{\frac{5}{2}}}{5}-\frac{4b^2cf(dx+c)^{\frac{5}{2}}}{5}+\frac{2b^2de(dx+c)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(dx+c)^{\frac{3}{2}}}{3}-\frac{4abcdf(dx+c)^{\frac{3}{2}}}{3}+\frac{4ab\,d^2e(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c^2}{}}{d^3}$
default	$\frac{\frac{2b^2f(dx+c)^{\frac{7}{2}}}{7}+\frac{4abdf(dx+c)^{\frac{5}{2}}}{5}-\frac{4b^2cf(dx+c)^{\frac{5}{2}}}{5}+\frac{2b^2de(dx+c)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(dx+c)^{\frac{3}{2}}}{3}-\frac{4abcdf(dx+c)^{\frac{3}{2}}}{3}+\frac{4ab\,d^2e(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c^2}{}}{d^3}$

```
[In] int((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-3*a^2*c^(1/2)*d^3*e*arctanh((d*x+c)^(1/2)/c^(1/2))+(d*x+c)^(1/2)*((3/
5*(5/7*f*x+e)*x^2*b^2+2*x*(3/5*f*x+e)*a*b+3*(1/3*f*x+e)*a^2)*d^3+c*(1/5*(3/
7*f*x+e)*x*b^2+2*(1/5*f*x+e)*a*b+a^2*f)*d^2-4/5*b*((1/7*f*x+1/2*e)*b+a*f)*c
^2*d+8/35*b^2*c^3*f))/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.77

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \left[\frac{105\,a^2\sqrt{c}d^3e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,(15\,b^2d^3fx^3+3\,(7\,b^2d^3e+(b^2cd^2+14\,abd^3)f)x^2-7\,(2\,b^2c^2d-10\cdots}{\cdots}\right.$$

[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")

[Out] [1/105*(105*a^2*sqrt(c)*d^3*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*(15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2*c*d^2 + 14*a*b*d^3)*f)*x^2 - 7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e + (8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3)*e - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3, 2/105*(105*a^2*sqrt(-c)*d^3*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2*c*d^2 + 14*a*b*d^3)*f)*x^2 - 7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e + (8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3)*e - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3]

Sympy [A] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \begin{cases} \frac{2a^2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^2e\sqrt{c+dx}+\frac{2b^2f(c+dx)^{\frac{7}{2}}}{7d^3}+\frac{2(c+dx)^{\frac{5}{2}}\cdot(2abdf-2b^2cf+b^2de)}{5d^3}+\frac{2(c+dx)^{\frac{3}{2}}(a^2d^2f-2abcdf+2abd^2e+b^2c^2\cdots}{3d^3} \\ \sqrt{c}\left(a^2e\log(x)+a^2fx+2abex+\frac{b^2fx^3}{3}+\frac{x^2\cdot(2abf+b^2e)}{2}\right) \end{cases}$$

[In] integrate((b*x+a)**2*(f*x+e)*(d*x+c)**(1/2)/x,x)

[Out] Piecewise((2*a**2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**2*e*sqrt(c + d*x) + 2*b**2*f*(c + d*x)**(7/2)/(7*d**3) + 2*(c + d*x)**(5/2)*(2*a*b*d*f - 2*b**2*c*f + b**2*d*e)/(5*d**3) + 2*(c + d*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f + 2*a*b*d**2*e + b**2*c**2*f - b**2*c*d*e)/(3*d**3), Ne(d, 0)), (sqrt(c)*(a**2*e*log(x) + a**2*f*x + 2*a*b*e*x + b**2*f*x**3/3 + x**2*(2*a*b*f + b**2*e)/2), True))

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = a^2\sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + \frac{2\left(105\sqrt{dx+c}a^2d^3e+15\,(dx+c)^{\frac{7}{2}}b^2f+21\,(b^2de-2\,(b^2c-abd)f)(dx+c)^{\frac{5}{2}}-35\,((b^2cd-2\,abd^2)e-\right.}{105\,d^3}$$

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^2*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
105*(105*sqrt(d*x + c)*a^2*d^3*e + 15*(d*x + c)^(7/2)*b^2*f + 21*(b^2*d*e -
 2*(b^2*c - a*b*d)*f)*(d*x + c)^(5/2) - 35*((b^2*c*d - 2*a*b*d^2)*e - (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(d*x + c)^(3/2))/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,a^2ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\left(21\,(dx+c)^{\frac{5}{2}}b^2d^{19}e-35\,(dx+c)^{\frac{3}{2}}b^2cd^{19}e+70\,(dx+c)^{\frac{3}{2}}abd^{20}e+105\sqrt{dx+c}a^2d^{21}e+15\,(dx+c)^{\frac{7}{2}}\right.}{}$$

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/105*(21*(d*x + c)^(5/
2)*b^2*d^19*e - 35*(d*x + c)^(3/2)*b^2*c*d^19*e + 70*(d*x + c)^(3/2)*a*b*d^
20*e + 105*sqrt(d*x + c)*a^2*d^21*e + 15*(d*x + c)^(7/2)*b^2*d^18*f - 42*(d
*x + c)^(5/2)*b^2*c*d^18*f + 35*(d*x + c)^(3/2)*b^2*c^2*d^18*f + 42*(d*x +
c)^(5/2)*a*b*d^19*f - 70*(d*x + c)^(3/2)*a*b*c*d^19*f + 35*(d*x + c)^(3/2)*
a^2*d^20*f)/d^21
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = \left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{5\,d^3}+\frac{2\,b^2\,c\,f}{5\,d^3}\right)\,{\left(c+d\,x\right)}^{5/2} + \left(c\,\left(c\,\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{d^3}+\frac{2\,b^2\,c\,f}{d^3}\right)+\frac{2\,\left(a\,d-b\,c\right)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{d^3}\right) - \frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(c\,f-d\,e\right)}{d^3}\right)\,\sqrt{c+d\,x} + \left(\frac{c\,\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{d^3}+\frac{2\,b^2\,c\,f}{d^3}\right)}{3} + \frac{2\,\left(a\,d-b\,c\right)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{3\,d^3}\right)\,{\left(c+d\,x\right)}^{3/2} + \frac{2\,b^2\,f\,{\left(c+d\,x\right)}^{7/2}}{7\,d^3}+a^2\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,2{}\mathrm{i}$$

```
[In] int(((e + f*x)*(a + b*x)^2*(c + d*x)^(1/2))/x,x)
```

```
[Out] ((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/(5*d^3) + (2*b^2*c*f)/(5*d^3))*(c + d*
x)^(5/2) + (c*(c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d^3 + (2*b^2*c*f)/d^3
) + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/d^3) - (2*(a*d - b*c)^2*(c*
f - d*e))/d^3)*(c + d*x)^(1/2) + ((c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d
^3 + (2*b^2*c*f)/d^3))/3 + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/(3*d
^3))*(c + d*x)^(3/2) + a^2*c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
+ (2*b^2*f*(c + d*x)^(7/2))/(7*d^3)
```

3.10 $\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = 2ae\sqrt{c+dx} - \frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} - 2a\sqrt{c}e\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] -2/15*(d*x+c)^(3/2)*(2*b*c*f-5*d*(a*f+b*e)-3*b*d*f*x)/d^2-2*a*e*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2)+2*a*e*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {152, 52, 65, 214}

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = -2a\sqrt{c}e\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) - \frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2} + 2ae\sqrt{c+dx}$$

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a*e*Sqrt[c + d*x] - (2*(c + d*x)^(3/2)*(2*b*c*f - 5*d*(b*e + a*f) - 3*b*d
*f*x))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+(ae)\int\frac{\sqrt{c+dx}}{x}\,dx$$

$$=2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+(ace)\int\frac{1}{x\sqrt{c+dx}}\,dx$$

$$=2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+\frac{(2ace)\text{Subst}\left(\int\frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d}$$

$$=2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}-2a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$=\frac{2\sqrt{c+dx}(-b(c+dx)(-5de+2cf-3dfx)+5ad(3de+cf+dfx))}{15d^2}$$
$$-2a\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*(c + d*x)*(-5*d*e + 2*c*f - 3*d*f*x)) + 5*a*d*(3*d*e
+ c*f + d*f*x)))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+\frac{2\sqrt{dx+c}\,\left(\left(x\left(\frac{3fx}{5}+e\right)b+3\left(\frac{fx}{3}+e\right)a\right)d^{2}+\left(\left(\frac{fx}{5}+e\right)b+af\right)cd-\frac{2c^{2}bf}{5}\right)}{3}}{d^{2}}$	83
derivativedivides	$\frac{\frac{2fb(dx+c)^{\frac{5}{2}}}{5}+\frac{2adf(dx+c)^{\frac{3}{2}}}{3}-\frac{2bcf(dx+c)^{\frac{3}{2}}}{3}+\frac{2bde(dx+c)^{\frac{3}{2}}}{3}+2a\,d^{2}e\sqrt{dx+c}-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^{2}}$	89
default	$\frac{\frac{2fb(dx+c)^{\frac{5}{2}}}{5}+\frac{2adf(dx+c)^{\frac{3}{2}}}{3}-\frac{2bcf(dx+c)^{\frac{3}{2}}}{3}+\frac{2bde(dx+c)^{\frac{3}{2}}}{3}+2a\,d^{2}e\sqrt{dx+c}-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^{2}}$	89

```
[In] int((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-3*a*c^(1/2)*d^2*e*arctanh((d*x+c)^(1/2)/c^(1/2))+(d*x+c)^(1/2)*((x*(3
/5*f*x+e)*b+3*(1/3*f*x+e)*a)*d^2+((1/5*f*x+e)*b+a*f)*c*d-2/5*c^2*b*f))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$=\left[\frac{15\,a\sqrt{c}d^2e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,(3\,bd^2fx^2+5\,(bcd+3\,ad^2)e-(2\,bc^2-5\,acd)f+(5\,bd^2e+(bcd+}{15\,d^2}\right.$$

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a*sqrt(c)*d^2*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*
(3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e
 + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/d^2, 2/15*(15*a*sqrt(-c)*d^2*e*ar
ctan(sqrt(d*x + c)*sqrt(-c)/c) + (3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (
2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/
d^2]
```

Sympy [A] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = \begin{cases} \frac{2ace\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2ae\sqrt{c+dx} + \frac{2bf(c+dx)^{\frac{5}{2}}}{5d^2} + \frac{2(c+dx)^{\frac{3}{2}}(adf-bcf+bde)}{3d^2} & \text{for } d \neq 0 \\ \sqrt{c}\left(ae\log(x) + afx + bex + \frac{bfx^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Piecewise((2*a*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a*e*sqrt(c + d
*x) + 2*b*f*(c + d*x)**(5/2)/(5*d**2) + 2*(c + d*x)**(3/2)*(a*d*f - b*c*f +
 b*d*e)/(3*d**2), Ne(d, 0)), (sqrt(c)*(a*e*log(x) + a*f*x + b*e*x + b*f*x**
2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = a\sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + \frac{2\left(15\sqrt{dx+c}ad^2e + 3(dx+c)^{\frac{5}{2}}bf + 5(bde-(bc-ad)f)(dx+c)^{\frac{3}{2}}\right)}{15d^2}$$

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/15
*(15*sqrt(d*x + c)*a*d^2*e + 3*(d*x + c)^(5/2)*b*f + 5*(b*d*e - (b*c - a*d)
*f)*(d*x + c)^(3/2))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,ace\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\left(5\,(dx+c)^{\frac{3}{2}}bd^9e + 15\,\sqrt{dx+c}ad^{10}e + 3\,(dx+c)^{\frac{5}{2}}bd^8f - 5\,(dx+c)^{\frac{3}{2}}bcd^8f + 5\,(dx+c)^{\frac{3}{2}}ad^9f\right)}{15\,d^{10}}$$

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/15*(5*(d*x + c)^(3/2)*b
*d^9*e + 15*sqrt(d*x + c)*a*d^10*e + 3*(d*x + c)^(5/2)*b*d^8*f - 5*(d*x + c
)^(3/2)*b*c*d^8*f + 5*(d*x + c)^(3/2)*a*d^9*f)/d^10
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = \left(c\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{d^2}+\frac{2\,b\,c\,f}{d^2}\right) - \frac{2\,(a\,d-b\,c)\,(c\,f-d\,e)}{d^2}\right)\sqrt{c+d\,x} + \left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{3\,d^2}+\frac{2\,b\,c\,f}{3\,d^2}\right)(c+d\,x)^{3/2} + \frac{2\,b\,f\,(c+d\,x)^{5/2}}{5\,d^2} + a\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,2\mathrm{i}$$

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] (c*((2*a*d*f - 4*b*c*f + 2*b*d*e)/d^2 + (2*b*c*f)/d^2) - (2*(a*d - b*c)*(c*
f - d*e))/d^2)*(c + d*x)^(1/2) + ((2*a*d*f - 4*b*c*f + 2*b*d*e)/(3*d^2) + (
2*b*c*f)/(3*d^2))*(c + d*x)^(3/2) + (2*b*f*(c + d*x)^(5/2))/(5*d^2) + a*c^(
1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
```

3.11 $\int \frac{\sqrt{c+dx}(e+fx)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{\sqrt{c+dx}(e+fx)}{x} dx = 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} - 2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] 2/3*f*(d*x+c)^(3/2)/d-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*e*(d*x+c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {81, 52, 65, 214}

$$\int \frac{\sqrt{c+dx}(e+fx)}{x} dx = -2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d}$$

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[c + d*x] + (2*f*(c + d*x)^(3/2))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2f(c+dx)^{3/2}}{3d}+e\int \frac{\sqrt{c+dx}}{x}\,dx\\
&= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}+(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx\\
&= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}+\frac{(2ce)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}-2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$
\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx=\frac{2\sqrt{c+dx}(3de+cf+dfx)}{3d}-2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)
$$

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/x,x]
[Out] (2*Sqrt[c + d*x]*(3*d*e + c*f + d*f*x))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c
+ d*x]/Sqrt[c]]
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{2f(dx+c)^{\frac{3}{2}}}{3}+2de\sqrt{dx+c}-2\sqrt{c}\,de\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d}$	46
default	$\frac{\frac{2f(dx+c)^{\frac{3}{2}}}{3}+2de\sqrt{dx+c}-2\sqrt{c}\,de\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d}$	46
pseudoelliptic	$\frac{-6\sqrt{c}\,de\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2((fx+3e)d+cf)\sqrt{dx+c}}{3d}$	48

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/d*(1/3*f*(d*x+c)^(3/2)+d*e*(d*x+c)^(1/2)-c^(1/2)*d*e*arctanh((d*x+c)^(1/2
)/c^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \left[\frac{3\sqrt{c}de\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\,(dfx+3\,de+cf)\sqrt{dx+c}}{3\,d}, \frac{2\left(3\sqrt{-c}de\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+(dfx+3\right.}{3\,d}\right.$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
[Out] [1/3*(3*sqrt(c)*d*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*(d*f*x
 + 3*d*e + c*f)*sqrt(d*x + c))/d, 2/3*(3*sqrt(-c)*d*e*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + (d*f*x + 3*d*e + c*f)*sqrt(d*x + c))/d]
```

Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \begin{cases} \frac{2ce\,\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2e\sqrt{c+dx} + \frac{2f(c+dx)^{\frac{3}{2}}}{3d} & \text{for } d \neq 0 \\ \sqrt{c}(e\log(fx)+fx) & \text{otherwise} \end{cases}$$

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x,x)
[Out] Piecewise((2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*e*sqrt(c + d*x)
+ 2*f*(c + d*x)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)*(e*log(f*x) + f*x), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(3\sqrt{dx+c}de+(dx+c)^{\frac{3}{2}}f\right)}{3\,d}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/3*(3
*sqrt(d*x + c)*d*e + (d*x + c)^(3/2)*f)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\left(3\sqrt{dx+c}d^3e+(dx+c)^{\frac{3}{2}}d^2f\right)}{3\,d^3}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/3*(3*sqrt(d*x + c)*d^3*e
+ (d*x + c)^(3/2)*d^2*f)/d^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = 2\,e\,\sqrt{c+d\,x}+\frac{2\,f\,{\left(c+d\,x\right)}^{3/2}}{3\,d}+\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,2\mathrm{i}$$

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] 2*e*(c + d*x)^(1/2) + c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*
f*(c + d*x)^(3/2))/(3*d)
```

3.12 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2f\sqrt{c+dx}}{b} - \frac{2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2\sqrt{bc-ad}(be-af)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a+2*(-a*f+b*e)*arctanh(b^(1/2)*
(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(3/2)+2*f*(d*x+c)^(1/2
)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {159, 162, 65, 214}

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2\sqrt{bc-ad}(be-af)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}} - \frac{2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2f\sqrt{c+dx}}{b}$$

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2f\sqrt{c+dx}}{b} + \frac{2\int \frac{\frac{bce}{2}+\frac{1}{2}(bde+bcf-adf)x}{x(a+bx)\sqrt{c+dx}}\,dx}{b} \\
&= \frac{2f\sqrt{c+dx}}{b} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a} - \frac{((bc-ad)(be-af))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{ab} \\
&= \frac{2f\sqrt{c+dx}}{b} + \frac{(2ce)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{ad} \\
&\quad - \frac{(2(bc-ad)(be-af))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{abd} \\
&= \frac{2f\sqrt{c+dx}}{b} - \frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2\sqrt{bc-ad}(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2f\sqrt{c+dx}}{b} + \frac{2\sqrt{-bc+ad}(be-af)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{ab^{3/2}} - \frac{2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}$$

[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]

[Out] (2*f*Sqrt[c + d*x])/b + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(a*b^(3/2)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{2f\sqrt{dx+c}}{b}-\frac{2e\,\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{a}+\frac{2\left(-a^2df+acfb+abde-b^2ce\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{ab\sqrt{(ad-bc)b}}$	103
default	$\frac{2f\sqrt{dx+c}}{b}-\frac{2e\,\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{a}+\frac{2\left(-a^2df+acfb+abde-b^2ce\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{ab\sqrt{(ad-bc)b}}$	103
pseudoelliptic	$\frac{-2(af-be)(ad-bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)+2\left(-\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}\,be+\sqrt{dx+c}\,af\right)\sqrt{(ad-bc)b}}{ab\sqrt{(ad-bc)b}}$	105

[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x,method=_RETURNVERBOSE)

[Out] 2*f*(d*x+c)^(1/2)/b-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a+2*(-a^2*d*f+a*b*c*f+a*b*d*e-b^2*c*e)/a/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.45

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \left[\frac{b\sqrt{c}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{dx+c}af-(be-af)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab}, \frac{b\sqrt{c}e\log \cdots}{\cdots}\right.$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c
)*a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d
*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), (b*sqrt(c)*e*log((d*x - 2
*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c)*a*f + 2*(b*e - a*f)*sqrt
(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))
/(a*b), (2*b*sqrt(-c)*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 2*sqrt(d*x + c)*
a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x
 + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), 2*(b*sqrt(-c)*e*arctan(sqrt
(d*x + c)*sqrt(-c)/c) + sqrt(d*x + c)*a*f + (b*e - a*f)*sqrt(-(b*c - a*d)/b
)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(90) = 180$.

Time = 15.02 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \begin{cases} \frac{2f\sqrt{c+dx}}{b}+\frac{2ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2(ad-bc)(af-be)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^2\sqrt{\frac{ad-bc}{b}}} \\ \sqrt{c}\left(\left(-f+\frac{be}{2a}\right)\left(\frac{2a\left(\begin{cases}-\frac{\frac{1}{x}+\frac{b}{2a}}{b} & \text{for } a=0\\ \frac{\log\left(2a\left(\frac{1}{x}+\frac{b}{2a}\right)-b\right)}{2a} & \text{otherwise}\end{cases}\right)}{b}-\frac{2a\left(\begin{cases}\frac{\frac{1}{x}+\frac{b}{2a}}{b} & \text{for } a=0\\ \frac{\log\left(2a\left(\frac{1}{x}+\frac{b}{2a}\right)+b\right)}{2a} & \text{otherwise}\end{cases}\right)}{b}\right)-\frac{e\log\left(\frac{a}{x^2}\cdots\right)}{2a}\cdots\right. \end{cases}$$

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a),x)
```

```
[Out] Piecewise((2*f*sqrt(c + d*x)/b + 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/(a*sqrt
(-c)) - 2*(a*d - b*c)*(a*f - b*e)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(
a*b**2*sqrt((a*d - b*c)/b)), Ne(d, 0)), (sqrt(c)*((-f + b*e/(2*a))*(2*a*Pie
cewise((-(1/x + b/(2*a))/b, Eq(a, 0)), (log(2*a*(1/x + b/(2*a)) - b)/(2*a),
 True))/b - 2*a*Piecewise(((1/x + b/(2*a))/b, Eq(a, 0)), (log(2*a*(1/x + b/
(2*a)) + b)/(2*a), True))/b) - e*log(a/x**2 + b/x)/(2*a)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}} + \frac{2\,\sqrt{dx+c}f}{b} - \frac{2\,(b^2ce-abde-abcf+a^2df)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ab}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2*sqrt(d*x + c)*f/b - 2
*(b^2*c*e - a*b*d*e - a*b*c*f + a^2*d*f)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 2368, normalized size of antiderivative = 23.45

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)),x)
[Out] (2*f*(c + d*x)^(1/2))/b - (c^(1/2)*e*atan(((c^(1/2)*e*((8*(c + d*x)^(1/2)*(
a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*
f + 4*a^2*b^2*c*d^3*e*f))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2
*d^2*f))/b + (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/
(a*b)))/a)*1i)/a + (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^
4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f)
)/b - (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*
e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a)*1i)/a)/((16*(
b^3*c^2*d^3*e^3 - a*b^2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3
*a*b^2*c^2*d^3*e^2*f - a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*
b*c*d^4*e^2*f))/b - (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d
^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f
))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b + (8*c^(1/2)
*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a + (c^(1/2
)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2
- 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3
*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (c^(1/2)*e*((8*(a^3*
b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4
*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a))*2i)/a - (atan(((((8*(c + d*x)^(1/2
)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2
*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f)
)/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/
2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))
*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3) + (((8*(c + d*x)^(1/2)*(a
^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^
2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f
 + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b
- (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*(
c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))*(a*
f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3))/((16*(b^3*c^2*d^3*e^3 - a*b^
2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3*a*b^2*c^2*d^3*e^2*f -
 a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*b*c*d^4*e^2*f))/b - ((
```

```
(8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a
^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2
- 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a
^2*b^3*c^2*d^2*f))/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3
*(a*d - b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c)
)^(1/2))/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3) + (((8*(c +
 d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^
4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b
^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*
c^2*d^2*f))/b - (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d -
 b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)
)/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3)))*(a*f - b*e)*(-b^
3*(a*d - b*c))^(1/2)*2i)/(a*b^3)
```

3.13 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2} dx = \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{(2b^2ce-ad(be+af))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}$$

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+(2*b^2*c*e-a*d*(a*f+b*e))*a
rctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(3/2)/(-a*d+b*c)^(1/2)
+(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {154, 162, 65, 214}

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2} dx = \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)(2b^2ce-ad(af+be))}{a^2b^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}$$

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(a*b*(a + b*x)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]])/a^2 + ((2*b^2*c*e - a*d*(b*e + a*f))*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[b*c - a*d]])/(a^2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{\int \frac{-bce-\frac{1}{2}d(be+af)x}{x(a+bx)\sqrt{c+dx}}\,dx}{ab} \\
&= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^2} + \frac{\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{a^2b} \\
&= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(2ce)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^2d} \\
&\quad + \frac{\left(2\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^2bd} \\
&= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{(2b^2ce-ad(be+af))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \frac{\frac{a(be-af)\sqrt{c+dx}}{b(a+bx)}+\frac{(-2b^2ce+abde+a^2df)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}-2\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}$$

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((a*(b*e - a*f)*Sqrt[c + d*x])/(b*(a + b*x)) + ((-2*b^2*c*e + a*b*d*e + a^2
*d*f)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(3/2)*Sqrt[-(b
*c) + a*d]) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a^2
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-(bx+a)(a^2df+abde-2b^2ce)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)+\left(2be\sqrt{c}\,(bx+a)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+a\sqrt{dx+c}\,(af-be)\right)\sqrt{(ad-bc}}{\sqrt{(ad-bc)b}\,a^2(bx+a)b}$
derivativedivides	$2d\left(-\frac{e\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d\,a^2}+\frac{-\frac{ad(af-be)\sqrt{dx+c}}{2b((dx+c)b+ad-bc)}+\frac{\left(a^2df+abde-2b^2ce\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}}{a^2d}\right)$
default	$2d\left(-\frac{e\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d\,a^2}+\frac{-\frac{ad(af-be)\sqrt{dx+c}}{2b((dx+c)b+ad-bc)}+\frac{\left(a^2df+abde-2b^2ce\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}}{a^2d}\right)$

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(b*x+a)*(a^2*d*f+a*b*d*e-2*b^2*c*e)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)
^(1/2))+(2*b*e*c^(1/2)*(b*x+a)*arctanh((d*x+c)^(1/2)/c^(1/2))+a*(d*x+c)^(1/
2)*(a*f-b*e))*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/a^2/(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(109) = 218$.

Time = 0.35 (sec) , antiderivative size = 1018, normalized size of antiderivative = 8.02

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \left[\frac{(a^3df-(2\,ab^2c-a^2bd)e+(a^2bdf-(2\,b^3c-ab^2d)e)x)\sqrt{b^2c-abd}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\,((\cdots}{2\,(a^3b^3c-a^4b^2d-\cdots}\right.$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)
*e)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)
*sqrt(d*x + c))/(b*x + a)) + 2*((b^4*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*
d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*((a*b^3*c -
a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b^2
*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2
*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
+ a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + ((b^4*c - a*b^3*d)*e*x + (a*b^3*c -
 a^2*b^2*d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + ((a*b
^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c -
a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), 1/2*(4*((b^4*c - a*b^3*d)*e*x + (a*
b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (a^3*d*f
- (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) + 2*((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*
x + c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2
*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + 2*((b^4
*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + ((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x
+ c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}} - \frac{(2\,b^2ce-abde-a^2df)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2b} + \frac{\sqrt{dx+c}bde-\sqrt{dx+c}adf}{((dx+c)b-bc+ad)ab}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/(a^2*sqrt(-c)) - (2*b^2*c*e - a*b*d*e
- a^2*d*f)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2*b) + (sqrt(d*x + c)*b*d*e - sqrt(d*x + c)*a*d*f)/(((d*x + c)*b - b*c
 + a*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 1827, normalized size of antiderivative = 14.39

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^2),x)
```

```
[Out] (atan((((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) + ((4*a^5*b^
3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f
- 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))
^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) + (2*(c
```

```
 + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*
d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b
*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d))
- (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) - ((4*a^5*b^3*d^3
 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b
^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2
)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) - (2*(c + d*
x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e
*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^
(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d)))/((4*
(a*b^2*c*d^4*e^3 - 2*b^3*c^2*d^3*e^3 + a^3*c*d^4*e*f^2 - 2*a*b^2*c^2*d^3*e^
2*f + 2*a^2*b*c*d^4*e^2*f))/(a^3*b) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2
*c*d^3*f))/(a^3*b) + ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^
(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c -
a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(
a^2*b^4*c - a^3*b^3*d)) + (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2
 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^
3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/
(2*(a^2*b^4*c - a^3*b^3*d)) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f
))/(a^3*b) - ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c
 + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*
d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*
c - a^3*b^3*d)) - (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4
*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/
(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*
b^4*c - a^3*b^3*d))))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d
*e)*1i)/(a^2*b^4*c - a^3*b^3*d) - (2*c^(1/2)*e*atanh((4*c^(1/2)*d^4*e*f^2*(
c + d*x)^(1/2))/(4*c*d^4*e*f^2 + (4*b^2*c*d^4*e^3)/a^2 - (16*b^2*c^2*d^3*e^
2*f)/a^2 + (8*b*c*d^4*e^2*f)/a) + (8*c^(1/2)*d^4*e^2*f*(c + d*x)^(1/2))/(8*
c*d^4*e^2*f + (4*b*c*d^4*e^3)/a - (16*b*c^2*d^3*e^2*f)/a + (4*a*c*d^4*e*f^2
)/b) + (4*b*c^(1/2)*d^4*e^3*(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2
*f - 16*b*c^2*d^3*e^2*f + (4*a^2*c*d^4*e*f^2)/b) - (16*b*c^(3/2)*d^3*e^2*f*
(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2*f - 16*b*c^2*d^3*e^2*f + (4
*a^2*c*d^4*e*f^2)/b)))/a^2 - ((a*d*f - b*d*e)*(c + d*x)^(1/2))/(a*b*(a*d -
b*c + b*(c + d*x)))
```

3.14 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3} dx = \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{(4b^2ce-3abde-a^2df)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3} + \frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*(a^3*d^2*f+3*a^2*b*d^2*e-12*a*b^2*c*d*e+8*b^3*c^2*e)*arctanh(b^(1/2)*(d
*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/b^(3/2)/(-a*d+b*c)^(3/2)-2*e*arctanh((d*x
+c)^(1/2)/c^(1/2))*c^(1/2)/a^3+1/2*(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)^2+1
/4*(-a^2*d*f-3*a*b*d*e+4*b^2*c*e)*(d*x+c)^(1/2)/a^2/b/(-a*d+b*c)/(b*x+a)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {154, 156, 162, 65, 214}

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3} dx = -\frac{2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3} + \frac{\sqrt{c+dx}(a^2(-d)f-3abde+4b^2ce)}{4a^2b(a+bx)(bc-ad)} + \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e)}{4a^3b^{3/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2}$$

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(2*a*b*(a + b*x)^2) + ((4*b^2*c*e - 3*a*b*d*e -
 a^2*d*f)*Sqrt[c + d*x])/(4*a^2*b*(b*c - a*d)*(a + b*x)) - (2*Sqrt[c]*e*Arc
Tanh[Sqrt[c + d*x]/Sqrt[c]])/a^3 + ((8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b
*d^2*e + a^3*d^2*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*a^
3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} - \frac{\int \frac{-2bce-\frac{1}{2}d(3be+af)x}{x(a+bx)^2\sqrt{c+dx}}\,dx}{2ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{(4b^2ce-3abde-a^2df)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{\int \frac{-2bc(bc-ad)e-\frac{1}{4}d(4b^2ce-ad(3be+af))x}{x(a+bx)\sqrt{c+dx}}\,dx}{2a^2b(bc-ad)}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{(4b^2ce-3abde-a^2df)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^3} - \frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8a^3b(bc-ad)}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{(4b^2ce-3abde-a^2df)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(2ce)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^3d} - \frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{4a^3bd(bc-ad)}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{(4b^2ce-3abde-a^2df)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{2\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3} + \frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \frac{\frac{a\sqrt{c+dx}(a^3df+4b^3cex+3ab^2e(2c-dx)-a^2b(5de+2cf+dfx))}{b(bc-ad)(a+bx)^2} + \frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}(-bc+ad)^{3/2}} - 8\sqrt{c}e\text{arctanh}}{4a^3}$$

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] ((a*Sqrt[c + d*x]*(a^3*d*f + 4*b^3*c*e*x + 3*a*b^2*e*(2*c - d*x) - a^2*b*(5
*d*e + 2*c*f + d*f*x)))/(b*(b*c - a*d)*(a + b*x)^2) + ((8*b^3*c^2*e - 12*a*
b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[
-(b*c) + a*d]])/(b^(3/2)*(-(b*c) + a*d)^(3/2)) - 8*Sqrt[c]*e*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(4*a^3)
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{(bx+a)^2\left(a^3d^2f+3a^2b\,d^2e-12a\,b^2cde+8b^3c^2e\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{4}+2\sqrt{(ad-bc)b}\left((bx+a)^2eb\left(c^{\frac{3}{2}}b-ad\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right.\right.}{\sqrt{(ad-bc)b}\,a^3(bx+a)^2(ad-bc)b}$
derivativedivides	$2d^2\left(-\frac{e\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^2a^3}+\frac{\frac{\frac{ad\left(a^2df+3abde-4b^2ce\right)(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(a^2df-5abde+4b^2ce\right)ad\sqrt{dx+c}}{8b}}{((dx+c)b+ad-bc)^2}+\frac{\left(a^3d^2f+3a^2b\,d^2e-12\right.}{8(}}{a^3d^2}\right.$
default	$2d^2\left(-\frac{e\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^2a^3}+\frac{\frac{\frac{ad\left(a^2df+3abde-4b^2ce\right)(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(a^2df-5abde+4b^2ce\right)ad\sqrt{dx+c}}{8b}}{((dx+c)b+ad-bc)^2}+\frac{\left(a^3d^2f+3a^2b\,d^2e-12\right.}{8(}}{a^3d^2}\right.$

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/((a*d-b*c)*b)^(1/2)*(1/8*(b*x+a)^2*(a^3*d^2*f+3*a^2*b*d^2*e-12*a*b^2*c*d*
e+8*b^3*c^2*e)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1
/2)*((b*x+a)^2*e*b*(c^(3/2)*b-a*d*c^(1/2))*arctanh((d*x+c)^(1/2)/c^(1/2))-1
/8*(a^3*d*f-2*b*(5/2*d*e+f*(1/2*d*x+c))*a^2+6*e*(-1/2*d*x+c)*a*b^2+4*b^3*c*
e*x)*(d*x+c)^(1/2)*a))/a^3/(b*x+a)^2/(a*d-b*c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(182) = 364$.

Time = 0.67 (sec) , antiderivative size = 2216, normalized size of antiderivative = 10.65

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*
d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d
^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^2*c - a*
b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x +
 a)) - 8*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^
2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*
e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*((6*a^2*b^4*c^2
 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5
*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d
```

```
 - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2
*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2
*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*
c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*
d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3
*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x +
 c)/(b*d*x + b*c)) - 4*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*
b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - ((6
*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b
^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e -
(a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c
*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4
*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), 1/8*(16*((b^6*c^2 - 2*a*b^5*c*d
 + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (
a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x + c)
*sqrt(-c)/c) - (a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*
a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*
(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a
^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d +
 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b
^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*
d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2
+ (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*
b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - 8*((b^6*c^2 - 2*a*b^5*
c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x
+ (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x +
 c)*sqrt(-c)/c) - ((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*
a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d
+ 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*
b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*
b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx$$
$$= -\frac{(8\,b^3c^2e - 12\,ab^2cde + 3\,a^2bd^2e + a^3d^2f)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(a^3b^2c - a^4bd)\sqrt{-b^2c+abd}} + \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{a^3\sqrt{-c}}$$
$$+ \frac{4\,(dx+c)^{\frac{3}{2}}b^3cde - 4\,\sqrt{dx+c}b^3c^2de - 3\,(dx+c)^{\frac{3}{2}}ab^2d^2e + 9\,\sqrt{dx+c}ab^2cd^2e - 5\,\sqrt{dx+c}a^2bd^3e - (dx -}{4\,(a^2b^2c - a^3bd)((dx+c)b - bc + ad)^2}$$

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*arctan(sqrt
(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c - a^4*b*d)*sqrt(-b^2*c + a*b*
d)) + 2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/(a^3*sqrt(-c)) + 1/4*(4*(d*x + c
)^(3/2)*b^3*c*d*e - 4*sqrt(d*x + c)*b^3*c^2*d*e - 3*(d*x + c)^(3/2)*a*b^2*d
^2*e + 9*sqrt(d*x + c)*a*b^2*c*d^2*e - 5*sqrt(d*x + c)*a^2*b*d^3*e - (d*x +
 c)^(3/2)*a^2*b*d^2*f - sqrt(d*x + c)*a^2*b*c*d^2*f + sqrt(d*x + c)*a^3*d^3
*f)/((a^2*b^2*c - a^3*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 4852, normalized size of antiderivative = 23.33

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^3),x)
[Out] (c^(1/2)*e*atan(((c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*
e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320
*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4
*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + (c^(1/2)*e
*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^
2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c
^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*
c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2
*c*d))))/a^3)*1i)/a^3 + (c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b
^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^
2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f -
 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - (c^
(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7
*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*
d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a
^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*
a^5*b^2*c*d))))/a^3)*1i)/a^3)/(((a^5*c*d^6*e*f^2)/4 - 12*a^2*b^3*c^2*d^5*e^
3 - 8*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2*c*d^6*e^3)/4 + 2*
a^2*b^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4*b*c*d^6*e^2*f)/2)/
(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(((c + d*x)^(1/2)*(a
^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 25
6*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a
^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2
*a^5*b^2*c*d)) + (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b
^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^
3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a
^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^
2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3 - (c^(1/2)*e*(((c + d*x)^(1/2
)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f
+ 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 +
16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2
 - 2*a^5*b^2*c*d)) - (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a
^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^
6*b^3*c^2 - 2*a^7*b^2*c*d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 2
56*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*
b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3))*2i)/a^3 - (((c + d*x)^(
```

```
1/2)*(a^2*d^2*f - 5*a*b*d^2*e + 4*b^2*c*d*e))/(4*a^2*b) - ((c + d*x)^(3/2)*
(a^2*d^2*f + 3*a*b*d^2*e - 4*b^2*c*d*e))/(4*a^2*(a*d - b*c)))/(b^2*(c + d*x
)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (ata
n((((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d
^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*
a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - ((-b^3*
(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*
d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 -
 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e
+ a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4
*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b
^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/
(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3
*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 -
a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(
1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e
^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72
*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^
6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*((5*a
^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*
e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + ((-b^3*(
a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*
e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d
^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a
^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*
e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*
d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2
*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^
2*d + 3*a^5*b^4*c*d^2)))/(((a^5*c*d^6*e*f^2)/4 - 12*a^2*b^3*c^2*d^5*e^3 - 8
*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2*c*d^6*e^3)/4 + 2*a^2*b
^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4*b*c*d^6*e^2*f)/2)/(a^8*
b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d
*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*
d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^
5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4
*b^3*c^2 - 2*a^5*b^2*c*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*
e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c
^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)
^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*
c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*
b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 -
a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f
 + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b
```

```
^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12
*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*
c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^
4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4
*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*
f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) +
((-b^3*(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b
^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^
3*c^2 - 2*a^7*b^2*c*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3
*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*
a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2
+ a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d
 + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c
*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))
*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^
3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2))))*(-b^3*(a*d - b*c)^3
)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(4*(
a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2))
```

3.15 $\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 226

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx \\
&= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} \\
&\quad - \frac{2(a+bx)^{3/2}\left(2(8a^3d^3f-12a^2bd^2(de+3cf)-5b^3c^2(27de+4cf)+3ab^2cd(21de+16cf))-3bd(21b^2cde\right.}{315b^4} \\
&\quad - 2\sqrt{a}c^3e\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

```
[Out] 2/21*(-2*a*d*f+2*b*c*f+3*b*d*e)*(b*x+a)^(3/2)*(d*x+c)^2/b^2+2/9*f*(b*x+a)^(
3/2)*(d*x+c)^3/b-2/315*(b*x+a)^(3/2)*(16*a^3*d^3*f-24*a^2*b*d^2*(3*c*f+d*e)
-10*b^3*c^2*(4*c*f+27*d*e)+6*a*b^2*c*d*(16*c*f+21*d*e)-3*b*d*(21*b^2*c*d*e+
4*(-a*d+b*c)*(-2*a*d*f+2*b*c*f+3*b*d*e))*x)/b^4-2*c^3*e*arctanh((b*x+a)^(1/
2)/a^(1/2))*a^(1/2)+2*c^3*e*(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{158, 152, 52, 65, 214\}$

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx =$$
$$-\frac{2(a+bx)^{3/2}\left(2(8a^3d^3f-12a^2bd^2(3cf+de)+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de))-3bdx(4(bc\right.}{315b^4}$$
$$-2\sqrt{a}c^3e\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2(a+bx)^{3/2}(c+dx)^2(-2adf+2bcf+3bde)}{21b^2}$$
$$+2c^3e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] 2*c^3*e*Sqrt[a + b*x] + (2*(3*b*d*e + 2*b*c*f - 2*a*d*f)*(a + b*x)^(3/2)*(c
 + d*x)^2)/(21*b^2) + (2*f*(a + b*x)^(3/2)*(c + d*x)^3)/(9*b) - (2*(a + b*x
)^(3/2)*(2*(8*a^3*d^3*f - 12*a^2*b*d^2*(d*e + 3*c*f) - 5*b^3*c^2*(27*d*e +
4*c*f) + 3*a*b^2*c*d*(21*d*e + 16*c*f)) - 3*b*d*(21*b^2*c*d*e + 4*(b*c - a*
d)*(3*b*d*e + 2*b*c*f - 2*a*d*f))*x))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
```

```
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{2\int \frac{\sqrt{a+bx}(c+dx)^2\left(\frac{9bce}{2}+\frac{3}{2}(3bde+2bcf-2adf)x\right)}{x}\,dx}{9b}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{4\int \frac{\sqrt{a+bx}(c+dx)\left(\frac{63}{4}b^2c^2e+\frac{3}{4}(21b^2cde+4(bc-ad)(3bde+2bcf-2adf))x\right)}{x}\,dx}{63b^2}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \frac{2(a+bx)^{3/2}\left(2(8a^3d^3f-12a^2bd^2(de+3cf)-5b^3c^2(27de+4cf)+3ab^2cd(21de+16cf))-3bd\right.}{315b^4} + \left(c^3e\right)\int \frac{\sqrt{a+bx}}{x}\,dx$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \frac{2(a+bx)^{3/2}\left(2(8a^3d^3f-12a^2bd^2(de+3cf)-5b^3c^2(27de+4cf)+3ab^2cd(21de+16cf))-3bd\right.}{315b^4} + \left(ac^3e\right)\int \frac{1}{x\sqrt{a+bx}}\,dx$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$
$$- \frac{2(a+bx)^{3/2}\left(2(8a^3d^3f-12a^2bd^2(de+3cf)-5b^3c^2(27de+4cf)+3ab^2cd(21de+16cf))-3b\right.}{315b^4}$$
$$+ \frac{(2ac^3e)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$
$$- \frac{2(a+bx)^{3/2}\left(2(8a^3d^3f-12a^2bd^2(de+3cf)-5b^3c^2(27de+4cf)+3ab^2cd(21de+16cf))-3b\right.}{315b^4}$$
$$- 2\sqrt{a}c^3e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$= \frac{2\sqrt{a+bx}(-16a^4d^3f+8a^3bd^2(3de+9cf+dfx)-6a^2b^2d(21c^2f+d^2x(2e+fx)+3cd(7e+2fx))+ab^3}{}$$
$$- 2\sqrt{a}c^3e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(-16*a^4*d^3*f + 8*a^3*b*d^2*(3*d*e + 9*c*f + d*f*x) - 6*a
^2*b^2*d*(21*c^2*f + d^2*x*(2*e + f*x) + 3*c*d*(7*e + 2*f*x)) + a*b^3*(105*
c^3*f + 63*c^2*d*(5*e + f*x) + 9*c*d^2*x*(7*e + 3*f*x) + d^3*x^2*(9*e + 5*f
*x)) + b^4*(105*c^3*(3*e + f*x) + 63*c^2*d*x*(5*e + 3*f*x) + 27*c*d^2*x^2*(
7*e + 5*f*x) + 5*d^3*x^3*(9*e + 7*f*x))))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-2\sqrt{a}\,b^4c^3e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{32\left(\frac{9\left(-5\left(\frac{7fx}{9}+e\right)x^3d^3-21\left(\frac{5fx}{7}+e\right)x^2c\,d^2-35x\left(\frac{3fx}{5}+e\right)c^2d-35\left(\frac{fx}{3}+e\right)c^3\right)b^4}{16}-\frac{105\left(\frac{3x^2\left(\frac{5f}{9}\right.}{3}\right.}{}\right.}{}$
derivativedivides	$\frac{2f\,d^3(bx+a)^{\frac{9}{2}}}{9}-\frac{6a\,d^3f(bx+a)^{\frac{7}{2}}}{7}+\frac{6bc\,d^2f(bx+a)^{\frac{7}{2}}}{7}+\frac{2b\,d^3e(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2d^3f(bx+a)^{\frac{5}{2}}}{5}-\frac{12abc\,d^2f(bx+a)^{\frac{5}{2}}}{5}-\frac{4ab\,d^3e(bx+a)^{\frac{5}{2}}}{5}+6$
default	$\frac{2f\,d^3(bx+a)^{\frac{9}{2}}}{9}-\frac{6a\,d^3f(bx+a)^{\frac{7}{2}}}{7}+\frac{6bc\,d^2f(bx+a)^{\frac{7}{2}}}{7}+\frac{2b\,d^3e(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2d^3f(bx+a)^{\frac{5}{2}}}{5}-\frac{12abc\,d^2f(bx+a)^{\frac{5}{2}}}{5}-\frac{4ab\,d^3e(bx+a)^{\frac{5}{2}}}{5}+6$

```
[In] int((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(-315*a^(1/2)*b^4*c^3*e*arctanh((b*x+a)^(1/2)/a^(1/2))-16*(9/16*(-5*(
7/9*f*x+e)*x^3*d^3-21*(5/7*f*x+e)*x^2*c*d^2-35*x*(3/5*f*x+e)*c^2*d-35*(1/3*
f*x+e)*c^3)*b^4-105/16*(3/35*x^2*(5/9*f*x+e)*d^3+3/5*(3/7*f*x+e)*x*c*d^2+3*
(1/5*f*x+e)*c^2*d+f*c^3)*a*b^3+63/8*(2/21*(1/2*f*x+e)*x*d^2+c*(2/7*f*x+e)*d
+c^2*f)*d*a^2*b^2-9/2*(1/3*(1/3*f*x+e)*d+c*f)*d^2*a^3*b+a^4*d^3*f)*(b*x+a)^
(1/2))/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$=\left[\frac{315\,\sqrt{a}b^4c^3e\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(35\,b^4d^3fx^4+5\,(9\,b^4d^3e+(27\,b^4cd^2+ab^3d^3)f)x^3+3\,(3\,(21\,b^4cd^2}{}\right.$$

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*sqrt(a)*b^4*c^3*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(35*b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3
+ 3*(3*(21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2
*b^2*d^3)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*
a^3*b*d^3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4
*d^3)*f + (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*
c^3 + 63*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))
/b^4, 2/315*(315*sqrt(-a)*b^4*c^3*e*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (35*
b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3 + 3*(3*(
21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2*b^2*d^3
)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*a^3*b*d^
```

```
3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4*d^3)*f
+ (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*c^3 + 63
*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 11.44 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$= \begin{cases} \frac{2ac^3e\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2c^3e\sqrt{a+bx} + \frac{2d^3f(a+bx)^{\frac{9}{2}}}{9b^4} + \frac{2(a+bx)^{\frac{7}{2}}(-3ad^3f+3bcd^2f+bd^3e)}{7b^4} + \frac{2(a+bx)^{\frac{5}{2}}\cdot(3a^2d^3f-6abcd^2f-2abd\cdots}{5b^4} \\ \sqrt{a}\left(c^3e\log(x) + c^3fx + 3c^2dex + \frac{d^3fx^4}{4} + \frac{x^3\cdot(3cd^2f+d^3e)}{3} + \frac{x^2\cdot(3c^2df+3cd^2e)}{2}\right) \end{cases}$$

```
[In] integrate((d*x+c)**3*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] Piecewise((2*a*c**3*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**3*e*sqrt
(a + b*x) + 2*d**3*f*(a + b*x)**(9/2)/(9*b**4) + 2*(a + b*x)**(7/2)*(-3*a*d
**3*f + 3*b*c*d**2*f + b*d**3*e)/(7*b**4) + 2*(a + b*x)**(5/2)*(3*a**2*d**3
*f - 6*a*b*c*d**2*f - 2*a*b*d**3*e + 3*b**2*c**2*d*f + 3*b**2*c*d**2*e)/(5*
b**4) + 2*(a + b*x)**(3/2)*(-a**3*d**3*f + 3*a**2*b*c*d**2*f + a**2*b*d**3*
e - 3*a*b**2*c**2*d*f - 3*a*b**2*c*d**2*e + b**3*c**3*f + 3*b**3*c**2*d*e)/
(3*b**4), Ne(b, 0)), (sqrt(a)*(c**3*e*log(x) + c**3*f*x + 3*c**2*d*e*x + d*
*3*f*x**4/4 + x**3*(3*c*d**2*f + d**3*e)/3 + x**2*(3*c**2*d*f + 3*c*d**2*e)
/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx = \sqrt{a}c^3e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)$$
$$+ \frac{2\left(315\sqrt{bx+a}b^4c^3e + 35(bx+a)^{\frac{9}{2}}d^3f + 45(bd^3e+3(bcd^2-ad^3)f)(bx+a)^{\frac{7}{2}} + 63((3b^2cd^2-2abd^3)\cdots\right.}{\cdots}$$

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^3*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
315*(315*sqrt(b*x + a)*b^4*c^3*e + 35*(b*x + a)^(9/2)*d^3*f + 45*(b*d^3*e +
 3*(b*c*d^2 - a*d^3)*f)*(b*x + a)^(7/2) + 63*((3*b^2*c*d^2 - 2*a*b*d^3)*e +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f)*(b*x + a)^(5/2) + 105*((3*b^3*c^2
*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*f)*(b*x + a)^(3/2))/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx = \frac{2\,ac^3e\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(315\,\sqrt{bx+a}b^{36}c^3e + 315\,(bx+a)^{\frac{3}{2}}b^{35}c^2de + 189\,(bx+a)^{\frac{5}{2}}b^{34}cd^2e - 315\,(bx+a)^{\frac{3}{2}}ab^{34}cd^2e + 45\,(bx+\right.}{}$$

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^3*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/315*(315*sqrt(b*x + a
)*b^36*c^3*e + 315*(b*x + a)^(3/2)*b^35*c^2*d*e + 189*(b*x + a)^(5/2)*b^34*
c*d^2*e - 315*(b*x + a)^(3/2)*a*b^34*c*d^2*e + 45*(b*x + a)^(7/2)*b^33*d^3*
e - 126*(b*x + a)^(5/2)*a*b^33*d^3*e + 105*(b*x + a)^(3/2)*a^2*b^33*d^3*e +
 105*(b*x + a)^(3/2)*b^35*c^3*f + 189*(b*x + a)^(5/2)*b^34*c^2*d*f - 315*(b
*x + a)^(3/2)*a*b^34*c^2*d*f + 135*(b*x + a)^(7/2)*b^33*c*d^2*f - 378*(b*x
+ a)^(5/2)*a*b^33*c*d^2*f + 315*(b*x + a)^(3/2)*a^2*b^33*c*d^2*f + 35*(b*x
+ a)^(9/2)*b^32*d^3*f - 135*(b*x + a)^(7/2)*a*b^32*d^3*f + 189*(b*x + a)^(5
/2)*a^2*b^32*d^3*f - 105*(b*x + a)^(3/2)*a^3*b^32*d^3*f)/b^36
```

Mupad [B] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx = \left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{7\,b^4}+\frac{2\,a\,d^3\,f}{7\,b^4}\right)\,{\left(a+b\,x\right)}^{7/2} + \left(\frac{a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)}{5} - \frac{6\,d\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{5\,b^4}\right)\,{\left(a+b\,x\right)}^{5/2} + \left(a\,\left(a\,\left(a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)-\frac{6\,d\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{b^4}\right)+\frac{2\,(a\,d-b\right.\right. + \left(\frac{a\,\left(a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)-\frac{6\,d\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{b^4}\right)}{3}+\frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(b\,c\,f-4\,a\,d\,f+3\,b\,d\right.}{3\,b^4}\right. + \frac{2\,d^3\,f\,{\left(a+b\,x\right)}^{9/2}}{9\,b^4}+\sqrt{a}\,c^3\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^3)/x,x)
```

```
[Out] ((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/(7*b^4) + (2*a*d^3*f)/(7*b^4))*(a +
b*x)^(7/2) + ((a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4))/5 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/(5*b^4))*(a + b*x)^(5/
2) + (a*(a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b^4)
 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4) + (2*(a*d - b*c)^2*(b*c
*f - 4*a*d*f + 3*b*d*e))/b^4) + (2*(a*d - b*c)^3*(a*f - b*e))/b^4)*(a + b*x
)^(1/2) + ((a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4) - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4))/3 + (2*(a*d - b*c)^
2*(b*c*f - 4*a*d*f + 3*b*d*e))/(3*b^4))*(a + b*x)^(3/2) + a^(1/2)*c^3*e*ata
n(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*d^3*f*(a + b*x)^(9/2))/(9*b^4)
```

3.16 $\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x} dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x} dx = 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f - 7abd(de+2cf) + 5b^2c(7de+2cf)) + 3bd(7bde+4bcf-4adf)x\right)}{105b^3} - 2\sqrt{a}c^2e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/7*f*(b*x+a)^(3/2)*(d*x+c)^2/b+2/105*(b*x+a)^(3/2)*(8*a^2*d^2*f-14*a*b*d*(
2*c*f+d*e)+10*b^2*c*(2*c*f+7*d*e)+3*b*d*(-4*a*d*f+4*b*c*f+7*b*d*e)*x)/b^3-2
*c^2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*c^2*e*(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {158, 152, 52, 65, 214}

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x} dx = \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f - 7abd(2cf+de) + 5b^2c(2cf+7de)) + 3bdx(-4adf+4bcf+7bde)\right)}{105b^3} - 2\sqrt{a}c^2e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] 2*c^2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + (2*(a + b
*x)^(3/2)*(2*(4*a^2*d^2*f - 7*a*b*d*(d*e + 2*c*f) + 5*b^2*c*(7*d*e + 2*c*f)
) + 3*b*d*(7*b*d*e + 4*b*c*f - 4*a*d*f)*x))/(105*b^3) - 2*Sqrt[a]*c^2*e*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2\int \frac{\sqrt{a+bx}(c+dx)\left(\frac{7bce}{2}+\frac{1}{2}(7bde+4bcf-4adf)x\right)}{x}\,dx}{7b}$$

$$= \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7abd(de+2cf)+5b^2c(7de+2cf))+3bd(7bde+4bcf-4adf)x\right)}{105b^3} + \left(c^2e\right)\int \frac{\sqrt{a+bx}}{x}\,dx$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7abd(de+2cf)+5b^2c(7de+2cf))+3bd(7bde+4bcf-4adf)x\right)}{105b^3} + \left(ac^2e\right)\int \frac{1}{x\sqrt{a+bx}}\,dx$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7abd(de+2cf)+5b^2c(7de+2cf))+3bd(7bde+4bcf-4adf)x\right)}{105b^3} + \frac{\left(2ac^2e\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7abd(de+2cf)+5b^2c(7de+2cf))+3bd(7bde+4bcf-4adf)x\right)}{105b^3} - 2\sqrt{a}c^2e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \frac{2\sqrt{a+bx}(8a^3d^2f-2a^2bd(7de+14cf+2dfx)+ab^2(35c^2f+14cd(5e+fx)+d^2x(7e+3fx))+b^3(35c^2}{105b^3} - 2\sqrt{a}c^2e\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^3*d^2*f - 2*a^2*b*d*(7*d*e + 14*c*f + 2*d*f*x) + a*b^
2*(35*c^2*f + 14*c*d*(5*e + f*x) + d^2*x*(7*e + 3*f*x)) + b^3*(35*c^2*(3*e
+ f*x) + 14*c*d*x*(5*e + 3*f*x) + 3*d^2*x^2*(7*e + 5*f*x))))/(105*b^3) - 2*
Sqrt[a]*c^2*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-210\sqrt{a}\,b^3c^2e\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+16\sqrt{bx+a}\left(\left(\frac{21\left(\frac{5fx}{7}+e\right)x^2d^2}{8}+\frac{35x\left(\frac{3fx}{5}+e\right)cd}{4}+\frac{105\left(\frac{fx}{3}+e\right)c^2}{8}\right)b^3+\frac{35\left(\frac{\left(\frac{3fx}{7}+e\right)x\,d^2}{5}\right.}{}\right.}{105b^3}$
derivativedivides	$\frac{\frac{2d^2f(bx+a)^{\frac{7}{2}}}{7}-\frac{4a\,d^2f(bx+a)^{\frac{5}{2}}}{5}+\frac{4bcdf(bx+a)^{\frac{5}{2}}}{5}+\frac{2b\,d^2e(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(bx+a)^{\frac{3}{2}}}{3}-\frac{4abcdf(bx+a)^{\frac{3}{2}}}{3}-\frac{2ab\,d^2e(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^2c}{}}{b^3}$
default	$\frac{\frac{2d^2f(bx+a)^{\frac{7}{2}}}{7}-\frac{4a\,d^2f(bx+a)^{\frac{5}{2}}}{5}+\frac{4bcdf(bx+a)^{\frac{5}{2}}}{5}+\frac{2b\,d^2e(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(bx+a)^{\frac{3}{2}}}{3}-\frac{4abcdf(bx+a)^{\frac{3}{2}}}{3}-\frac{2ab\,d^2e(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^2c}{}}{b^3}$

```
[In] int((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(-210*a^(1/2)*b^3*c^2*e*arctanh((b*x+a)^(1/2)/a^(1/2))+16*(b*x+a)^(1/
2)*((21/8*(5/7*f*x+e)*x^2*d^2+35/4*x*(3/5*f*x+e)*c*d+105/8*(1/3*f*x+e)*c^2)
*b^3+35/8*(1/5*(3/7*f*x+e)*x*d^2+2*(1/5*f*x+e)*c*d+c^2*f)*a*b^2-7/2*((1/7*f
*x+1/2*e)*d+c*f)*d*a^2*b+a^3*d^2*f))/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx$$

$$= \left[\frac{105\sqrt{a}b^3c^2e\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15b^3d^2fx^3+3\left(7b^3d^2e+(14b^3cd+ab^2d^2)f\right)x^2+7\left(15b^3c^2+10\right.\right.}{}\right.$$

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(a)*b^3*c^2*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*b^3*c*d + a*b^2*d^2)*f)*x^2 +
7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e + (35*a*b^2*c^2 - 28*a^2*b*c*
d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2)*e + (35*b^3*c^2 + 14*a*b^2*c
*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3, 2/105*(105*sqrt(-a)*b^3*c^2*e*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*
b^3*c*d + a*b^2*d^2)*f)*x^2 + 7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e
 + (35*a*b^2*c^2 - 28*a^2*b*c*d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2
)*e + (35*b^3*c^2 + 14*a*b^2*c*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx$$

$$= \begin{cases} \frac{2ac^2e\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2c^2e\sqrt{a+bx}+\frac{2d^2f(a+bx)^{\frac{7}{2}}}{7b^3}+\frac{2(a+bx)^{\frac{5}{2}}\left(-2ad^2f+2bcdf+bd^2e\right)}{5b^3}+\frac{2(a+bx)^{\frac{3}{2}}\left(a^2d^2f-2abcdf-abd^2e+b^2c^2\right.}{3b^3} \\ \sqrt{a}\left(c^2e\log(x)+c^2fx+2cdex+\frac{d^2fx^3}{3}+\frac{x^2\cdot(2cdf+d^2e)}{2}\right) \end{cases}$$

```
[In] integrate((d*x+c)**2*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] Piecewise((2*a*c**2*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**2*e*sqrt
(a + b*x) + 2*d**2*f*(a + b*x)**(7/2)/(7*b**3) + 2*(a + b*x)**(5/2)*(-2*a*d
**2*f + 2*b*c*d*f + b*d**2*e)/(5*b**3) + 2*(a + b*x)**(3/2)*(a**2*d**2*f -
2*a*b*c*d*f - a*b*d**2*e + b**2*c**2*f + 2*b**2*c*d*e)/(3*b**3), Ne(b, 0)),
 (sqrt(a)*(c**2*e*log(x) + c**2*f*x + 2*c*d*e*x + d**2*f*x**3/3 + x**2*(2*c
*d*f + d**2*e)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \sqrt{a}c^2e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2\left(105\sqrt{bx+a}b^3c^2e+15\,(bx+a)^{\frac{7}{2}}d^2f+21\,(bd^2e+2\,(bcd-ad^2)f)(bx+a)^{\frac{5}{2}}+35\,((2\,b^2cd-abd^2)e+\right.}{105\,b^3}$$

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^2*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
105*(105*sqrt(b*x + a)*b^3*c^2*e + 15*(b*x + a)^(7/2)*d^2*f + 21*(b*d^2*e +
 2*(b*c*d - a*d^2)*f)*(b*x + a)^(5/2) + 35*((2*b^2*c*d - a*b*d^2)*e + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(b*x + a)^(3/2))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \frac{2\,ac^2e\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(105\sqrt{bx+a}b^{21}c^2e+70\,(bx+a)^{\frac{3}{2}}b^{20}cde+21\,(bx+a)^{\frac{5}{2}}b^{19}d^2e-35\,(bx+a)^{\frac{3}{2}}ab^{19}d^2e+35\,(bx+a)^{\frac{3}{2}}b\right.}{}$$

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^2*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/105*(105*sqrt(b*x + a
)*b^21*c^2*e + 70*(b*x + a)^(3/2)*b^20*c*d*e + 21*(b*x + a)^(5/2)*b^19*d^2*
e - 35*(b*x + a)^(3/2)*a*b^19*d^2*e + 35*(b*x + a)^(3/2)*b^20*c^2*f + 42*(b
*x + a)^(5/2)*b^19*c*d*f - 70*(b*x + a)^(3/2)*a*b^19*c*d*f + 15*(b*x + a)^(
7/2)*b^18*d^2*f - 42*(b*x + a)^(5/2)*a*b^18*d^2*f + 35*(b*x + a)^(3/2)*a^2*
b^18*d^2*f)/b^21
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{5\,b^3}+\frac{2\,a\,d^2\,f}{5\,b^3}\right)\,{\left(a+b\,x\right)}^{5/2} + \left(a\,\left(a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)-\frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{b^3}\right) - \frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(a\,f-b\,e\right)}{b^3}\right)\,\sqrt{a+b\,x} + \left(\frac{a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)}{3}-\frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{3\,b^3}\right)\,{\left(a+b\,x\right)}^{3/2} + \frac{2\,d^2\,f\,{\left(a+b\,x\right)}^{7/2}}{7\,b^3}+\sqrt{a}\,c^2\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^2)/x,x)
```

```
[Out] ((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/(5*b^3) + (2*a*d^2*f)/(5*b^3))*(a + b*
x)^(5/2) + (a*(a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b^3 + (2*a*d^2*f)/b^3
) - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/b^3) - (2*(a*d - b*c)^2*(a*
f - b*e))/b^3)*(a + b*x)^(1/2) + ((a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b
^3 + (2*a*d^2*f)/b^3))/3 - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/(3*b
^3))*(a + b*x)^(3/2) + a^(1/2)*c^2*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
+ (2*d^2*f*(a + b*x)^(7/2))/(7*b^3)
```

3.17 $\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x} dx = 2ce\sqrt{a+bx} - \frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} - 2\sqrt{a}ce\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2/15*(b*x+a)^(3/2)*(2*a*d*f-5*b*(c*f+d*e)-3*b*d*f*x)/b^2-2*c*e*arctanh((b*
x+a)^(1/2)/a^(1/2))*a^(1/2)+2*c*e*(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {152, 52, 65, 214}

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x} dx = -2\sqrt{a}ce\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) - \frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2} + 2ce\sqrt{a+bx}$$

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] - (2*(a + b*x)^(3/2)*(2*a*d*f - 5*b*(d*e + c*f) - 3*b*d
*f*x))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+(ce)\int\frac{\sqrt{a+bx}}{x}\,dx$$

$$=2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+(ace)\int\frac{1}{x\sqrt{a+bx}}\,dx$$

$$=2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+\frac{(2ace)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}$$

$$=2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}-2\sqrt{a}ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx = \frac{2\sqrt{a+bx}(15b^2ce+5bde(a+bx)+5bcf(a+bx)-5adf(a+bx)+3df(a+bx)^2)}{15b^2} - 2\sqrt{a}ce\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(15*b^2*c*e + 5*b*d*e*(a + b*x) + 5*b*c*f*(a + b*x) - 5*a*
d*f*(a + b*x) + 3*d*f*(a + b*x)^2))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

method	result	size
pseudoelliptic	$\frac{-2\sqrt{a}\,b^2ce\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{4\sqrt{bx+a}\left(\frac{5\left(-x\left(\frac{3fx}{5}+e\right)d-3\left(\frac{fx}{3}+e\right)c\right)b^2}{2}-\frac{5\left(\left(\frac{fx}{5}+e\right)d+cf\right)ab}{2}+a^2df\right)}{15}}{b^2}$	86
derivativedivides	$\frac{\frac{2df(bx+a)^{\frac{5}{2}}}{5}-\frac{2adf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bcf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bde(bx+a)^{\frac{3}{2}}}{3}+2b^2ce\sqrt{bx+a}-2\sqrt{a}\,b^2ce\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2}$	89
default	$\frac{\frac{2df(bx+a)^{\frac{5}{2}}}{5}-\frac{2adf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bcf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bde(bx+a)^{\frac{3}{2}}}{3}+2b^2ce\sqrt{bx+a}-2\sqrt{a}\,b^2ce\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2}$	89

```
[In] int((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(-15*a^(1/2)*b^2*c*e*arctanh((b*x+a)^(1/2)/a^(1/2))-2*(b*x+a)^(1/2)*(5
/2*(-x*(3/5*f*x+e)*d-3*(1/3*f*x+e)*c)*b^2-5/2*((1/5*f*x+e)*d+c*f)*a*b+a^2*d
*f))/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$$
$$= \left[\frac{15\sqrt{a}b^2ce\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3b^2dfx^2+5\left(3b^2c+abd\right)e+\left(5abc-2a^2d\right)f+\left(5b^2de+\left(5b^2c+c\right.\right.\right.}{15b^2}\right.$$

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(a)*b^2*c*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e
 + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/b^2, 2/15*(15*sqrt(-a)*b^2*c*e*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (
5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/
b^2]
```

Sympy [A] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$$
$$= \begin{cases} \frac{2ace\,\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2ce\sqrt{a+bx}+\frac{2df(a+bx)^{\frac{5}{2}}}{5b^2}+\frac{2(a+bx)^{\frac{3}{2}}(-adf+bcf+bde)}{3b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(ce\log(x)+cfx+dex+\frac{dfx^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] Piecewise((2*a*c*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c*e*sqrt(a + b
*x) + 2*d*f*(a + b*x)**(5/2)/(5*b**2) + 2*(a + b*x)**(3/2)*(-a*d*f + b*c*f
+ b*d*e)/(3*b**2), Ne(b, 0)), (sqrt(a)*(c*e*log(x) + c*f*x + d*e*x + d*f*x*
*2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$$
$$=\sqrt{a}ce\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)$$
$$+\frac{2\left(15\sqrt{bx+a}b^2ce+3(bx+a)^{\frac{5}{2}}df+5(bde+(bc-ad)f)(bx+a)^{\frac{3}{2}}\right)}{15\,b^2}$$

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/15
*(15*sqrt(b*x + a)*b^2*c*e + 3*(b*x + a)^(5/2)*d*f + 5*(b*d*e + (b*c - a*d)
*f)*(b*x + a)^(3/2))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx=\frac{2\,ace\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$+\frac{2\left(15\sqrt{bx+a}b^{10}ce+5(bx+a)^{\frac{3}{2}}b^9de+5(bx+a)^{\frac{3}{2}}b^9cf+3(bx+a)^{\frac{5}{2}}b^8df-5(bx+a)^{\frac{3}{2}}ab^8df\right)}{15\,b^{10}}$$

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/15*(15*sqrt(b*x + a)*b^
10*c*e + 5*(b*x + a)^(3/2)*b^9*d*e + 5*(b*x + a)^(3/2)*b^9*c*f + 3*(b*x + a
)^(5/2)*b^8*d*f - 5*(b*x + a)^(3/2)*a*b^8*d*f)/b^10
```

Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx = \left(a\left(\frac{2\,b\,c\,f-4\,a\,d\,f+2\,b\,d\,e}{b^2}+\frac{2\,a\,d\,f}{b^2}\right)+\frac{2\,(a\,d-b\,c)\,(a\,f-b\,e)}{b^2}\right)\sqrt{a+b\,x}+\left(\frac{2\,b\,c\,f-4\,a\,d\,f+2\,b\,d\,e}{3\,b^2}+\frac{2\,a\,d\,f}{3\,b^2}\right)(a+b\,x)^{3/2}+\frac{2\,d\,f\,(a+b\,x)^{5/2}}{5\,b^2}+\sqrt{a}\,c\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x))/x,x)
```

```
[Out] (a*((2*b*c*f - 4*a*d*f + 2*b*d*e)/b^2 + (2*a*d*f)/b^2) + (2*(a*d - b*c)*(a*
f - b*e))/b^2)*(a + b*x)^(1/2) + ((2*b*c*f - 4*a*d*f + 2*b*d*e)/(3*b^2) + (
2*a*d*f)/(3*b^2))*(a + b*x)^(3/2) + (2*d*f*(a + b*x)^(5/2))/(5*b^2) + a^(1/
2)*c*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

3.18 $\int \frac{\sqrt{a+bx}(e+fx)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{\sqrt{a+bx}(e+fx)}{x} dx = 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} - 2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*f*(b*x+a)^(3/2)/b-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*e*(b*x+a
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {81, 52, 65, 214}

$$\int \frac{\sqrt{a+bx}(e+fx)}{x} dx = -2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b}$$

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]]
```

<u>Rule 52</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2f(a+bx)^{3/2}}{3b} + e\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} + (ae)\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b} \\
&= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} - 2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \frac{2\sqrt{a+bx}(3be+af+bfx)}{3b} - 2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(3*b*e + a*f + b*f*x))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{2f(bx+a)^{\frac{3}{2}}}{3}+2be\sqrt{bx+a}-2\sqrt{a}\,be\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46
default	$\frac{\frac{2f(bx+a)^{\frac{3}{2}}}{3}+2be\sqrt{bx+a}-2\sqrt{a}\,be\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46
pseudoelliptic	$\frac{-6\sqrt{a}\,be\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2((fx+3e)b+af)\sqrt{bx+a}}{3b}$	48

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/b*(1/3*f*(b*x+a)^(3/2)+b*e*(b*x+a)^(1/2)-a^(1/2)*b*e*arctanh((b*x+a)^(1/2
)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx$$
$$= \left[\frac{3\sqrt{a}be\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(bfx+3be+af)\sqrt{bx+a}}{3b}, \frac{2\left(3\sqrt{-a}be\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(bfx+\right.}{3b}\right.$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
[Out] [1/3*(3*sqrt(a)*b*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(b*f*x
 + 3*b*e + a*f)*sqrt(b*x + a))/b, 2/3*(3*sqrt(-a)*b*e*arctan(sqrt(b*x + a)*
sqrt(-a)/a) + (b*f*x + 3*b*e + a*f)*sqrt(b*x + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \begin{cases} \frac{2ae\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2e\sqrt{a+bx}+\frac{2f(a+bx)^{\frac{3}{2}}}{3b} & \text{for } b\neq 0 \\ \sqrt{a}(e\log(fx)+fx) & \text{otherwise} \end{cases}$$

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x,x)
[Out] Piecewise((2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*e*sqrt(a + b*x)
+ 2*f*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*(e*log(f*x) + f*x), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \sqrt{a}e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2\left(3\sqrt{bx+a}be+(bx+a)^{\frac{3}{2}}f\right)}{3\,b}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(3
*sqrt(b*x + a)*b*e + (b*x + a)^(3/2)*f)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(3\sqrt{bx+a}b^3e+(bx+a)^{\frac{3}{2}}b^2f\right)}{3\,b^3}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(3*sqrt(b*x + a)*b^3*e
+ (b*x + a)^(3/2)*b^2*f)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = 2\,e\,\sqrt{a+b\,x} + \frac{2\,f\,{(a+b\,x)}^{3/2}}{3\,b} + \sqrt{a}\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/x,x)
```

```
[Out] 2*e*(a + b*x)^(1/2) + a^(1/2)*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*
f*(a + b*x)^(3/2))/(3*b)
```

3.19 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \frac{2f\sqrt{a+bx}}{d} + \frac{2\sqrt{bc-ad}(de-cf)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

```
[Out] -2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c+2*(-c*f+d*e)*arctan(d^(1/2)*(
b*x+a)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c/d^(3/2)+2*f*(b*x+a)^(1/2)
/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {159, 162, 65, 214, 211}

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \frac{2\sqrt{bc-ad}(de-cf)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c} + \frac{2f\sqrt{a+bx}}{d}$$

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d + (2*Sqrt[b*c - a*d]*(d*e - c*f)*ArcTan[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2f\sqrt{a+bx}}{d}+\frac{2\int \frac{\frac{ade}{2}+\frac{1}{2}(bde-bcf+adf)x}{x\sqrt{a+bx}(c+dx)}\,dx}{d}\\
&= \frac{2f\sqrt{a+bx}}{d}+\frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c}+\frac{((bc-ad)(de-cf))\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx}{cd}
\end{aligned}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc} + \frac{(2(bc-ad)(de-cf))\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bcd}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{2\sqrt{bc-ad}(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \frac{2f\sqrt{a+bx}}{d} - \frac{2\sqrt{bc-ad}(-de+cf)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d - (2*Sqrt[b*c - a*d]*(-(d*e) + c*f)*ArcTan[(Sqrt[d]*S
qrt[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/c
```

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{2f\sqrt{bx+a}}{d} - \frac{2e\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}}{c} - \frac{2\left(acdf-ae\,d^2-c^2bf+bcde\right)\text{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{dc\sqrt{(ad-bc)d}}$	103
default	$\frac{2f\sqrt{bx+a}}{d} - \frac{2e\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}}{c} - \frac{2\left(acdf-ae\,d^2-c^2bf+bcde\right)\text{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{dc\sqrt{(ad-bc)d}}$	103
pseudoelliptic	$\frac{-2(cf-de)(ad-bc)\,\text{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)+2\left(-\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\,de+\sqrt{bx+a}\,cf\right)\sqrt{(ad-bc)d}}{dc\sqrt{(ad-bc)d}}$	105

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(b*x+a)^(1/2)/d-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c-2/d*(a*c*d
*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/((a*d-b*c)*d)^(1/2)*arctanh(d*(b*x+a)^(1/2)/(
(a*d-b*c)*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.46

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{a}de\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}cf-(de-cf)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{bdx-bc+2ad-2\sqrt{bx+a}d\sqrt{-\frac{bc-ad}{d}}}{dx+c}\right)}{cd}, \frac{2\sqrt{-\ldots}}{\ldots}\right.$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a
)*c*f - (d*e - c*f)*sqrt(-(b*c - a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(
b*x + a)*d*sqrt(-(b*c - a*d)/d))/(d*x + c)))/(c*d), (2*sqrt(-a)*d*e*arctan(
sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt(-(b*c -
a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(b*x + a)*d*sqrt(-(b*c - a*d)/d))/
(d*x + c)))/(c*d), (sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x
) + 2*sqrt(b*x + a)*c*f - 2*(d*e - c*f)*sqrt((b*c - a*d)/d)*arctan(-sqrt(b*
x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d), 2*(sqrt(-a)*d*e*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt((b*c - a*d)/
d)*arctan(-sqrt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(90) = 180$.

Time = 12.59 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$$

$$= \begin{cases} \frac{2ae\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{c\sqrt{-a}}+\frac{2f\sqrt{a+bx}}{d}+\frac{2(ad-bc)(cf-de)\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-\frac{ad-bc}{d}}}\right)}{cd^2\sqrt{-\frac{ad-bc}{d}}} \\ \sqrt{a}\left(\left(-f+\frac{de}{2c}\right)\left(\frac{2c\left(\begin{cases}-\frac{\frac{1}{x}+\frac{d}{2c}}{d} & \text{for } c=0\\ \frac{\log\left(2c\left(\frac{1}{x}+\frac{d}{2c}\right)-d\right)}{2c} & \text{otherwise}\end{cases}\right)}{d}-\frac{2c\left(\begin{cases}\frac{\frac{1}{x}+\frac{d}{2c}}{d} & \text{for } c=0\\ \frac{\log\left(2c\left(\frac{1}{x}+\frac{d}{2c}\right)+d\right)}{2c} & \text{otherwise}\end{cases}\right)}{d}\right)-\frac{e\log\left(\frac{c}{x^2}+\frac{d}{x}\ldots\right.}{2c}\ldots\right. \end{cases}$$

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c),x)
```

```
[Out] Piecewise((2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/(c*sqrt(-a)) + 2*f*sqrt(a + b
*x)/d + 2*(a*d - b*c)*(c*f - d*e)*atan(sqrt(a + b*x)/sqrt(-(a*d - b*c)/d))/
(c*d**2*sqrt(-(a*d - b*c)/d)), Ne(b, 0)), (sqrt(a)*((-f + d*e/(2*c))*(2*c*P
iecewise((-(1/x + d/(2*c))/d, Eq(c, 0)), (log(2*c*(1/x + d/(2*c)) - d)/(2*c
), True))/d - 2*c*Piecewise(((1/x + d/(2*c))/d, Eq(c, 0)), (log(2*c*(1/x +
d/(2*c)) + d)/(2*c), True))/d) - e*log(c/x**2 + d/x)/(2*c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c} + \frac{2\,\sqrt{bx+a}f}{d} + \frac{2\,(bcde-ad^2e-bc^2f+acdf)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}cd}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="giac")
```

```
[Out] 2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c) + 2*sqrt(b*x + a)*f/d + 2
*(b*c*d*e - a*d^2*e - b*c^2*f + a*c*d*f)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d
- a*d^2))/(sqrt(b*c*d - a*d^2)*c*d)
```

Mupad [B] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 2355, normalized size of antiderivative = 23.32

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)),x)
```

```
[Out] (2*f*(a + b*x)^(1/2))/d - (a^(1/2)*e*atan(((a^(1/2)*e*((8*(a + b*x)^(1/2)*(
b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*
f - 2*a^2*b^2*c*d^3*e*f))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2
*d^3*f))/d + (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/
(c*d)))/c)*1i)/c + (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*
d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f)
)/d - (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*
e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c)*1i)/c)/((16*(
a^2*b^3*d^3*e^3 - a*b^4*c*d^2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3
*a^2*b^3*c*d^2*e^2*f + 2*a^2*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^
4*c^2*d*e^2*f))/d - (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2
*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f
))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d + (8*a^(1/2)
*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c + (a^(1/2
)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2
- 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d
*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (a^(1/2)*e*((8*(a*b^
3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c
^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c))*2i)/c - (atan(((((8*(a + b*x)^(1/2
)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2
*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f)
)/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2
)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(
c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3) + (((8*(a + b*x)^(1/2)*(b^4*
c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c
^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f -
2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (
8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*(a +
b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(c*f - d
*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3))/((16*(a^2*b^3*d^3*e^3 - a*b^4*c*d^
2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3*a^2*b^3*c*d^2*e^2*f + 2*a^2
*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^4*c^2*d*e^2*f))/d - (((8*(a
```

```
+ b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3
*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*
b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2
*c^2*d^3*f))/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d -
 b*c))^(1/2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))
/(c*d^3))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3) + (((8*(a + b*x)^(1/
2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a
^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^
2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f
))/d - (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/
2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*
(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3)))*(c*f - d*e)*(d^3*(a*d - b*c)
)^(1/2)*2i)/(c*d^3)
```

3.20 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx = \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{(2ad^2e-bc(de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

```
[Out] -2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c^2-(2*a*d^2*e-b*c*(c*f+d*e))*a
rctan(d^(1/2)*(b*x+a)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(3/2)/(-a*d+b*c)^(1/2)+
(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {154, 162, 65, 214, 211}

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx = -\frac{\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)(2ad^2e-bc(cf+de))}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2} + \frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(c*d*(c + d*x)) - ((2*a*d^2*e - b*c*(d*e + c*f)
)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2)*Sqrt[b*c -
a*d]) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\int \frac{-ade-\frac{1}{2}b(de+cf)x}{x\sqrt{a+bx}(c+dx)}\,dx}{cd} \\
&= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^2} - \frac{(2ad^2e-bc(de+cf))\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx}{2c^2d}
\end{aligned}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^2} - \frac{(2ad^2e-bc(de+cf))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^2d}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{(2ad^2e-bc(de+cf))\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \frac{\frac{c(de-cf)\sqrt{a+bx}}{d(c+dx)} + \frac{(-2ad^2e+bc(de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}} - 2\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
[Out] ((c*(d*e - c*f)*Sqrt[a + b*x])/(d*(c + d*x)) + ((-2*a*d^2*e + b*c*(d*e + c*
f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[b*c - a*
d]) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{-2e\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+\frac{-\frac{c(cf-de)\sqrt{bx+a}}{dx+c}+\frac{\left(2ae\,d^2-c^2bf-bcde\right)\text{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{\sqrt{(ad-bc)d}}}{d}}{c^2}$	110
derivativedivides	$2b\left(-\frac{e\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^2}+\frac{\frac{bc(cf-de)\sqrt{bx+a}}{2d(-d(bx+a)+ad-bc)}+\frac{\left(2ae\,d^2-c^2bf-bcde\right)\text{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{2d\sqrt{(ad-bc)d}}}{c^2b}\right)$	137
default	$2b\left(-\frac{e\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^2}+\frac{\frac{bc(cf-de)\sqrt{bx+a}}{2d(-d(bx+a)+ad-bc)}+\frac{\left(2ae\,d^2-c^2bf-bcde\right)\text{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{2d\sqrt{(ad-bc)d}}}{c^2b}\right)$	137

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+1/d*(-c*(c*f-d*e)*(b*x+a
)^(1/2)/(d*x+c)+(2*a*d^2*e-b*c^2*f-b*c*d*e)/((a*d-b*c)*d)^(1/2)*arctanh(d*(
b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(110) = 220$.

Time = 0.32 (sec) , antiderivative size = 1008, normalized size of antiderivative = 7.88

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx$$
$$= \Bigg[-\frac{(bc^3f + (bc^2d - 2\,acd^2)e + (bc^2df + (bcd^2 - 2\,ad^3)e)x)\sqrt{-bcd+ad^2}\log\left(\frac{bdx-bc+2\,ad-2\,\sqrt{-bcd+ad^2}\sqrt{bx+a}}{dx+c}\right)}{2\,(bc^4d^2 - a}$$
$$-\frac{(bc^3f + (bc^2d - 2\,acd^2)e + (bc^2df + (bcd^2 - 2\,ad^3)e)x)\sqrt{bcd-ad^2}\arctan\left(\frac{\sqrt{bcd-ad^2}\sqrt{bx+a}}{bdx+ad}\right) - ((bcd^3 - }{bc^4d^2 - ac^3d^3 + (b}$$
$$-\frac{(bc^3f + (bc^2d - 2\,acd^2)e + (bc^2df + (bcd^2 - 2\,ad^3)e)x)\sqrt{bcd-ad^2}\arctan\left(\frac{\sqrt{bcd-ad^2}\sqrt{bx+a}}{bdx+ad}\right) - 2\,((bcd^3 - }{bc^4d^2 - ac^3d^3 + (b}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c^3*f + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3
)*e)*x)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d
^2)*sqrt(b*x + a))/(d*x + c)) - 2*((b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c
*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*((b*c^2*d
^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x + a))/(b*c^4*d^2 - a*c^
3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), 1/2*(4*((b*c*d^3 - a*d^4)*e*x + (b*c^2*
d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (b*c^3*f + (b
*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(-b*c*d
+ a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(
d*x + c)) + 2*((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x
+ a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f + (b*
c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*c*d -
a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - ((b*c*d^3
- a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*
sqrt(a) + 2*a)/x) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqr
t(b*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f
 + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*
c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 2*((
b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b
*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c^2} + \frac{(bcde-2\,ad^2e+bc^2f)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}c^2d} + \frac{\sqrt{bx+a}bde-\sqrt{bx+a}bcf}{(bc+(bx+a)d-ad)cd}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c^2) + (b*c*d*e - 2*a*d^2*e
+ b*c^2*f)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)
*c^2*d) + (sqrt(b*x + a)*b*d*e - sqrt(b*x + a)*b*c*f)/((b*c + (b*x + a)*d -
 a*d)*c*d)
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 1814, normalized size of antiderivative = 14.17

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^2),x)
```

```
[Out] (atan((((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) + ((4*b^3*c^
5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f -
 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(
1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) + (2*(a +
 b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*
d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c)
)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)) - (
((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) - ((4*b^3*c^5*d^3 -
8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2
*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(1/2)*(b*
c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) - (2*(a + b*x)^(1
/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f -
4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c))^(1/2)*
(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)))/((4*(a*b^4
*c*d^2*e^3 - 2*a^2*b^3*d^3*e^3 + a*b^4*c^3*e*f^2 - 2*a^2*b^3*c*d^2*e^2*f +
2*a*b^4*c^2*d*e^2*f))/(c^3*d) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2
*f))/(c^3*d) + ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(
a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d
^3)))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^
4 - b*c^3*d^3)) + (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4
*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/
(c^2*d))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2
*d^4 - b*c^3*d^3)) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d
) - ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1
/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*
(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d
^3)) - (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^
2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d
^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^
3*d^3))))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(a*c^
2*d^4 - b*c^3*d^3) - (2*a^(1/2)*e*atanh((4*a^(1/2)*b^4*e*f^2*(a + b*x)^(1/2
))/(4*a*b^4*e*f^2 + (4*a*b^4*d^2*e^3)/c^2 - (16*a^2*b^3*d^2*e^2*f)/c^2 + (8
*a*b^4*d*e^2*f)/c) + (8*a^(1/2)*b^4*e^2*f*(a + b*x)^(1/2))/(8*a*b^4*e^2*f +
 (4*a*b^4*d*e^3)/c - (16*a^2*b^3*d*e^2*f)/c + (4*a*b^4*c*e*f^2)/d) + (4*a^(
1/2)*b^4*d*e^3*(a + b*x)^(1/2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b
^3*d*e^2*f + (4*a*b^4*c^2*e*f^2)/d) - (16*a^(3/2)*b^3*d*e^2*f*(a + b*x)^(1/
```

```
2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b^3*d*e^2*f + (4*a*b^4*c^2*e*
f^2)/d)))/c^2 - ((b*c*f - b*d*e)*(a + b*x)^(1/2))/(c*d*(b*c - a*d + d*(a +
b*x)))
```

3.21 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 205

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3} dx = \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} - \frac{(12abcd^2e-8a^2d^3e-b^2c^2(3de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}} - \frac{2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}$$

```
[Out] -1/4*(12*a*b*c*d^2*e-8*a^2*d^3*e-b^2*c^2*(c*f+3*d*e))*arctan(d^(1/2)*(b*x+a
)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3/2)/(-a*d+b*c)^(3/2)-2*e*arctanh((b*x+a)^
(1/2)/a^(1/2))*a^(1/2)/c^3+1/2*(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)^2-1/4*(
4*a*d^2*e-b*c*(c*f+3*d*e))*(b*x+a)^(1/2)/c^2/d/(-a*d+b*c)/(d*x+c)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {154, 156, 162, 65, 214, 211}

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3} dx = -\frac{\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de))}{4c^3d^{3/2}(bc-ad)^{3/2}} - \frac{2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3} - \frac{\sqrt{a+bx}(4ad^2e-bc(cf+3de))}{4c^2d(c+dx)(bc-ad)} + \frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(2*c*d*(c + d*x)^2) - ((4*a*d^2*e - b*c*(3*d*e
+ c*f))*Sqrt[a + b*x])/(4*c^2*d*(b*c - a*d)*(c + d*x)) - ((12*a*b*c*d^2*e -
 8*a^2*d^3*e - b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b
*c - a*d]])/(4*c^3*d^(3/2)*(b*c - a*d)^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\int \frac{-2ade-\frac{1}{2}b(3de+cf)x}{x\sqrt{a+bx}(c+dx)^2}\,dx}{2cd}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{\int \frac{2ad(bc-ad)e-\frac{1}{4}b(4ad^2e-bc(3de+cf))x}{x\sqrt{a+bx}(c+dx)}\,dx}{2c^2d(bc-ad)}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^3} - \frac{(12abcd^2e-8a^2d^3e-b^2c^2(3de+cf))\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx}{8c^3d(bc-ad)}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^3} - \frac{(12abcd^2e-8a^2d^3e-b^2c^2(3de+cf))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4bc^3d(bc-ad)}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} - \frac{(12abcd^2e-8a^2d^3e-b^2c^2(3de+cf))\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}} - \frac{2\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx$$
$$= \frac{\frac{c\sqrt{a+bx}(2ad(3cde-c^2f+2d^2ex)+bc(c^2f-3d^2ex-cd(5e+fx)))}{d(-bc+ad)(c+dx)^2} + \frac{(-12abcd^2e+8a^2d^3e+b^2c^2(3de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{3/2}} - 8\sqrt{a}e\text{arcta}}{4c^3}$$

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] ((c*Sqrt[a + b*x]*(2*a*d*(3*c*d*e - c^2*f + 2*d^2*e*x) + b*c*(c^2*f - 3*d^2
*e*x - c*d*(5*e + f*x))))/(d*(-(b*c) + a*d)*(c + d*x)^2) + ((-12*a*b*c*d^2*
e + 8*a^2*d^3*e + b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqr
t[b*c - a*d]])/(d^(3/2)*(b*c - a*d)^(3/2)) - 8*Sqrt[a]*e*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/(4*c^3)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{2\left(-(dx+c)^2\left(a^2d^3e-\frac{3}{2}abc\,d^2e+\frac{1}{8}b^2c^3f+\frac{3}{8}b^2c^2de\right)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)+\sqrt{(ad-bc)d}\,\left((dx+c)^2e\left(a^{\frac{3}{2}}d-bc\sqrt{a}\right)d\ \operatorname{arc}\right.\right.}{\sqrt{(ad-bc)d}\,(ad-bc)d(dx+c)^2c^3}$
derivativedivides	$2b^2\left(-\frac{e\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2c^3}+\frac{\frac{\frac{bc\left(4ae\,d^2-c^2bf-3bcde\right)(bx+a)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(4ae\,d^2+c^2bf-5bcde\right)bc\sqrt{bx+a}}{8d}}{(-d(bx+a)+ad-bc)^2}+\frac{\left(8a^2d^3e-12abc\,d^2e+\right.}{8}}{c^3b^2}\right.$
default	$2b^2\left(-\frac{e\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2c^3}+\frac{\frac{\frac{bc\left(4ae\,d^2-c^2bf-3bcde\right)(bx+a)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(4ae\,d^2+c^2bf-5bcde\right)bc\sqrt{bx+a}}{8d}}{(-d(bx+a)+ad-bc)^2}+\frac{\left(8a^2d^3e-12abc\,d^2e+\right.}{8}}{c^3b^2}\right.$

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/((a*d-b*c)*d)^(1/2)*(-(d*x+c)^2*(a^2*d^3*e-3/2*a*b*c*d^2*e+1/8*b^2*c^3*f
+3/8*b^2*c^2*d*e)*arctanh(d*(b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2))+((a*d-b*c)*d
)^(1/2)*((d*x+c)^2*e*(a^(3/2)*d-b*c*a^(1/2))*d*arctanh((b*x+a)^(1/2)/a^(1/2
))+1/4*(b*x+a)^(1/2)*c*(-2*a*d^3*e*x-3*e*(-1/2*b*x+a)*c*d^2+(1/2*(f*x+5*e)*
b+a*f)*c^2*d-1/2*b*c^3*f)))/(a*d-b*c)/d/(d*x+c)^2/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(179) = 358$.

Time = 0.71 (sec) , antiderivative size = 2211, normalized size of antiderivative = 10.79

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d
^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*
d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*
```

d^2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x + c)) + 8*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), 1/8*(16*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x + c)) + 2*((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 4*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 8*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx$$
$$= \frac{(3\,b^2c^2de - 12\,abcd^2e + 8\,a^2d^3e + b^2c^3f)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{4\,(bc^4d - ac^3d^2)\sqrt{bcd-ad^2}} + \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c^3}$$
$$+ \frac{5\,\sqrt{bx+a}b^3c^2de + 3\,(bx+a)^{\frac{3}{2}}b^2cd^2e - 9\,\sqrt{bx+a}ab^2cd^2e - 4\,(bx+a)^{\frac{3}{2}}abd^3e + 4\,\sqrt{bx+a}a^2bd^3e - \sqrt{bx}}{4\,(bc^3d - ac^2d^2)(bc+(bx+a)d-ad)^2}$$

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^2*c^2*d*e - 12*a*b*c*d^2*e + 8*a^2*d^3*e + b^2*c^3*f)*arctan(sqrt(
b*x + a)*d/sqrt(b*c*d - a*d^2))/((b*c^4*d - a*c^3*d^2)*sqrt(b*c*d - a*d^2))
 + 2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c^3) + 1/4*(5*sqrt(b*x +
a)*b^3*c^2*d*e + 3*(b*x + a)^(3/2)*b^2*c*d^2*e - 9*sqrt(b*x + a)*a*b^2*c*d^
2*e - 4*(b*x + a)^(3/2)*a*b*d^3*e + 4*sqrt(b*x + a)*a^2*b*d^3*e - sqrt(b*x
+ a)*b^3*c^3*f + (b*x + a)^(3/2)*b^2*c^2*d*f + sqrt(b*x + a)*a*b^2*c^2*d*f)
/((b*c^3*d - a*c^2*d^2)*(b*c + (b*x + a)*d - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 4839, normalized size of antiderivative = 23.60

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3} \, dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^3),x)
```

```
[Out] (atan((((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*
b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2
 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f -
24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) - ((d^
3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^
7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3
 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e
 + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*
c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*
c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a
^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))
/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^
2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 -
 b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) + ((d^3*(a*d - b*c)^3)^(
1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e
^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320
*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^
2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*
b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e
 + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + ((d^3*(a*
d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e
- 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5
 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3
*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e
+ b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^
3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c
^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^
4 - 3*a^2*b*c^4*d^5)))/(((a*b^6*c^5*e*f^2)/4 - 12*a^2*b^5*c^2*d^3*e^3 - 8*a
^4*b^3*d^5*e^3 + (9*a*b^6*c^3*d^2*e^3)/4 + 18*a^3*b^4*c*d^4*e^3 - 4*a^2*b^5
*c^3*d^2*e^2*f + 2*a^3*b^4*c^2*d^3*e^2*f + (3*a*b^6*c^4*d*e^2*f)/2)/(b^2*c^
8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)
^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d
*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e
^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)) - ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e -
a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d
```

^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2
)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e
)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c
^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c
^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*
b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*
d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*
c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5
)) + ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^
2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24
*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*
(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*
d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 -
2*a*b*c^7*d^2) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e +
b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^
8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2
*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(
8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*
d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*
c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5))))*(d^3*(a*d - b*c)^3)^(1/2)*(
8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(4*(a^3*c^3*d
^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) - (((a + b*x)^(1/2)*
(b^2*c^2*f + 4*a*b*d^2*e - 5*b^2*c*d*e))/(4*c^2*d) + ((a + b*x)^(3/2)*(b^2*
c^2*f - 4*a*b*d^2*e + 3*b^2*c*d*e))/(4*c^2*(a*d - b*c)))/(d^2*(a + b*x)^2 -
(2*a*d^2 - 2*b*c*d)*(a + b*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (a^(1/2)*
e*atan(((a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9
*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3
*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^
2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + (a^(1/2)*e*((5*a*b^
5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e +
a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + (a^(1/2)*e*
(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 -
128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2))))/
c^3)*1i)/c^3 + (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*
e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*
b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5
*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) - (a^(1/2)*e*(
(5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*
d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - (a^(
1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^
7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d
^2))))/c^3)*1i)/c^3)/(((a*b^6*c^5*e*f^2)/4 - 12*a^2*b^5*c^2*d^3*e^3 - 8*a^4
*b^3*d^5*e^3 + (9*a*b^6*c^3*d^2*e^3)/4 + 18*a^3*b^4*c*d^4*e^3 - 4*a^2*b^5*c

```
^3*d^2*e^2*f + 2*a^3*b^4*c^2*d^3*e^2*f + (3*a*b^6*c^4*d*e^2*f)/2)/(b^2*c^8*
d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^
2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4
*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^
3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*
d^2)) + (a^(1/2)*e*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^
4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2
*a*b*c^7*d^2) + (a^(1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*
d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c
^4*d^3 - 2*a*b*c^5*d^2))))/c^3))/c^3 - (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^
6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2
*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^
4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*
c^5*d^2)) - (a^(1/2)*e*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^
7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3
 - 2*a*b*c^7*d^2) - (a^(1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*
c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a
^2*c^4*d^3 - 2*a*b*c^5*d^2))))/c^3))/c^3))*2i)/c^3
```

3.22 $\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 111

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4}-\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4}-\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4}-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4}-\frac{75\arcsin(1-2ax)}{128a^4}$$

```
[Out] 75/128*arcsin(2*a*x-1)/a^4-25/32*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^4-5/8*(a*x)^(
5/2)*(-a*x+1)^(1/2)/a^4-1/4*(a*x)^(7/2)*(-a*x+1)^(1/2)/a^4-75/64*(a*x)^(1/2
)*(-a*x+1)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {16, 81, 52, 55, 633, 222}

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{75\arcsin(1-2ax)}{128a^4}-\frac{\sqrt{1-ax}(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\sqrt{ax}}{64a^4}$$

```
[In] Int[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-75*Sqrt[a*x]*Sqrt[1 - a*x])/(64*a^4) - (25*(a*x)^(3/2)*Sqrt[1 - a*x])/(32
*a^4) - (5*(a*x)^(5/2)*Sqrt[1 - a*x])/(8*a^4) - ((a*x)^(7/2)*Sqrt[1 - a*x])
/(4*a^4) - (75*ArcSin[1 - 2*a*x])/(128*a^4)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(ax)^{5/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^3}$$

$$= -\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{15\int \frac{(ax)^{5/2}}{\sqrt{1-ax}}\,dx}{8a^3}$$

$$= -\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{25\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{16a^3}$$

$$= -\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{64a^3}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{128a^3}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{128a^3}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{128a^5}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\sin^{-1}(1-2ax)}{128a^4}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\sqrt{a}x(-75+25ax+10a^2x^2+24a^3x^3+16a^4x^4)+150\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{64a^{7/2}\sqrt{-ax(-1+ax)}}$$

```
[In] Integrate[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-75 + 25*a*x + 10*a^2*x^2 + 24*a^3*x^3 + 16*a^4*x^4) + 150*Sqrt
[x]*Sqrt[1 - a*x]*ArcTan[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])/(64*a^(7/
2)*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\sqrt{-ax+1}\,x\left(32\,\mathrm{csgn}(a)a^3x^3\sqrt{-x(ax-1)a}+80\,\mathrm{csgn}(a)x^2a^2\sqrt{-x(ax-1)a}+100\,\mathrm{csgn}(a)\sqrt{-x(ax-1)a}\,ax+150\,\mathrm{csgn}(a)\sqrt{-x(ax-}\right.}{128a^3\sqrt{ax}\,\sqrt{-x(ax-1)a}}$
risch	$\frac{(16a^3x^3+40a^2x^2+50ax+75)x(ax-1)\sqrt{ax(-ax+1)}}{64a^3\sqrt{-x(ax-1)a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{75\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax(-ax+1)}}{128a^3\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{9}{2}}\left(144a^3x^3+168a^2x^2+210ax+315\right)\sqrt{-ax+1}}{576a^4}+\frac{35\sqrt{\pi}\,(-a)^{\frac{9}{2}}\arcsin(\sqrt{a}\,\sqrt{x})}{64a^{\frac{9}{2}}}\right)}{(-a)^{\frac{7}{2}}\sqrt{ax}\,\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{7}{2}}\left(56a^2x^2+70ax+\right.\right.}{168a^3}}{(-a)^{\frac{5}{2}}}$

```
[In] int(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(-a*x+1)^(1/2)*x*(32*csgn(a)*a^3*x^3*(-x*(a*x-1)*a)^(1/2)+80*csgn(a)
*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+100*csgn(a)*(-x*(a*x-1)*a)^(1/2)*a*x+150*csgn
(a)*(-x*(a*x-1)*a)^(1/2)-75*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/
2)))*csgn(a)/a^3/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(16\,a^3x^3+40\,a^2x^2+50\,ax+75)\sqrt{ax}\sqrt{-ax+1}+75\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{64\,a^4}$$

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*((16*a^3*x^3 + 40*a^2*x^2 + 50*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) + 7
5*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 41.97 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.36

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= a\left(\begin{cases} -\frac{35i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{64a^5} - \frac{ix^{\frac{9}{2}}}{4\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{7ix^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{ax-1}} - \frac{35ix^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{ax-1}} + \frac{35i\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{ax-1}} & \text{for } |ax| > 1 \\ \frac{35\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{64a^5} + \frac{x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{7x^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{-ax+1}} + \frac{35x^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{-ax+1}} - \frac{35\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right)$$
$$+ \begin{cases} -\frac{5i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}} + \frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for } |ax| > 1 \\ \frac{5\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}} - \frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-35*I*acosh(sqrt(a)*sqrt(x))/(64*a**5) - I*x**(9/2)/(4*sqrt(a)
*sqrt(a*x - 1)) - I*x**(7/2)/(24*a**(3/2)*sqrt(a*x - 1)) - 7*I*x**(5/2)/(96
*a**(5/2)*sqrt(a*x - 1)) - 35*I*x**(3/2)/(192*a**(7/2)*sqrt(a*x - 1)) + 35*
I*sqrt(x)/(64*a**(9/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (35*asin(sqrt(a)*sqrt
(x))/(64*a**5) + x**(9/2)/(4*sqrt(a)*sqrt(-a*x + 1)) + x**(7/2)/(24*a**(3/2
)*sqrt(-a*x + 1)) + 7*x**(5/2)/(96*a**(5/2)*sqrt(-a*x + 1)) + 35*x**(3/2)/(
192*a**(7/2)*sqrt(-a*x + 1)) - 35*sqrt(x)/(64*a**(9/2)*sqrt(-a*x + 1)), Tru
e)) + Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(
a)*sqrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(
24*a**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*
x) > 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x
+ 1)) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqr
t(-a*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{-a^2x^2+ax}\,x^3}{4\,a} - \frac{5\,\sqrt{-a^2x^2+ax}\,x^2}{8\,a^2} - \frac{25\,\sqrt{-a^2x^2+ax}\,x}{32\,a^3} - \frac{75\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{128\,a^4} - \frac{75\,\sqrt{-a^2x^2+ax}}{64\,a^4}$$

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + a*x)*x^3/a - 5/8*sqrt(-a^2*x^2 + a*x)*x^2/a^2 - 25/32*
sqrt(-a^2*x^2 + a*x)*x/a^3 - 75/128*arcsin(-(2*a^2*x - a)/a)/a^4 - 75/64*sq
rt(-a^2*x^2 + a*x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(2\,(4\,(2\,ax+5)ax+25)ax+75)\sqrt{ax}\sqrt{-ax+1}-75\,\arcsin\left(\sqrt{ax}\right)}{64\,a^4}$$

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*((2*(4*(2*a*x + 5)*a*x + 25)*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) - 75*
arcsin(sqrt(a*x)))/a^4
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.11

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{75\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{32\,a^4} - \frac{\frac{5\,\sqrt{a\,x}}{4\,(\sqrt{1-a\,x}-1)}+\frac{85\,(a\,x)^{3/2}}{12\,(\sqrt{1-a\,x}-1)^3}+\frac{33\,(a\,x)^{5/2}}{2\,(\sqrt{1-a\,x}-1)^5}-\frac{33\,(a\,x)^{7/2}}{2\,(\sqrt{1-a\,x}-1)^7}-\frac{85\,(a\,x)^{9/2}}{12\,(\sqrt{1-a\,x}-1)^9}-\frac{5\,(a\,x)^{11/2}}{4\,(\sqrt{1-a\,x}-1)^{11}}}{a^4\left(\frac{a\,x}{(\sqrt{1-a\,x}-1)^2}+1\right)^6} - \frac{\frac{35\,\sqrt{a\,x}}{32\,(\sqrt{1-a\,x}-1)}+\frac{805\,(a\,x)^{3/2}}{96\,(\sqrt{1-a\,x}-1)^3}+\frac{2681\,(a\,x)^{5/2}}{96\,(\sqrt{1-a\,x}-1)^5}+\frac{5053\,(a\,x)^{7/2}}{96\,(\sqrt{1-a\,x}-1)^7}-\frac{5053\,(a\,x)^{9/2}}{96\,(\sqrt{1-a\,x}-1)^9}-\frac{2681\,(a\,x)^{11/2}}{96\,(\sqrt{1-a\,x}-1)^{11}}-\frac{805\,(a\,x)\cdots}{96\,(\sqrt{1-a\,x}\cdots}}{a^4\left(\frac{a\,x}{(\sqrt{1-a\,x}-1)^2}+1\right)^8}$$

```
[In] int((x^3*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (75*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(32*a^4) - ((5*(a*x)^(1/2))/(4
*((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (
33*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x
)^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^
(11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
 1)^6) - ((35*(a*x)^(1/2))/(32*((1 - a*x)^(1/2) - 1)) + (805*(a*x)^(3/2))/(
96*((1 - a*x)^(1/2) - 1)^3) + (2681*(a*x)^(5/2))/(96*((1 - a*x)^(1/2) - 1)^
5) + (5053*(a*x)^(7/2))/(96*((1 - a*x)^(1/2) - 1)^7) - (5053*(a*x)^(9/2))/(
96*((1 - a*x)^(1/2) - 1)^9) - (2681*(a*x)^(11/2))/(96*((1 - a*x)^(1/2) - 1)
^11) - (805*(a*x)^(13/2))/(96*((1 - a*x)^(1/2) - 1)^13) - (35*(a*x)^(15/2))
/(32*((1 - a*x)^(1/2) - 1)^15))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 + 1)^8)
```

3.23 $\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}} \, dx = -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} - \frac{11\arcsin(1-2ax)}{16a^3}$$

```
[Out] 11/16*arcsin(2*a*x-1)/a^3-11/12*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^3-1/3*(a*x)^(5
/2)*(-a*x+1)^(1/2)/a^3-11/8*(a*x)^(1/2)*(-a*x+1)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {16, 81, 52, 55, 633, 222}

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}} \, dx = -\frac{11\arcsin(1-2ax)}{16a^3} - \frac{\sqrt{1-ax}(ax)^{5/2}}{3a^3} - \frac{11\sqrt{1-ax}(ax)^{3/2}}{12a^3} - \frac{11\sqrt{1-ax}\sqrt{ax}}{8a^3}$$

```
[In] Int[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-11*Sqrt[a*x]*Sqrt[1 - a*x])/(8*a^3) - (11*(a*x)^(3/2)*Sqrt[1 - a*x])/(12*
a^3) - ((a*x)^(5/2)*Sqrt[1 - a*x])/(3*a^3) - (11*ArcSin[1 - 2*a*x])/(16*a^3
)
```

<u>Rule 16</u>

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{(ax)^{3/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^2}\\ &= -\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{6a^2}\end{aligned}$$

$$= -\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} + \frac{11\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{8a^2}$$

$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} + \frac{11\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{16a^2}$$

$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} + \frac{11\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{16a^2}$$

$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} - \frac{11\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{16a^4}$$

$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} - \frac{11\sin^{-1}(1-2ax)}{16a^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\sqrt{a}x(-33+11ax+14a^2x^2+8a^3x^3)+66\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{24a^{5/2}\sqrt{-ax(-1+ax)}}$$

```
[In] Integrate[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-33 + 11*a*x + 14*a^2*x^2 + 8*a^3*x^3) + 66*Sqrt[x]*Sqrt[1 - a*
x]*ArcTan[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])/(24*a^(5/2)*Sqrt[-(a*x*(
-1 + a*x))])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{-ax+1}\, x \left(16 \,\mathrm{csgn}(a) x^2 a^2 \sqrt{-x(ax-1)a}+44 \,\mathrm{csgn}(a) \sqrt{-x(ax-1)a}\, ax+66 \,\mathrm{csgn}(a) \sqrt{-x(ax-1)a}-33 \arctan\left(\frac{\mathrm{csgn}(a)(2ax-1)}{2\sqrt{-x(ax-1)a}}\right)\right.}{48a^2 \sqrt{ax}\, \sqrt{-x(ax-1)a}}$
risch	$\frac{(8a^2x^2+22ax+33)x(ax-1)\sqrt{ax(-ax+1)}}{24a^2\sqrt{-x(ax-1)a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{11\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax(-ax+1)}}{16a^2\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{7}{2}}\left(56a^2x^2+70ax+105\right)\sqrt{-ax+1}}{168a^3}+\frac{5\sqrt{\pi}\,(-a)^{\frac{7}{2}}\arcsin(\sqrt{a}\,\sqrt{x})}{8a^{\frac{7}{2}}}\right)}{(-a)^{\frac{5}{2}}\sqrt{ax}\,\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{5}{2}}(10ax+15)\sqrt{-ax+1}}{20a^2}+\frac{3\sqrt{\pi}\,(-a\right.}{(-a)^{\frac{3}{2}}\sqrt{ax}\,\sqrt{\pi}\,a}$

[In] int(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/48*(-a*x+1)^(1/2)*x*(16*csgn(a)*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+44*csgn(a)*(-x*(a*x-1)*a)^(1/2)*a*x+66*csgn(a)*(-x*(a*x-1)*a)^(1/2)-33*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/a^2/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\, dx = -\frac{(8\,a^2x^2+22\,ax+33)\sqrt{ax}\sqrt{-ax+1}+33\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{24\,a^3}$$

[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")

[Out] -1/24*((8*a^2*x^2 + 22*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) + 33*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a^3

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.52

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= a\left(\begin{cases} -\frac{5i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}} + \frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for } |ax|>1 \\ \frac{5\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}} - \frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right)$$
$$+ \begin{cases} -\frac{3i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}} + \frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for } |ax|>1 \\ \frac{3\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}} - \frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(a)*s
qrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(24*a
**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*x) >
 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x + 1)
) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqrt(-a
*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True)) + Piecewise((-3*I*
acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*sqrt(a*x - 1)) - I*
x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(5/2)*sqrt(a*x - 1)
), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**(5/2)/(2*sqrt(a)*s
qrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3*sqrt(x)/(4*a**(5/
2)*sqrt(-a*x + 1)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{-a^2x^2+ax}\,x^2}{3\,a} - \frac{11\,\sqrt{-a^2x^2+ax}\,x}{12\,a^2} - \frac{11\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{16\,a^3} - \frac{11\,\sqrt{-a^2x^2+ax}}{8\,a^3}$$

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + a*x)*x^2/a - 11/12*sqrt(-a^2*x^2 + a*x)*x/a^2 - 11/16*
arcsin(-(2*a^2*x - a)/a)/a^3 - 11/8*sqrt(-a^2*x^2 + a*x)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(2\,(4\,ax+11)ax+33)\sqrt{ax}\sqrt{-ax+1}-33\,\arcsin\left(\sqrt{ax}\right)}{24\,a^3}$$

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(4*a*x + 11)*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) - 33*arcsin(sqrt(
a*x)))/a^3
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.09

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{11\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{4\,a^3} - \frac{\frac{5\,\sqrt{a\,x}}{4\,\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,{\left(a\,x\right)}^{3/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^3}+\frac{33\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{33\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}-\frac{85\,{\left(a\,x\right)}^{9/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^9}-\frac{5\,{\left(a\,x\right)}^{11/2}}{4\,{\left(\sqrt{1-a\,x}-1\right)}^{11}}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^6} - \frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)}+\frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3}-\frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

```
[In] int((x^2*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (11*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(4*a^3) - ((5*(a*x)^(1/2))/(4*
((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (3
3*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x)
^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^(
11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
1)^6) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1)) + (11*(a*x)^(3/2))/(2*((
1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (3*
(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)
^2 + 1)^4)
```

3.24 $\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\arcsin(1-2ax)}{8a^2}$$

```
[Out] 7/8*arcsin(2*a*x-1)/a^2-1/2*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^2-7/4*(a*x)^(1/2)*
(-a*x+1)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {16, 81, 52, 55, 633, 222}

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{7\arcsin(1-2ax)}{8a^2} - \frac{\sqrt{1-ax}(ax)^{3/2}}{2a^2} - \frac{7\sqrt{1-ax}\sqrt{ax}}{4a^2}$$

```
[In] Int[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-7*Sqrt[a*x]*Sqrt[1 - a*x])/(4*a^2) - ((a*x)^(3/2)*Sqrt[1 - a*x])/(2*a^2)
- (7*ArcSin[1 - 2*a*x])/(8*a^2)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{ax}(1+ax)}{\sqrt{1-ax}}\,dx}{a} \\
&= -\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{4a} \\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{8a} \\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{8a} \\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{8a^3}
\end{aligned}
$$

$$= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\sin^{-1}(1-2ax)}{8a^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\sqrt{a}x(-7+5ax+2a^2x^2)+14\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{4a^{3/2}\sqrt{-ax(-1+ax)}}$$

```
[In] Integrate[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-7 + 5*a*x + 2*a^2*x^2) + 14*Sqrt[x]*Sqrt[1 - a*x]*ArcTan[(Sqrt
[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])/(4*a^(3/2)*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

method	result	s
default	$-\frac{\sqrt{-ax+1}\,x\left(4\,\mathrm{csgn}(a)\sqrt{-x(ax-1)a}\,ax+14\,\mathrm{csgn}(a)\sqrt{-x(ax-1)a}-7\arctan\left(\frac{\mathrm{csgn}(a)(2ax-1)}{2\sqrt{-x(ax-1)a}}\right)\right)\mathrm{csgn}(a)}{8a\sqrt{ax}\,\sqrt{-x(ax-1)a}}$	9
risch	$\frac{(2ax+7)x(ax-1)\sqrt{ax(-ax+1)}}{4a\sqrt{-x(ax-1)a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{7\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax(-ax+1)}}{8a\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$	[illegible]
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{5}{2}}(10ax+15)\sqrt{-ax+1}}{20a^2}+\frac{3\sqrt{\pi}\,(-a)^{\frac{5}{2}}\arcsin(\sqrt{a}\,\sqrt{x})}{4a^{\frac{5}{2}}}\right)}{(-a)^{\frac{3}{2}}\sqrt{ax}\,\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{3}{2}}\sqrt{-ax+1}}{a}+\frac{\sqrt{\pi}\,(-a)^{\frac{3}{2}}\arcsin(\sqrt{a}\,\sqrt{x})}{a^{\frac{3}{2}}}\right)}{\sqrt{-a}\,\sqrt{ax}\,\sqrt{\pi}\,a}$	[illegible]

```
[In] int(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-a*x+1)^(1/2)*x/a*(4*csgn(a)*(-x*(a*x-1)*a)^(1/2)*a*x+14*csgn(a)*(-x*
(a*x-1)*a)^(1/2)-7*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn
(a)/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(2\,ax+7)\sqrt{ax}\sqrt{-ax+1}+7\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{4\,a^2}$$

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) + 7*arctan(sqrt(a*x)*sqrt(-a*x +
 1)/(a*x)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.27

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{3i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}} + \frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for } |ax|>1 \\ \frac{3\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}} - \frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2} - \frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax|>1 \\ \frac{\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2} + \frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}} - \frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-3*I*acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*s
qrt(a*x - 1)) - I*x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(
5/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**
(5/2)/(2*sqrt(a)*sqrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3
*sqrt(x)/(4*a**(5/2)*sqrt(-a*x + 1)), True)) + Piecewise((-I*acosh(sqrt(a)*
sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3/2), Abs(a*x) > 1), (asin(sqrt
(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a*x + 1)) - sqrt(x)/(a**(3/2)*s
qrt(-a*x + 1)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{-a^2x^2+axx}}{2\,a} - \frac{7\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{8\,a^2} - \frac{7\,\sqrt{-a^2x^2+ax}}{4\,a^2}$$

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-a^2*x^2 + a*x)*x/a - 7/8*arcsin(-(2*a^2*x - a)/a)/a^2 - 7/4*sqrt
(-a^2*x^2 + a*x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(2\,ax+7)\sqrt{ax}\sqrt{-ax+1} - 7\,\arcsin\left(\sqrt{ax}\right)}{4\,a^2}$$

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) - 7*arcsin(sqrt(a*x)))/a^2
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.03

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{7\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{2\,a^2} - \frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1} - \frac{2\,{\left(a\,x\right)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2} - \frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)} + \frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3} - \frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5} - \frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

```
[In] int((x*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
[Out] (7*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(2*a^2) - ((2*(a*x)^(1/2))/((1
- a*x)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a^2*((a*x)/((
1 - a*x)^(1/2) - 1)^2 + 1)^2) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1))
+ (11*(a*x)^(3/2))/(2*((1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 -
a*x)^(1/2) - 1)^5) - (3*(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^2*((a*
x)/((1 - a*x)^(1/2) - 1)^2 + 1)^4)
```

3.25 $\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 37

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx=-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\arcsin(1-2ax)}{2a}$$

```
[Out] 3/2*arcsin(2*a*x-1)/a-(a*x)^(1/2)*(-a*x+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {81, 55, 633, 222}

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx=-\frac{3\arcsin(1-2ax)}{2a}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}$$

```
[In] Int[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (3*ArcSin[1 - 2*a*x])/(2*a)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{2a^2} \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$
\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx=\frac{\sqrt{a}x(-1+ax)+6\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{\sqrt{a}\sqrt{-ax(-1+ax)}}
$$

```
[In] Integrate[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-1 + a*x) + 6*Sqrt[x]*Sqrt[1 - a*x]*ArcTan[(Sqrt[a]*Sqrt[x])/(-
1 + Sqrt[1 - a*x])])/(Sqrt[a]*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

method	result	size
default	$-\frac{\sqrt{-ax+1}\,x\left(2\,\mathrm{csgn}(a)\sqrt{-x(ax-1)a}-3\arctan\left(\frac{\mathrm{csgn}(a)(2ax-1)}{2\sqrt{-x(ax-1)a}}\right)\right)\mathrm{csgn}(a)}{2\sqrt{ax}\,\sqrt{-x(ax-1)a}}$	70
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{3}{2}}\sqrt{-ax+1}}{a}+\frac{\sqrt{\pi}\,(-a)^{\frac{3}{2}}\arcsin(\sqrt{a}\,\sqrt{x})}{a^{\frac{3}{2}}}\right)}{\sqrt{-a}\,\sqrt{ax}\,\sqrt{\pi}}+\frac{2\sqrt{x}\,\arcsin(\sqrt{a}\,\sqrt{x})}{\sqrt{a}\,\sqrt{ax}}$	86
risch	$\frac{x(ax-1)\sqrt{ax(-ax+1)}}{\sqrt{-x(ax-1)a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax(-ax+1)}}{2\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$	103

```
[In] int((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-a*x+1)^(1/2)*x*(2*csgn(a)*(-x*(a*x-1)*a)^(1/2)-3*arctan(1/2*csgn(a)*
(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{ax}\sqrt{-ax+1}+3\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{a}$$

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) + 3*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.59

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases}-\frac{i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2}-\frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax|>1\\ \frac{\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right) + \begin{cases}-\frac{2i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for } |ax|>1\\ \frac{2\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}$$

```
[In] integrate((a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-I*acosh(sqrt(a)*sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3
/2), Abs(a*x) > 1), (asin(sqrt(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a
*x + 1)) - sqrt(x)/(a**(3/2)*sqrt(-a*x + 1)), True)) + Piecewise((-2*I*acos
h(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*sqrt(x))/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{3\ \arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{2\,a} - \frac{\sqrt{-a^2x^2+ax}}{a}$$

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*arcsin(-(2*a^2*x - a)/a)/a - sqrt(-a^2*x^2 + a*x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{ax}\sqrt{-ax+1} - 3\ \arcsin\left(\sqrt{ax}\right)}{a}$$

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) - 3*arcsin(sqrt(a*x)))/a
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.19

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{a} - \frac{4\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}} - \frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1} - \frac{2\,{\left(a\,x\right)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2}$$

```
[In] int((a*x + 1)/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (2*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/a - (4*atan((a*((1 - a*x)^(1/2)
 - 1))/((a*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2) - ((2*(a*x)^(1/2))/((1 - a*x
)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a*((a*x)/((1 - a*x
)^(1/2) - 1)^2 + 1)^2)
```

3.26 $\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \arcsin(1-2ax)$$

```
[Out] arcsin(2*a*x-1)-2*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {16, 79, 55, 633, 222}

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\arcsin(1-2ax) - \frac{2\sqrt{1-ax}}{\sqrt{ax}}$$

```
[In] Int[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x])/Sqrt[a*x] - ArcSin[1 - 2*a*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\int \frac{1+ax}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{a}\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \sin^{-1}(1-2ax)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 68 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{2\left(-1+ax+2\sqrt{a}\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)\right)}{\sqrt{-ax(-1+ax)}}$$

```
[In] Integrate[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (2*(-1 + a*x + 2*Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*ArcTan[(Sqrt[a]*Sqrt[x])/(-1
 + Sqrt[1 - a*x])]))/Sqrt[-(a*x*(-1 + a*x))]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

method	result	size
meijerg	$\frac{2\sqrt{a}\,\sqrt{x}\,\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{ax}}-\frac{2\sqrt{-ax+1}}{\sqrt{ax}}$	38
default	$\frac{\sqrt{-ax+1}\,\left(\arctan\left(\frac{\operatorname{csgn}(a)(2ax-1)}{2\sqrt{-x(ax-1)a}}\right)ax-2\,\operatorname{csgn}(a)\sqrt{-x(ax-1)a}\right)\operatorname{csgn}(a)}{\sqrt{ax}\,\sqrt{-x(ax-1)a}}$	69
risch	$\frac{2(ax-1)\sqrt{ax(-ax+1)}}{\sqrt{-x(ax-1)a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{a\arctan\left(\frac{\sqrt{a^2}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax(-ax+1)}}{\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$	103

```
[In] int((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^(1/2)/(a*x)^(1/2)*x^(1/2)*arcsin(a^(1/2)*x^(1/2))-2*(-a*x+1)^(1/2)/(a*x
)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\left(ax\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)+\sqrt{ax}\sqrt{-ax+1}\right)}{ax}$$

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(a*x*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)) + sqrt(a*x)*sqrt(-a*x + 1))/
(a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{2i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for}\ |ax|>1 \\ \frac{2\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise} \end{cases}\right) + \begin{cases} -2\sqrt{-1+\frac{1}{ax}} & \text{for}\ \frac{1}{|ax|}>1 \\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x+1)/x/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*I*acosh(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*
sqrt(x))/a, True)) + Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2
*I*sqrt(1 - 1/(a*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-a^2x^2+ax}}{ax} - \arcsin\left(-\frac{2a^2x-a}{a}\right)$$

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + a*x)/(a*x) - arcsin(-(2*a^2*x - a)/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{2a\arcsin\left(\sqrt{ax}\right) - \frac{a\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} + \frac{\sqrt{ax}a}{\sqrt{-ax+1}-1}}{a}$$

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*arcsin(sqrt(a*x)) - a*(sqrt(-a*x + 1) - 1)/sqrt(a*x) + sqrt(a*x)*a/(sq
rt(-a*x + 1) - 1))/a
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-a\,x}}{\sqrt{a\,x}} - \frac{4\,a\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

```
[In] int((a*x + 1)/(x*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
[Out] - (2*(1 - a*x)^(1/2))/(a*x)^(1/2) - (4*a*atan((a*((1 - a*x)^(1/2) - 1))/((a
*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2)
```

3.27 $\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 45

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}$$

```
[Out] -2/3*a*(-a*x+1)^(1/2)/(a*x)^(3/2)-10/3*a*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {16, 79, 37}

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

```
[In] Int[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a*Sqrt[1 - a*x])/(3*(a*x)^(3/2)) - (10*a*Sqrt[1 - a*x])/(3*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$\begin{aligned}\text{integral} &= a^2 \int \frac{1+ax}{(ax)^{5/2}\sqrt{1-ax}}\,dx \\ &= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}} + \frac{1}{3}\left(5a^2\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\ &= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}} - \frac{10a\sqrt{1-ax}}{3\sqrt{ax}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax(-1+ax)}(1+5ax)}{3ax^2}$$

```
[In] Integrate[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 5*a*x))/(3*a*x^2)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{2(5ax+1)\sqrt{-ax+1}}{3x\sqrt{ax}}$	25
default	$-\frac{2\sqrt{-ax+1}\ \mathrm{csgn}(a)^2(5ax+1)}{3x\sqrt{ax}}$	29
meijerg	$-\frac{2a\sqrt{-ax+1}}{\sqrt{ax}}-\frac{2(2ax+1)\sqrt{-ax+1}}{3\sqrt{ax}\,x}$	42
risch	$\frac{2\sqrt{ax(-ax+1)}\,(5a^2x^2-4ax-1)}{3\sqrt{ax}\,\sqrt{-ax+1}\,x\sqrt{-x(ax-1)a}}$	55

```
[In] int((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/x/(a*x)^(1/2)*(5*a*x+1)*(-a*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\,(5\,ax+1)\sqrt{ax}\sqrt{-ax+1}}{3\,ax^2}$$

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2/3*(5*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases}-2\sqrt{-1+\frac{1}{ax}} & \text{for } \frac{1}{|ax|}>1\\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise}\end{cases}\right) + \begin{cases}-\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for } \frac{1}{|ax|}>1\\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise}\end{cases}$$

```
[In] integrate((a*x+1)/x**2/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
[Out] a*Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2*I*sqrt(1 - 1/(a*x)
), True)) + Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*
x), 1/Abs(a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3
*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{10\sqrt{-a^2x^2+ax}}{3\,x} - \frac{2\sqrt{-a^2x^2+ax}}{3\,ax^2}$$

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -10/3*sqrt(-a^2*x^2 + a*x)/x - 2/3*sqrt(-a^2*x^2 + a*x)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(33) = 66.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\frac{a^2\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}} + \frac{21\,a^2\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} - \frac{\left(a^2+\frac{21\,a\left(\sqrt{-ax+1}-1\right)^2}{x}\right)(ax)^{\frac{3}{2}}}{\left(\sqrt{-ax+1}-1\right)^3}}{12\,a}$$

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -1/12*(a^2*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 21*a^2*(sqrt(-a*x + 1) - 1)
/sqrt(a*x) - (a^2 + 21*a*(sqrt(-a*x + 1) - 1)^2/x)*(a*x)^(3/2)/(sqrt(-a*x +
 1) - 1)^3)/a
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\,\left(\frac{10\,a\,x}{3}+\frac{2}{3}\right)}{x\,\sqrt{a\,x}}$$

```
[In] int((a*x + 1)/(x^2*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
[Out] -((1 - a*x)^(1/2)*((10*a*x)/3 + 2/3))/(x*(a*x)^(1/2))
```

3.28 $\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}} - \frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}} - \frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}$$

```
[Out] -2/5*a^2*(-a*x+1)^(1/2)/(a*x)^(5/2)-6/5*a^2*(-a*x+1)^(1/2)/(a*x)^(3/2)-12/5
*a^2*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 79, 47, 37}

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}} - \frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}} - \frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

```
[In] Int[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^2*Sqrt[1 - a*x])/(5*(a*x)^(5/2)) - (6*a^2*Sqrt[1 - a*x])/(5*(a*x)^(3/
2)) - (12*a^2*Sqrt[1 - a*x])/(5*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a^3 \int \frac{1+ax}{(ax)^{7/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}} + \frac{1}{5}\left(9a^3\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}} - \frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}} + \frac{1}{5}\left(6a^3\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}} - \frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}} - \frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax(-1+ax)}(1+3ax+6a^2x^2)}{5ax^3}$$

```
[In] Integrate[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 3*a*x + 6*a^2*x^2))/(5*a*x^3)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{2\sqrt{-ax+1}\,(6a^2x^2+3ax+1)}{5x^2\sqrt{ax}}$	33
default	$-\frac{2\sqrt{-ax+1}\,\operatorname{csgn}(a)^2(6a^2x^2+3ax+1)}{5x^2\sqrt{ax}}$	37
meijerg	$-\frac{2a(2ax+1)\sqrt{-ax+1}}{3\sqrt{ax}\,x}-\frac{2\left(\frac{8}{3}a^2x^2+\frac{4}{3}ax+1\right)\sqrt{-ax+1}}{5\sqrt{ax}\,x^2}$	59
risch	$\frac{2\sqrt{ax(-ax+1)}\,(6a^3x^3-3a^2x^2-2ax-1)}{5\sqrt{ax}\,\sqrt{-ax+1}\,x^2\sqrt{-x(ax-1)a}}$	63

```
[In] int((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/5/x^2/(a*x)^(1/2)*(-a*x+1)^(1/2)*(6*a^2*x^2+3*a*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.48

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\,(6\,a^2x^2+3\,ax+1)\sqrt{ax}\sqrt{-ax+1}}{5\,ax^3}$$

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.59

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3} - \frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3} - \frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15} - \frac{8a\sqrt{-1+\frac{1}{ax}}}{15x} - \frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15} - \frac{8ia\sqrt{1-\frac{1}{ax}}}{15x} - \frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x+1)/x**3/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*x), 1/Abs(
a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3*x), True)
) + Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*
x) - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1
/(a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{12\sqrt{-a^2x^2+ax}\,a}{5\,x} - \frac{6\sqrt{-a^2x^2+ax}}{5\,x^2} - \frac{2\sqrt{-a^2x^2+ax}}{5\,ax^3}$$

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -12/5*sqrt(-a^2*x^2 + a*x)*a/x - 6/5*sqrt(-a^2*x^2 + a*x)/x^2 - 2/5*sqrt(-a
^2*x^2 + a*x)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(55) = 110$.

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\frac{a^3\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}} + \frac{15\,a^3\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}} + \frac{110\,a^3\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} - \frac{\left(a^3 + \frac{15\,a^2\left(\sqrt{-ax+1}-1\right)^2}{x} + \frac{110\,a\left(\sqrt{-ax+1}-1\right)^4}{x^2}\right)(ax)^{\frac{5}{2}}}{\left(\sqrt{-ax+1}-1\right)^5}}{80\,a}$$

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80*(a^3*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 15*a^3*(sqrt(-a*x + 1) - 1)
^3/(a*x)^(3/2) + 110*a^3*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (a^3 + 15*a^2*(sq
rt(-a*x + 1) - 1)^2/x + 110*a*(sqrt(-a*x + 1) - 1)^4/x^2)*(a*x)^(5/2)/(sqrt
(-a*x + 1) - 1)^5)/a
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\left(\frac{12\,a^2\,x^2}{5}+\frac{6\,a\,x}{5}+\frac{2}{5}\right)}{x^2\,\sqrt{a\,x}}$$

```
[In] int((a*x + 1)/(x^3*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((6*a*x)/5 + (12*a^2*x^2)/5 + 2/5))/(x^2*(a*x)^(1/2))
```

3.29 $\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}$$

```
[Out] -2/7*a^3*(-a*x+1)^(1/2)/(a*x)^(7/2)-26/35*a^3*(-a*x+1)^(1/2)/(a*x)^(5/2)-10
4/105*a^3*(-a*x+1)^(1/2)/(a*x)^(3/2)-208/105*a^3*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 79, 47, 37}

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

```
[In] Int[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^3*Sqrt[1 - a*x])/(7*(a*x)^(7/2)) - (26*a^3*Sqrt[1 - a*x])/(35*(a*x)^(
5/2)) - (104*a^3*Sqrt[1 - a*x])/(105*(a*x)^(3/2)) - (208*a^3*Sqrt[1 - a*x])
/(105*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a^4 \int \frac{1+ax}{(ax)^{9/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} + \frac{1}{7}\left(13a^4\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} + \frac{1}{35}\left(52a^4\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} - \frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}} + \frac{1}{105}\left(104a^4\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} - \frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}} - \frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax(-1+ax)}(15+39ax+52a^2x^2+104a^3x^3)}{105ax^4}$$

```
[In] Integrate[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*
a*x^4)
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

method	result	size
gosper	$-\frac{2\sqrt{-ax+1}\,(104a^3x^3+52a^2x^2+39ax+15)}{105x^3\sqrt{ax}}$	41
default	$-\frac{2\sqrt{-ax+1}\,\operatorname{csgn}(a)^2\,(104a^3x^3+52a^2x^2+39ax+15)}{105x^3\sqrt{ax}}$	45
risch	$\frac{2\sqrt{ax(-ax+1)}\,(104a^4x^4-52a^3x^3-13a^2x^2-24ax-15)}{105\sqrt{ax}\,\sqrt{-ax+1}\,x^3\,\sqrt{-x(ax-1)a}}$	71
meijerg	$-\frac{2a\left(\frac{8}{3}a^2x^2+\frac{4}{3}ax+1\right)\sqrt{-ax+1}}{5\sqrt{ax}\,x^2}-\frac{2\left(\frac{16}{5}a^3x^3+\frac{8}{5}a^2x^2+\frac{6}{5}ax+1\right)\sqrt{-ax+1}}{7\sqrt{ax}\,x^3}$	75

```
[In] int((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/105/x^3/(a*x)^(1/2)*(-a*x+1)^(1/2)*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.44

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\,(104\,a^3x^3+52\,a^2x^2+39\,ax+15)\sqrt{ax}\sqrt{-ax+1}}{105\,ax^4}$$

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt(a*x)*sqrt(-a*x + 1)/(a
*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.00 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.82

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= a\left(\begin{cases} -\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15} - \frac{8a\sqrt{-1+\frac{1}{ax}}}{15x} - \frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15} - \frac{8ia\sqrt{1-\frac{1}{ax}}}{15x} - \frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise} \end{cases}\right)$$
$$+ \begin{cases} -\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35} - \frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x} - \frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2} - \frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35} - \frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x} - \frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2} - \frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x+1)/x**4/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*x)
 - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1/(
a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2),
 True)) + Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(
a*x))/(35*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*
x**3), 1/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1
 - 1/(a*x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a
*x))/(7*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{208\sqrt{-a^2x^2+ax}a^2}{105\,x} - \frac{104\sqrt{-a^2x^2+ax}a}{105\,x^2} - \frac{26\sqrt{-a^2x^2+ax}}{35\,x^3} - \frac{2\sqrt{-a^2x^2+ax}}{7\,ax^4}$$

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -208/105*sqrt(-a^2*x^2 + a*x)*a^2/x - 104/105*sqrt(-a^2*x^2 + a*x)*a/x^2 -
26/35*sqrt(-a^2*x^2 + a*x)/x^3 - 2/7*sqrt(-a^2*x^2 + a*x)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.80

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx =$$
$$-\frac{\frac{15\,a^4\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{231\,a^4\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{1435\,a^4\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{7875\,a^4\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(15\,a^4+\frac{231\,a^3\left(\sqrt{-ax+1}-1\right)^2}{x}+143\cdots\right.}{(\cdots}}{6720\,a}$$

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6720*(15*a^4*(sqrt(-a*x + 1) - 1)^7/(a*x)^(7/2) + 231*a^4*(sqrt(-a*x + 1
) - 1)^5/(a*x)^(5/2) + 1435*a^4*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 7875*a
^4*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (15*a^4 + 231*a^3*(sqrt(-a*x + 1) - 1)^
2/x + 1435*a^2*(sqrt(-a*x + 1) - 1)^4/x^2 + 7875*a*(sqrt(-a*x + 1) - 1)^6/x
^3)*(a*x)^(7/2)/(sqrt(-a*x + 1) - 1)^7)/a
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\,\left(\frac{208\,a^3\,x^3}{105}+\frac{104\,a^2\,x^2}{105}+\frac{26\,a\,x}{35}+\frac{2}{7}\right)}{x^3\,\sqrt{a\,x}}$$

```
[In] int((a*x + 1)/(x^4*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((26*a*x)/35 + (104*a^2*x^2)/105 + (208*a^3*x^3)/105 + 2/
7))/(x^3*(a*x)^(1/2))
```

3.30 $\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} - \frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}$$

```
[Out] -2/9*a^4*(-a*x+1)^(1/2)/(a*x)^(9/2)-34/63*a^4*(-a*x+1)^(1/2)/(a*x)^(7/2)-68
/105*a^4*(-a*x+1)^(1/2)/(a*x)^(5/2)-272/315*a^4*(-a*x+1)^(1/2)/(a*x)^(3/2)-
544/315*a^4*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 79, 47, 37}

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

```
[In] Int[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^4*Sqrt[1 - a*x])/(9*(a*x)^(9/2)) - (34*a^4*Sqrt[1 - a*x])/(63*(a*x)^(
7/2)) - (68*a^4*Sqrt[1 - a*x])/(105*(a*x)^(5/2)) - (272*a^4*Sqrt[1 - a*x])/
(315*(a*x)^(3/2)) - (544*a^4*Sqrt[1 - a*x])/(315*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a^5 \int \frac{1+ax}{(ax)^{11/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} + \frac{1}{9}\left(17a^5\right)\int \frac{1}{(ax)^{9/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} + \frac{1}{21}\left(34a^5\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} + \frac{1}{105}\left(136a^5\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx
\end{aligned}
$$

$$= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} + \frac{1}{315}\left(272a^5\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx$$

$$= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} - \frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax(-1+ax)}(35+85ax+102a^2x^2+136a^3x^3+272a^4x^4)}{315ax^5}$$

```
[In] Integrate[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*
a^4*x^4))/(315*a*x^5)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

method	result	size
gosper	$-\frac{2\sqrt{-ax+1}\,(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)}{315x^4\sqrt{ax}}$	49
default	$-\frac{2\sqrt{-ax+1}\,\operatorname{csgn}(a)^2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)}{315x^4\sqrt{ax}}$	53
risch	$\frac{2\sqrt{ax(-ax+1)}\,(272a^5x^5-136a^4x^4-34a^3x^3-17a^2x^2-50ax-35)}{315\sqrt{ax}\,\sqrt{-ax+1}\,x^4\,\sqrt{-x(ax-1)a}}$	79
meijerg	$-\frac{2a(\frac{16}{5}a^3x^3+\frac{8}{5}a^2x^2+\frac{6}{5}ax+1)\sqrt{-ax+1}}{7\sqrt{ax}\,x^3}-\frac{2(\frac{128}{35}a^4x^4+\frac{64}{35}a^3x^3+\frac{48}{35}a^2x^2+\frac{8}{7}ax+1)\sqrt{-ax+1}}{9\sqrt{ax}\,x^4}$	91

```
[In] int((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/315/x^4/(a*x)^(1/2)*(-a*x+1)^(1/2)*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+
85*a*x+35)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\,(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35)\sqrt{ax}\sqrt{-ax+1}}{315\,ax^5}$$

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt(a*x)*sq
rt(-a*x + 1)/(a*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.14 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.97

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for } \frac{1}{|ax|}>1 \\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{256a^4\sqrt{-1+\frac{1}{ax}}}{315}-\frac{128a^3\sqrt{-1+\frac{1}{ax}}}{315x}-\frac{32a^2\sqrt{-1+\frac{1}{ax}}}{105x^2}-\frac{16a\sqrt{-1+\frac{1}{ax}}}{63x^3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{9x^4} & \text{for } \frac{1}{|ax|}>1 \\ -\frac{256ia^4\sqrt{1-\frac{1}{ax}}}{315}-\frac{128ia^3\sqrt{1-\frac{1}{ax}}}{315x}-\frac{32ia^2\sqrt{1-\frac{1}{ax}}}{105x^2}-\frac{16ia\sqrt{1-\frac{1}{ax}}}{63x^3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{9x^4} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x+1)/x**5/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(a*x))/(3
5*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*x**3), 1
/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1 - 1/(a*
x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a*x))/(7*
x**3), True)) + Piecewise((-256*a**4*sqrt(-1 + 1/(a*x))/315 - 128*a**3*sqrt
(-1 + 1/(a*x))/(315*x) - 32*a**2*sqrt(-1 + 1/(a*x))/(105*x**2) - 16*a*sqrt(
-1 + 1/(a*x))/(63*x**3) - 2*sqrt(-1 + 1/(a*x))/(9*x**4), 1/Abs(a*x) > 1), (
-256*I*a**4*sqrt(1 - 1/(a*x))/315 - 128*I*a**3*sqrt(1 - 1/(a*x))/(315*x) -
32*I*a**2*sqrt(1 - 1/(a*x))/(105*x**2) - 16*I*a*sqrt(1 - 1/(a*x))/(63*x**3)
 - 2*I*sqrt(1 - 1/(a*x))/(9*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{544\,\sqrt{-a^2x^2+ax}a^3}{315\,x} - \frac{272\,\sqrt{-a^2x^2+ax}a^2}{315\,x^2} - \frac{68\,\sqrt{-a^2x^2+ax}a}{105\,x^3} - \frac{34\,\sqrt{-a^2x^2+ax}}{63\,x^4} - \frac{2\,\sqrt{-a^2x^2+ax}}{9\,ax^5}$$

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -544/315*sqrt(-a^2*x^2 + a*x)*a^3/x - 272/315*sqrt(-a^2*x^2 + a*x)*a^2/x^2
- 68/105*sqrt(-a^2*x^2 + a*x)*a/x^3 - 34/63*sqrt(-a^2*x^2 + a*x)/x^4 - 2/9*
sqrt(-a^2*x^2 + a*x)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(91) = 182.

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\frac{35\,a^5\left(\sqrt{-ax+1}-1\right)^9}{(ax)^{\frac{9}{2}}} + \frac{585\,a^5\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}} + \frac{4032\,a^5\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}} + \frac{17640\,a^5\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}} + \frac{83790\,a^5\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} - \frac{\left(35\,a^5 + \cdots\right)\cdots}{\cdots}}{80640\,a}$$

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80640*(35*a^5*(sqrt(-a*x + 1) - 1)^9/(a*x)^(9/2) + 585*a^5*(sqrt(-a*x +
1) - 1)^7/(a*x)^(7/2) + 4032*a^5*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 17640
*a^5*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 83790*a^5*(sqrt(-a*x + 1) - 1)/sq
rt(a*x) - (35*a^5 + 585*a^4*(sqrt(-a*x + 1) - 1)^2/x + 4032*a^3*(sqrt(-a*x
+ 1) - 1)^4/x^2 + 17640*a^2*(sqrt(-a*x + 1) - 1)^6/x^3 + 83790*a*(sqrt(-a*x
 + 1) - 1)^8/x^4)*(a*x)^(9/2)/(sqrt(-a*x + 1) - 1)^9)/a
```

Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.40

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\,\left(\frac{544\,a^4\,x^4}{315}+\frac{272\,a^3\,x^3}{315}+\frac{68\,a^2\,x^2}{105}+\frac{34\,a\,x}{63}+\frac{2}{9}\right)}{x^4\,\sqrt{a\,x}}$$

```
[In] int((a*x + 1)/(x^5*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
[Out] -((1 - a*x)^(1/2)*((34*a*x)/63 + (68*a^2*x^2)/105 + (272*a^3*x^3)/315 + (54
4*a^4*x^4)/315 + 2/9))/(x^4*(a*x)^(1/2))
```

3.31 $\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + 2a\arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

```
[Out] 2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {156, 12, 94, 209}

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = 2a\arctan\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[In] Int[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \int \frac{2a}{\sqrt{-1+x}x\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + (2a)\int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + (2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\sqrt{1+x}\right) \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + 2a\tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + 4a\arctan\left(\sqrt{\frac{-1+x}{1+x}}\right)$$

```
[In] Integrate[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 4*a*ArcTan[Sqrt[(-1 + x)/(1 + x)]]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{-1+x}\,\sqrt{1+x}}{x\sqrt{x^2-1}}$	44
risch	$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}-\frac{2a\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(-1+x)(1+x)}}{\sqrt{-1+x}\,\sqrt{1+x}}$	47

```
[In] int((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/(
x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx=\frac{4\,ax\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.00

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}} + \frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}}$$

```
[In] integrate((2*a*x-1)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -2\,a\arcsin\left(\frac{1}{|x|}\right) - \frac{\sqrt{x^2-1}}{x}$$

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right) - \frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}$$

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{x-1}\,\sqrt{x+1}}{x} - a\left(\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right) - \ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)\right)2\mathrm{i}$$

```
[In] int((2*a*x - 1)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] - a*(log(((x - 1)^(1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1) - log(((x - 1)^(
1/2) - 1i)/((x + 1)^(1/2) - 1)))*2i - ((x - 1)^(1/2)*(x + 1)^(1/2))/x
```

3.32 $\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 39

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + 2a\arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

```
[Out] 2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {194, 156, 12, 94, 209}

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = 2a\arctan\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[In] Int[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 194

```
Int[(u_)^(m_.)*(v_)^(n_.)*(w_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[ExpandToS
um[u, x]^m*ExpandToSum[v, x]^n*ExpandToSum[w, x]^p*ExpandToSum[z, x]^q, x]
/; FreeQ[{m, n, p, q}, x] && LinearQ[{u, v, w, z}, x] &&  !LinearMatchQ[{u,
 v, w, z}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \int \frac{2a}{\sqrt{-1+x}x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + (2a)\int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + (2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\sqrt{1+x}\right)\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + 2a\tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+4a\arctan\left(\sqrt{\frac{-1+x}{1+x}}\right)$$

```
[In] Integrate[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 4*a*ArcTan[Sqrt[(-1 + x)/(1 + x)]]
```

Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{-1+x}\,\sqrt{1+x}}{x\sqrt{x^2-1}}$	44
risch	$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}-\frac{2a\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(-1+x)(1+x)}}{\sqrt{-1+x}\,\sqrt{1+x}}$	47

```
[In] int((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBO
SE)
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/(
x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = \frac{4\,ax\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="f
ricas")
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.00

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\,{\frac{1}{x^2}}\right)}{2\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\,{\frac{e^{2i\pi}}{x^2}}\right)}{2\pi^{\frac{3}{2}}} + \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\,{\frac{1}{x^2}}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\,{\frac{e^{2i\pi}}{x^2}}\right)}{4\pi^{\frac{3}{2}}}$$

```
[In] integrate((a**2*x**2-(-a*x+1)**2)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -2\,a\arcsin\left(\frac{1}{|x|}\right) - \frac{\sqrt{x^2-1}}{x}$$

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}$$

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 444, normalized size of antiderivative = 11.38

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = a\,\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)2\mathrm{i}-a^2\,\mathrm{atan}\left(\frac{1024\,a^6}{1024\,a^5+1024\,a^7+\frac{a^6\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}}+\frac{1024\,a^8}{1024\,a^5+1024\,a^7+\frac{a^6\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}}-\frac{a^5\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}\right)}-\frac{a^7\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\,\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}\right)}\right)4\mathrm{i}-a\,\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)2\mathrm{i}-\frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)}+a^2\,\mathrm{acosh}(x)-\frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{4\left(\sqrt{x+1}-1\right)^2}+\frac{1}{4}}{\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}}$$

```
[In] int(-((a*x - 1)^2 - a^2*x^2)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] a*log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*2i - a^2*atan((1024*a^6)/(1
024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) +
 (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) + (1024*a^8)/(1024*a
```

```
^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8
*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) - (a^5*((x - 1)^(1/2) - 1
i)*1024i)/(((x + 1)^(1/2) - 1)*(1024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) -
 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)
^(1/2) - 1))) - (a^7*((x - 1)^(1/2) - 1i)*1024i)/(((x + 1)^(1/2) - 1)*(1024
*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a
^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1))))*4i - a*log(((x - 1)^(
1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1)*2i - ((x - 1)^(1/2) - 1i)/(4*((x +
1)^(1/2) - 1)) + a^2*acosh(x) - ((5*((x - 1)^(1/2) - 1i)^2)/(4*((x + 1)^(1/
2) - 1)^2) + 1/4)/(((x - 1)^(1/2) - 1i)^3/((x + 1)^(1/2) - 1)^3 + ((x - 1)^
(1/2) - 1i)/((x + 1)^(1/2) - 1))
```

3.33 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 145

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{2a^{3/2}BE\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)} + \frac{2\sqrt{a}(aBe+A(b-be))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right),\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}$$

```
[Out] -2*a^(3/2)*B*EllipticE((1-c)^(1/2)*(b*x+a)^(1/2)/a^(1/2),((1-e)/(1-c))^(1/2
))/b^2/(1-e)/(1-c)^(1/2)+2*(a*B*e+A*(-b*e+b))*EllipticF((1-c)^(1/2)*(b*x+a)
^(1/2)/a^(1/2),((1-e)/(1-c))^(1/2))*a^(1/2)/b^2/(1-e)/(1-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {164, 114, 120}

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = \frac{2\sqrt{a}(aBe+A(b-be))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right),\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)} - \frac{2a^{3/2}BE\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}$$

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(-1 + e
)*x)/a]),x]
```

```
[Out] (-2*a^(3/2)*B*EllipticE[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e
)/(1 - c)])/(b^2*Sqrt[1 - c]*(1 - e)) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*El
lipticF[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e)/(1 - c)])/(b^2
*Sqrt[1 - c]*(1 - e))
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\text{integral} = -\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}}\,dx}{b(1-e)} + \left(A+\frac{aBe}{b-be}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$$

$$= -\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)} + \frac{2\sqrt{a}\left(A+\frac{aBe}{b-be}\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b\sqrt{1-c}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.59 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx =$$

$$-\frac{2\sqrt{\frac{a}{-1+c}}(a+bx)^{3/2}\left(-B\sqrt{\frac{a}{-1+c}}\left(-1+c+\frac{a}{a+bx}\right)\left(-1+e+\frac{a}{a+bx}\right)-\frac{iaB(-1+e)\sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+c}}\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}E\Big(}{\sqrt{a+bx}}\right.}{ab^2(-1+e)\sqrt{c+\frac{b(-1}{}}}$$

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(
-1 + e)*x)/a]),x]
```

```
[Out] (-2*Sqrt[a/(-1 + c)]*(a + b*x)^(3/2)*(-(B*Sqrt[a/(-1 + c)]*(-1 + c + a/(a +
 b*x))*(-1 + e + a/(a + b*x))) - (I*a*B*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x)
)/(-1 + c)]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[
a/(-1 + c)]/Sqrt[a + b*x]], (-1 + c)/(-1 + e)])/Sqrt[a + b*x] + (I*(a*B*c +
 A*(b - b*c))*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x))/(-1 + c)]*Sqrt[(-1 + e +
 a/(a + b*x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + c)]/Sqrt[a + b*x]]
, (-1 + c)/(-1 + e)])/Sqrt[a + b*x]))/(a*b^2*(-1 + e)*Sqrt[c + (b*(-1 + c)*
x)/a]*Sqrt[e + (b*(-1 + e)*x)/a])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(127) = 254$.

Time = 5.52 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.17

method	result
default	$\frac{2\left(AF\left(\sqrt{\frac{(c-1)(bex+ae-bx)}{a(c-e)}},\sqrt{\frac{c-e}{c-1}}\right)bce-AF\left(\sqrt{\frac{(c-1)(bex+ae-bx)}{a(c-e)}},\sqrt{\frac{c-e}{c-1}}\right)be^2-BF\left(\sqrt{\frac{(c-1)(bex+ae-bx)}{a(c-e)}},\sqrt{\frac{c-e}{c-1}}\right)ace+BF\left(\sqrt{\frac{(c-1}{}}\right.\right.}{}$
elliptic	$\frac{\sqrt{\frac{(bx+a)(bcx+ac-bx)(bex+ae-bx)}{a^2}}\left(\frac{2A\left(-\frac{ae}{b(-1+e)}+\frac{ac}{b(c-1)}\right)\sqrt{\frac{x+\frac{ac}{b(c-1)}}{-\frac{ae}{b(-1+e)}+\frac{ac}{b(c-1)}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{ac}{b(c-1)}+\frac{a}{b}}}\sqrt{\frac{x+\frac{ae}{b(-1+e)}}{\frac{ae}{b(-1+e)}-\frac{ac}{b(c-1)}}}F\left(\sqrt{\frac{x+\frac{ac}{b(c-}}{-\frac{ae}{b(-1+e)}+}}\right.}{\sqrt{\frac{b^3cex^3}{a^2}+\frac{3b^2cex^2}{a}-\frac{b^3cx^3}{a^2}-\frac{b^3ex^3}{a^2}+3bcex-\frac{2b^2cx^2}{a}-\frac{2b^2ex^2}{a}+\frac{b^3x^3}{a^2}+ace-bcx-}}\right.}{}$

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(c+b*(c-1)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2*(A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^(1/2))*b
*c*e-A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^(1/2))
*b*e^2-B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^(1/2
))*a*c*e+B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^(1
/2))*a*e^2-A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^
(1/2))*b*c+A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^
(1/2))*b*e+B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^
(1/2))*a*c-B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^
(1/2))*a*e-B*EllipticE(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^
(1/2))*a*c+B*EllipticE(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^
(1/2))*a*e)*(-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^(1/2)*(-(b*x+a)*(-1+e)/a)^(1/
2)*((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2)*a/(b*x+a)^(1/2)/((b*c*x+a*c-b*x)/a
)^(1/2)/((b*e*x+a*e-b*x)/a)^(1/2)/(-1+e)^2/(c-1)/b^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 1228, normalized size of antiderivative = 8.47

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="fricas")
```

```
[Out] -2/3*((B*a^3 - 3*A*a^2*b - (2*B*a^3 - 3*A*a^2*b)*c - (2*B*a^3 - 3*A*a^2*b -
 3*(B*a^3 - A*a^2*b)*c)*e)*sqrt(-(b^3*c - b^3 - (b^3*c - b^3)*e)/a^2)*weier
strassPInverse(4/3*(a^2*c^2 + a^2*e^2 - a^2*c + a^2 - (a^2*c + a^2)*e)/(b^2
*c^2 - 2*b^2*c + (b^2*c^2 - 2*b^2*c + b^2)*e^2 + b^2 - 2*(b^2*c^2 - 2*b^2*c
 + b^2)*e), 4/27*(2*a^3*c^3 + 2*a^3*e^3 - 3*a^3*c^2 - 3*a^3*c + 2*a^3 - 3*(
a^3*c + a^3)*e^2 - 3*(a^3*c^2 - 4*a^3*c + a^3)*e)/(b^3*c^3 - 3*b^3*c^2 + 3*
b^3*c - (b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^3 - b^3 + 3*(b^3*c^3 - 3*b^
3*c^2 + 3*b^3*c - b^3)*e^2 - 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e), 1/
3*(2*a*c - (3*a*c - 2*a)*e + 3*(b*c - (b*c - b)*e - b)*x - a)/(b*c - (b*c -
 b)*e - b)) - 3*(B*a^2*b*c - B*a^2*b - (B*a^2*b*c - B*a^2*b)*e)*sqrt(-(b^3*
c - b^3 - (b^3*c - b^3)*e)/a^2)*weierstrassZeta(4/3*(a^2*c^2 + a^2*e^2 - a^
2*c + a^2 - (a^2*c + a^2)*e)/(b^2*c^2 - 2*b^2*c + (b^2*c^2 - 2*b^2*c + b^2)
*e^2 + b^2 - 2*(b^2*c^2 - 2*b^2*c + b^2)*e), 4/27*(2*a^3*c^3 + 2*a^3*e^3 -
3*a^3*c^2 - 3*a^3*c + 2*a^3 - 3*(a^3*c + a^3)*e^2 - 3*(a^3*c^2 - 4*a^3*c +
a^3)*e)/(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - (b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b
^3)*e^3 - b^3 + 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^2 - 3*(b^3*c^3 -
3*b^3*c^2 + 3*b^3*c - b^3)*e), weierstrassPInverse(4/3*(a^2*c^2 + a^2*e^2 -
```

```
a^2*c + a^2 - (a^2*c + a^2)*e)/(b^2*c^2 - 2*b^2*c + (b^2*c^2 - 2*b^2*c + b
^2)*e^2 + b^2 - 2*(b^2*c^2 - 2*b^2*c + b^2)*e), 4/27*(2*a^3*c^3 + 2*a^3*e^3
 - 3*a^3*c^2 - 3*a^3*c + 2*a^3 - 3*(a^3*c + a^3)*e^2 - 3*(a^3*c^2 - 4*a^3*c
 + a^3)*e)/(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - (b^3*c^3 - 3*b^3*c^2 + 3*b^3*c
- b^3)*e^3 - b^3 + 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^2 - 3*(b^3*c^3
 - 3*b^3*c^2 + 3*b^3*c - b^3)*e), 1/3*(2*a*c - (3*a*c - 2*a)*e + 3*(b*c - (
b*c - b)*e - b)*x - a)/(b*c - (b*c - b)*e - b))))/(b^4*c^2 - 2*b^4*c + b^4
+ (b^4*c^2 - 2*b^4*c + b^4)*e^2 - 2*(b^4*c^2 - 2*b^4*c + b^4)*e)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{bcx}{a}-\frac{bx}{a}}\sqrt{e+\frac{bex}{a}-\frac{bx}{a}}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(c+b*(-1+c)*x/a)**(1/2)/(e+b*(-1+e)*x/a)**
(1/2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + b*c*x/a - b*x/a)*sqrt(e + b*e*x/
a - b*x/a)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*(e - 1)*x
/a + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Recursive assumption sageVARx>=(-sageVARa) ignoredsym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+B\,x}{\sqrt{c+\frac{b\,x\,(c-1)}{a}}\,\sqrt{e+\frac{b\,x\,(e-1)}{a}}\,\sqrt{a+b\,x}}\,dx$$

```
[In] int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a +
 b*x)^(1/2)),x)
[Out] int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a +
 b*x)^(1/2)), x)
```

3.34 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 221

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{2aB\sqrt{-bc+ad}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}(1-e)\sqrt{c+dx}} + \frac{2\sqrt{a}(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right),-\frac{ad}{(bc-ad)(1-e)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}$$

```
[Out] 2*(a*B*e+A*(-b*e+b))*EllipticF((1-e)^(1/2)*(b*x+a)^(1/2)/a^(1/2),(-a*d/(-a*
d+b*c)/(1-e))^(1/2))*a^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)/b^2/(1-e)^(3/2)/(
d*x+c)^(1/2)-2*a*B*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),(-(-a*d+
b*c)*(1-e)/a/d)^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)/b^2/(1-
e)/d^(1/2)/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used

$= \{164, 115, 114, 122, 120\}$

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$$
$$= \frac{2\sqrt{a}(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right),-\frac{ad}{(bc-ad)(1-e)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}$$
$$-\frac{2aB\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}(1-e)\sqrt{c+dx}}$$

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]),x]
```

```
[Out] (-2*a*B*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticE[ArcSin
[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], -(((b*c - a*d)*(1 - e))/(a*d)
)])/(b^2*Sqrt[d]*(1 - e)*Sqrt[c + d*x]) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticF[ArcSin[(Sqrt[1 - e]*Sqrt[a + b*x]
)/Sqrt[a]], -((a*d)/((b*c - a*d)*(1 - e)))])/(b^2*(1 - e)^(3/2)*Sqrt[c + d*
x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
```

```
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\text{integral} = -\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b(1-e)} + \left(A+\frac{aBe}{b-be}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$$

$$= \frac{\left(\left(A+\frac{aBe}{b-be}\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx}{\sqrt{c+dx}} - \frac{\left(aB\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+\frac{b(-1+e)x}{a}}\right)\int \frac{\sqrt{\frac{be}{-b(-1+e)+be}+\frac{b^2(-1+e)x}{a(-b(-1+e)+be)}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{b(1-e)\sqrt{c+dx}\sqrt{\frac{b\left(e+\frac{b(-1+e)x}{a}\right)}{-b(-1+e)+be}}}$$

$$= -\frac{2aB\sqrt{-bc+ad}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}(1-e)\sqrt{c+dx}} + \frac{2\sqrt{a}(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right)|-\frac{ad}{(bc-ad)(1-e)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx =$$

$$-\frac{2\sqrt{\frac{a}{-1+e}}(a+bx)^{3/2}\left(-\frac{bB\sqrt{\frac{a}{-1+e}}(c+dx)(ae+b(-1+e)x)}{(a+bx)^2}-\frac{iaBd\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+bx}}\right)|\frac{(bc-ad)(-1+e}{ad}\right.}{\sqrt{a+bx}}\right.}{ab^2d\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}$$

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]
),x]
```

```
[Out] (-2*Sqrt[a/(-1 + e)]*(a + b*x)^(3/2)*(-((b*B*Sqrt[a/(-1 + e)]*(c + d*x)*(a*
e + b*(-1 + e)*x))/(a + b*x)^2) - (I*a*B*d*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[a/(-1 + e)
]/Sqrt[a + b*x]], ((b*c - a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x] + (I*d*(a*B*
e + A*(b - b*e))*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(-1 + e + a/(a + b*
x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + e)]/Sqrt[a + b*x]], ((b*c -
a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x]))/(a*b^2*d*Sqrt[c + d*x]*Sqrt[e + (b*(
-1 + e)*x)/a])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(195) = 390.

Time = 2.96 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.30

method	result
elliptic	$\frac{\sqrt{\frac{(bx+a)(dx+c)(bex+ae-bx)}{a}}\left(\frac{2A\left(\frac{ae}{b(-1+e)}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{ae}{b(-1+e)}}{\frac{ae}{b(-1+e)}-\frac{c}{d}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{ae}{b(-1+e)}+\frac{a}{b}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{ae}{b(-1+e)}+\frac{c}{d}}}F\left(\sqrt{\frac{x+\frac{ae}{b(-1+e)}}{\frac{ae}{b(-1+e)}-\frac{c}{d}}},\sqrt{\frac{-\frac{ae}{b(-1+e)}+\frac{c}{d}}{-\frac{ae}{b(-1+e)}+\frac{a}{b}}}\right)}{\sqrt{\frac{b^2dex^3}{a}+2bdex^2+\frac{b^2cex^2}{a}-\frac{dx^3b^2}{a}+adex+2bcex-bdx^2-\frac{b^2cx^2}{a}+ace-bcx}}+\right.}{\sqrt{}}$
default	$\frac{2\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d(bex+ae-bx)}{ade-bce+bc}}\sqrt{-\frac{(bx+a)(-1+e)}{a}}\sqrt{-\frac{(dx+c)b(-1+e)}{ade-bce+bc}}\left(AF\left(\sqrt{\frac{d(bex+ae-bx)}{ade-bce+bc}},\sqrt{\frac{ade-bce+bc}{da}}\right)abde^2-AF\left(\sqrt{\frac{d(bex+a}{ade-bc}}\right.\right.}{}$

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/((b*e*x+a*e-b*x)/a)^(1/2)*((b*x+a)*(d*x+c)*(b
*e*x+a*e-b*x)/a)^(1/2)*(2*A*(a*e/b/(-1+e)-c/d)*((x+a*e/b/(-1+e))/(a*e/b/(-1
+e)-c/d))^(1/2)*((x+a/b)/(-a*e/b/(-1+e)+a/b))^(1/2)*((x+c/d)/(-a*e/b/(-1+e)
+c/d))^(1/2)/(1/a*b^2*d*e*x^3+2*b*d*e*x^2+1/a*b^2*c*e*x^2-1/a*d*x^3*b^2+a*d
*e*x+2*b*c*e*x-b*d*x^2-1/a*b^2*c*x^2+a*c*e-b*c*x)^(1/2)*EllipticF(((x+a*e/b
/(-1+e))/(a*e/b/(-1+e)-c/d))^(1/2),((-a*e/b/(-1+e)+c/d)/(-a*e/b/(-1+e)+a/b)
)^(1/2))+2*B*(a*e/b/(-1+e)-c/d)*((x+a*e/b/(-1+e))/(a*e/b/(-1+e)-c/d))^(1/2)
*((x+a/b)/(-a*e/b/(-1+e)+a/b))^(1/2)*((x+c/d)/(-a*e/b/(-1+e)+c/d))^(1/2)/(1
/a*b^2*d*e*x^3+2*b*d*e*x^2+1/a*b^2*c*e*x^2-1/a*d*x^3*b^2+a*d*e*x+2*b*c*e*x-
b*d*x^2-1/a*b^2*c*x^2+a*c*e-b*c*x)^(1/2)*((-a*e/b/(-1+e)+a/b)*EllipticE(((x
+a*e/b/(-1+e))/(a*e/b/(-1+e)-c/d))^(1/2),((-a*e/b/(-1+e)+c/d)/(-a*e/b/(-1+e
)+a/b))^(1/2))-a/b*EllipticF(((x+a*e/b/(-1+e))/(a*e/b/(-1+e)-c/d))^(1/2),((
-a*e/b/(-1+e)+c/d)/(-a*e/b/(-1+e)+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 1126, normalized size of antiderivative = 5.10

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/3*((B*a*b*c + (B*a^2 - 3*A*a*b)*d - (B*a*b*c + (2*B*a^2 - 3*A*a*b)*d)*e)*
sqrt((b^2*d*e - b^2*d)/a)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*
d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2*d^
2)*e)/(b^2*d^2*e^2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*e^3 + 3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*e^
2 - 3*(2*b^3*c^3 - 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^3*e^3 -
 3*b^3*d^3*e^2 + 3*b^3*d^3*e - b^3*d^3), -1/3*(b*c + a*d - (b*c + 2*a*d)*e
- 3*(b*d*e - b*d)*x)/(b*d*e - b*d)) - 3*(B*a*b*d*e - B*a*b*d)*sqrt((b^2*d*e
 - b^2*d)/a)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2 + (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*e)/(b^2*d^2*e^
2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*e^3 +
3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*e^2 - 3*(2*b^3*c^3
- 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^3*e^3 - 3*b^3*d^3*e^2 +
3*b^3*d^3*e - b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^
2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)
```

```
*e)/(b^2*d^2*e^2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*e^3 + 3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*e^2
- 3*(2*b^3*c^3 - 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^3*e^3 - 3
*b^3*d^3*e^2 + 3*b^3*d^3*e - b^3*d^3), -1/3*(b*c + a*d - (b*c + 2*a*d)*e -
3*(b*d*e - b*d)*x)/(b*d*e - b*d))))/(b^3*d^2*e^2 - 2*b^3*d^2*e + b^3*d^2)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{bex}{a}-\frac{bx}{a}}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(e+b*(-1+e)*x/a)**(1/2),x)
[Out] Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + b*e*x/a - b*x/a)),
 x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="maxima")
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="giac")
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+B\,x}{\sqrt{e+\frac{b\,x\,(e-1)}{a}}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))
,x)
[Out] int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))
, x)
```

3.35 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$

Optimal result

Integrand size = 35, antiderivative size = 281

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 - \frac{6489123157\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{699840\sqrt{5-2x}} + \frac{522167393\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{23328\sqrt{-5+2x}}$$

```
[Out] 522167393/139968*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2
)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)-6489123157/699840*EllipticE(2/11*(2-3*x)^(1/
2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-1182926269
/1603800*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-12243139/356400*(7+5*x)
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-17561/8910*(7+5*x)^2*(2-3*x)^(1
/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-427/2970*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^
(1/2)*(1+4*x)^(1/2)+2/55*(7+5*x)^4*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/
2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {167, 1614, 1629, 164, 115, 114, 122, 120}

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$$
$$= \frac{522167393\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{23328\sqrt{2x-5}} - \frac{6489123157\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{699840\sqrt{5-2x}} + \frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4 - \frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3}{2970} - \frac{17561\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}{8910} - \frac{12243139\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}{356400} - \frac{1182926269\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1603800}$$

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

```
[Out] (-1182926269*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1603800 - (1224313
9*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/356400 - (17561*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/8910 - (427*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/2970 + (2*Sqrt[2 - 3*x]*Sqrt[
-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^4)/55 - (6489123157*Sqrt[11]*Sqrt[-5 + 2*
x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(699840*Sqrt[5 - 2*
x]) + (522167393*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(23328*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
```

```
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) && !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && !LtQ[m,
-1]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
```

```
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4+\frac{1}{55}\int \frac{(7+5x)^3\left(-3-1190x+854x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&= -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970}\\
&\quad+\frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4-\frac{\int \frac{(7+5x)^2\left(386274+1593290x-1966832x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{11880}\\
&= -\frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910}-\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970}\\
&\quad+\frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4+\frac{\int \frac{(7+5x)\left(-1136748928-1303270640x+4113694704x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1995840}
\end{aligned}$$

$$= -\frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 - \frac{\int \frac{1970951691408-958810283760x-6359411622144x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{239500800}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 - \frac{\int \frac{413184248769600-1439027951296320x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{25866086400}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 + \frac{6489123157\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{233280} + \frac{5743841323\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{46656}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 + \frac{\left(522167393\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int\frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{23328\sqrt{-5+2x}} + \frac{\left(6489123157\sqrt{-5+2x}\right)\int\frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{233280\sqrt{5-2x}}$$

$$= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 - \frac{6489123157\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{699840\sqrt{5-2x}} + \frac{522167393\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{23328\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.48

$$\int\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$$

$$= \frac{24\sqrt{2-3x}\sqrt{1+4x}(3325071575 - 797747975x - 670058262x^2 - 167736600x^3 + 67338000x^4 + 2916000}{}$$

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(3325071575 - 797747975*x - 670058262*x^2 -
 167736600*x^3 + 67338000*x^4 + 29160000*x^5) - 71380354727*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 57438413230*Sqrt
[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(15396
480*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.55

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(-8398080000x^{7}-15894144000x^{6}+57788380800x^{5}+29554530236\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44}}{11}\right.\right.}{15}$
risch	$-\frac{\left(14580000x^{4}+70119000x^{3}+91429200x^{2}-106456131x-665014315\right)\left(-2+3x\right)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{\left(2-3x\right)\left(-5+2x\right)\left(1+4x\right)}}{641520\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{2-3x}}-\frac{\left(-\frac{10}{}\right.}{}$
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\left(-\frac{11828459x\sqrt{-24x^{3}+70x^{2}-21x-10}}{71280}-\frac{133002863\sqrt{-24x^{3}+70x^{2}-21x-10}}{128304}-\frac{1026559\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-4}}{7776\sqrt{-24x^{3}+70x^{2}-}}\right.}{}$

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/15396480*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-8398080000*x^7-158
94144000*x^6+57788380800*x^5+29554530236*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/
2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-71380354727*(1+4*x
)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2)
,3^(1/2))+176080611456*x^4+141293068560*x^3-1085513167176*x^2+360716686200*
x+159603435600)/(24*x^3-70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.25

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$$
$$= \frac{1}{641520}\left(14580000\,x^4+70119000\,x^3+91429200\,x^2-106456131\,x-665014315\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}$$
$$-\frac{32008789087}{5038848}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$+\frac{6489123157}{699840}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/641520*(14580000*x^4 + 70119000*x^3 + 91429200*x^2 - 106456131*x - 665014
315)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 32008789087/5038848*sqrt(
-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 6489123157/699840*
sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6
655/2916, x - 35/36))
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^3\,dx$$

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**3, x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \int (5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \int (5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^3\,dx$$

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3,x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3, x)
```

3.36 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 243

$$
\begin{aligned}
&\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx \\
&= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200}-\frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700} \\
&\quad -\frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 \\
&\quad -\frac{17746949\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{29160\sqrt{5-2x}} \\
&\quad +\frac{5592499\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{3888\sqrt{-5+2x}}
\end{aligned}
$$

```
[Out] 5592499/23328*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)-17746949/29160*EllipticE(2/11*(2-3*x)^(1/2)*11^
(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-5256763/97200*(2
-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-8141/2700*(7+5*x)*(2-3*x)^(1/2)*(-
5+2*x)^(1/2)*(1+4*x)^(1/2)-61/270*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1
+4*x)^(1/2)+2/45*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {167, 1614, 1629, 164, 115, 114, 122, 120}

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx$$
$$= \frac{5592499\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{3888\sqrt{2x-5}} - \frac{17746949\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{29160\sqrt{5-2x}} + \frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3 - \frac{61}{270}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2 - \frac{8141\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}{2700} - \frac{5256763\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{97200}$$

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

```
[Out] (-5256763*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/97200 - (8141*Sqrt[2
- 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/2700 - (61*Sqrt[2 - 3*x]*Sqr
t[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/270 + (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/45 - (17746949*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipti
cE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(29160*Sqrt[5 - 2*x]) + (5592
499*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3])/(3888*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
```

```
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3+\frac{1}{45}\int\frac{(7+5x)^2\left(-3-1190x+854x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&= -\frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad+\frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3-\frac{\int\frac{(7+5x)\left(299866+1013390x-1367688x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{7560}\\
&= -\frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700}-\frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad+\frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3+\frac{\int\frac{-589706376-121654680x+1766272368x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{907200}\\
&= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200}-\frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700}\\
&\quad-\frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad+\frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3+\frac{\int\frac{-119325868200+357778491840x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{97977600}
\end{aligned}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700} - \frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 + \frac{17746949\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{9720} + \frac{61517489\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{7776}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700} - \frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 + \frac{\left(5592499\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int\frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{3888\sqrt{-5+2x}} + \frac{\left(17746949\sqrt{-5+2x}\right)\int\frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{9720\sqrt{5-2x}}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700} - \frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 - \frac{17746949\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{29160\sqrt{5-2x}} + \frac{5592499\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{3888\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 4.89 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.53

$$\int\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx$$

$$= \frac{6\sqrt{2-3x}\sqrt{1+4x}(6902575-2933650x-1649952x^2+147600x^3+216000x^4)-35493898\sqrt{66}\sqrt{5-2x}}{116640\sqrt{-5+}}$$

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(6902575 - 2933650*x - 1649952*x^2 + 147600*
x^3 + 216000*x^4) - 35493898*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3] + 27962495*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(116640*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.61

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(-15552000x^6-4147200x^5+12899689\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-35493898\sqrt{1+4x}\,\sqrt{\ldots}\right.}{116640(24x^3-70x^2+21x+\ldots}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{959x\sqrt{-24x^3+70x^2-21x-10}}{540}-\frac{276103\sqrt{-24x^3+70x^2-21x-10}}{3888}-\frac{26089\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+4\ldots}}{11}\ldots\right.}{2592\sqrt{-24x^3+70x^2-21x-10}}\ldots\right.}{\sqrt{2-\ldots}}$
risch	$-\frac{(108000x^3+343800x^2+34524x-1380515)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{19440\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{26089\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{\ldots}}{7776\sqrt{-24x^3+\ldots}}\ldots\right.}{\ldots}$

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/116640*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-15552000*x^6-4147200
*x^5+12899689*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(
1/11*(11+44*x)^(1/2),3^(1/2))-35493898*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)
*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+125816544*x^4+163495
440*x^3-604794324*x^2+171873450*x+82830900)/(24*x^3-70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx$$
$$= \frac{1}{19440}\left(108000\,x^3+343800\,x^2+34524\,x-1380515\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}$$
$$-\frac{163224523}{419904}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$+\frac{17746949}{29160}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/19440*(108000*x^3 + 343800*x^2 + 34524*x - 1380515)*sqrt(4*x + 1)*sqrt(2*
x - 5)*sqrt(-3*x + 2) - 163224523/419904*sqrt(-6)*weierstrassPInverse(847/1
08, 6655/2916, x - 35/36) + 17746949/29160*sqrt(-6)*weierstrassZeta(847/108
, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx=\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2\,dx$$

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2, x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx=\int (5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \int (5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^2\,dx$$

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2,x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2, x)
```

3.37 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 193

$$\begin{aligned}\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx &= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780}+\frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}\\ &\quad+\frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2}-\frac{954811\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{22680\sqrt{5-2x}}\\ &\quad+\frac{72479\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{756\sqrt{-5+2x}}\end{aligned}$$

```
[Out] 5/28*(-5+2*x)^(3/2)*(1+4*x)^(3/2)*(2-3*x)^(1/2)+72479/4536*EllipticF(1/11*3
3^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+13
6/105*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)-954811/22680*EllipticE(2/1
1*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/
2)-20911/3780*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{159, 164, 115, 114, 122, 120\}$

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx$$
$$= \frac{72479\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{756\sqrt{2x-5}} - \frac{954811\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{22680\sqrt{5-2x}} + \frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2} + \frac{136}{105}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2} - \frac{20911\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{3780}$$

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

```
[Out] (-20911*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3780 + (136*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/105 + (5*Sqrt[2 - 3*x]*(-5 + 2*x)^(3/2)*
(1 + 4*x)^(3/2))/28 - (954811*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*S
qrt[2 - 3*x])/Sqrt[11]], -1/2])/(22680*Sqrt[5 - 2*x]) + (72479*Sqrt[11/6]*S
qrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(756*Sqrt[-5
 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
```

```
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2}+\frac{1}{28}\int \frac{\left(\frac{1249}{2}-1088x\right)\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{2-3x}}\,dx\\
&= \frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}\\
&\quad +\frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2}-\frac{1}{840}\int \frac{(38731-41822x)\sqrt{1+4x}}{\sqrt{2-3x}\sqrt{-5+2x}}\,dx\\
&= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780}+\frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}\\
&\quad +\frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2}+\frac{\int \frac{-787710+1909622x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{7560}
\end{aligned}$$

$$= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2} + \frac{954811\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{7560} + \frac{797269\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1512}$$

$$= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2} + \frac{\left(72479\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{756\sqrt{-5+2x}} + \frac{\left(954811\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{7560\sqrt{5-2x}}$$

$$= -\frac{20911\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{5}{28}\sqrt{2-3x}(-5+2x)^{3/2}(1+4x)^{3/2} - \frac{954811\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{22680\sqrt{5-2x}} + \frac{72479\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{756\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.65

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx$$

$$= \frac{24\sqrt{2-3x}\sqrt{1+4x}(48475-37975x-6066x^2+5400x^3) - 954811\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)\right)}{45360\sqrt{-5+2x}}$$

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(48475 - 37975*x - 6066*x^2 + 5400*x^3) - 9
54811*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3] + 724790*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(45360*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(-1555200x^{5}+264748\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-954811\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2 \cdots}\right.}{45360(24x^{3}-70x^{2}+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{59x\sqrt{-24x^{3}+70x^{2}-21x-10}}{30}-\frac{277\sqrt{-24x^{3}+70x^{2}-21x-10}}{54}-\frac{31\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{\cdots}\right.}{36\sqrt{-24x^{3}+70x^{2}-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+\cdots}}$
risch	$-\frac{(2700x^{2}+3717x-9695)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{1890\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{31\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33\cdots}}{11}\right.}{108\sqrt{-24x^{3}+70x^{2}-21x-10}}\right.}{\cdots}$

```
[In] int((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/45360*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-1555200*x^5+264748*(1
+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(
1/2),3^(1/2))-954811*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ell
ipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2395008*x^4+10468080*x^3-18808968*x^2+
3994200*x+2326800)/(24*x^3-70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$
\begin{aligned}
&\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx \\
&= \frac{1}{1890}\left(2700\,x^{2}+3717\,x-9695\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\
&\quad -\frac{549703}{23328}\,\sqrt{-6}\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\
&\quad +\frac{954811}{22680}\,\sqrt{-6}\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)
\end{aligned}
$$

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
fricas")
```

[Out] 1/1890*(2700*x^2 + 3717*x - 9695)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2
) - 549703/23328*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36
) + 954811/22680*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPI
nverse(847/108, 6655/2916, x - 35/36))

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\cdot(5x+7)\,dx$$

[In] integrate((7+5*x)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)

[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7), x)

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \int (5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
maxima")

[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \int (5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
giac")

[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx=\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)\;dx$$

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7),x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7), x)
```

3.38 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 162

$$
\begin{aligned}
&\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx \\
&= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} \\
&\quad -\frac{847\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{270\sqrt{5-2x}} \\
&\quad +\frac{121\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{18\sqrt{-5+2x}}
\end{aligned}
$$

```
[Out] 121/108*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+1/10*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)-847/2
70*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(
1/2)/(5-2*x)^(1/2)-22/45*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {103, 159, 164, 115, 114, 122, 120}

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \frac{121\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{18\sqrt{2x-5}} - \frac{847\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{270\sqrt{5-2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2} - \frac{22}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

```
[Out] (-22*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/45 + (Sqrt[2 - 3*x]*Sqrt[-
5 + 2*x]*(1 + 4*x)^(3/2))/10 - (847*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(270*Sqrt[5 - 2*x]) + (121*Sqrt[11/6]
*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(18*Sqrt[-
5 + 2*x])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\text{integral} = \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{1}{10}\int \frac{\left(\frac{99}{2}-44x\right)\sqrt{1+4x}}{\sqrt{2-3x}\sqrt{-5+2x}}\,dx$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{1}{90}\int \frac{-\frac{1815}{2}+1694x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{847}{90}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx + \frac{1331}{36}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{\left(121\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{18\sqrt{-5+2x}} + \frac{\left(847\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{90\sqrt{5-2x}}$$

$$= -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{847\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{270\sqrt{5-2x}} + \frac{121\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{18\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \frac{6\sqrt{2-3x}\sqrt{1+4x}(175-250x+72x^2) - 847\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right) + 605\sqrt{66}\sqrt{5-}}{540\sqrt{-5+2x}}$$

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(175 - 250*x + 72*x^2) - 847*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 605*Sqrt[66]*Sqr
t[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(540*Sqrt[-5 +
 2*x])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(121\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-847\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-51}{540(24x^3-70x^2+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{2x\sqrt{-24x^3+70x^2-21x-10}}{5}-\frac{7\sqrt{-24x^3+70x^2-21x-10}}{18}-\frac{\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{12\sqrt{-24x^3+70x^2-21x-10}}+\frac{7\sqrt{}}{}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{(-35+36x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{90\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{36\sqrt{-24x^3+70x^2-21x-10}}+\frac{7\sqrt{22-}}{}\right.}{}$

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/540*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(121*(1+4*x)^(1/2)*(2-3*x
)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-847*
(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)
^(1/2),3^(1/2))-5184*x^4+20160*x^3-19236*x^2+2250*x+2100)/(24*x^3-70*x^2+21
*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\begin{aligned}
&\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx \\
&= \frac{1}{90}\,(36\,x-35)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\
&\quad - \frac{1331}{972}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\
&\quad + \frac{847}{270}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)
\end{aligned}$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/90*(36*x - 35)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 1331/972*sqrt
(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 847/270*sqrt(-6)*
weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916,
 x - 35/36))
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1), x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,dx$$

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2),x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2), x)
```

3.39 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 182

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx &= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{427\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}}\\ &\quad-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{375\sqrt{-5+2x}}\\ &\quad-\frac{2691\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{125\sqrt{11}\sqrt{-5+2x}}\end{aligned}$$

```
[Out] -1253/12375*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-
2*x)^(1/2)/(-5+2*x)^(1/2)-2691/1375*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),
55/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-427/225*Ellipti
cE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*
x)^(1/2)+2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules

used = {167, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx$$
$$= -\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{375\sqrt{2x-5}}$$
$$-\frac{427\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}}$$
$$-\frac{2691\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{125\sqrt{11}\sqrt{2x-5}}+\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/15 - (427*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(225*Sqrt[5 - 2*
x]) - (1253*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(375*Sqrt[-5 + 2*x]) - (2691*Sqrt[5 - 2*x]*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(125*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
```

```
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{15}\int\frac{-3-1190x+854x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx\\
&= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{15}\int\frac{-\frac{11928}{25}+\frac{854x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&\quad+\frac{27807}{125}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx\\
&= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{1253}{375}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&\quad+\frac{427}{75}\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx\\
&\quad-\frac{55614}{125}\text{Subst}\left(\int\frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx,x,\sqrt{2-3x}\right)\\
&= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{\left(1253\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int\frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{375\sqrt{-5+2x}}\\
&\quad-\frac{\left(55614\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int\frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx,x,\sqrt{2-3x}\right)}{125\sqrt{-5+2x}}\\
&\quad+\frac{\left(427\sqrt{-5+2x}\right)\int\frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{75\sqrt{5-2x}}
\end{aligned}
$$

$$= \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{427\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}} - \frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{375\sqrt{-5+2x}} - \frac{2691\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{125\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \frac{\sqrt{-5+2x}\left(1650\sqrt{2-3x}\sqrt{5-2x}\sqrt{1+4x} - 23485\sqrt{11}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 3759\sqrt{11}\,\text{EllipticF}\left(\text{ar}\ldots\right.\right.}{12375\sqrt{5-2x}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
[Out] (Sqrt[-5 + 2*x]*(1650*Sqrt[2 - 3*x]*Sqrt[5 - 2*x]*Sqrt[1 + 4*x] - 23485*Sqr
t[11]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 3759*Sqrt[11]*E
llipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 24219*Sqrt[11]*Ellipti
cPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(12375*Sqrt[5 - 2*x
])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(54488\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+23485\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-\ldots\right.}{594000x^{3}-1732500x^{2}+519750x+24750\ldots}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{2\sqrt{-24x^{3}+70x^{2}-21x-10}}{15}-\frac{3976\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{15125\sqrt{-24x^{3}+70x^{2}-21x-10}}+\frac{854\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110\ldots}}{\ldots}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\ldots}$
risch	$-\frac{2(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{15\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{3976\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{45375\sqrt{-24x^{3}+70x^{2}-21x-10}}+\frac{854\sqrt{22-33x}\ldots}{\ldots}\right.}{\ldots}$

[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x,method=_RETURNVERBOSE)

[Out] 1/24750*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(54488*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+23485*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))-87048*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))+79200*x^3-231000*x^2+69300*x+33000)/(24*x^3-70*x^2+21*x+10)

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{5\,x+7}\,dx$$

[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="fricas")

[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5x+7}\,dx$$

[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x),x)

[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7), x)

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{5\,x+7}\,dx$$

[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="maxima")

[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{5x+7}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}}{5x+7}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7), x)
```

3.40 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 189

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2} dx \\
&= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)} + \frac{6\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}} \\
&+ \frac{152\sqrt{\frac{2}{33}}\sqrt{5-2x}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{125\sqrt{-5+2x}} \\
&+ \frac{26859\sqrt{5-2x}\operatorname{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{7750\sqrt{11}\sqrt{-5+2x}}
\end{aligned}$$

```
[Out] 152/4125*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x
)^(1/2)/(-5+2*x)^(1/2)+26859/85250*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+6/25*EllipticE(2
/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(
1/2)-1/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules

used = {166, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \frac{152\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{125\sqrt{2x-5}} + \frac{6\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}} + \frac{26859\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{7750\sqrt{11}\sqrt{2x-5}} - \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] -1/5*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x) + (6*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt
[5 - 2*x]) + (152*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt
[1 + 4*x]], 1/3])/(125*Sqrt[-5 + 2*x]) + (26859*Sqrt[5 - 2*x]*EllipticPi[55
/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(7750*Sqrt[11]*Sqrt[-5 + 2
*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
```

```
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{1}{10}\int\frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx\\
&= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{1}{10}\int\frac{\frac{1204}{25}-\frac{72x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&\quad -\frac{8953}{250}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx\\
&= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}-\frac{18}{25}\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx\\
&\quad +\frac{152}{125}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&\quad +\frac{8953}{125}\text{Subst}\left(\int\frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx,x,\sqrt{2-3x}\right)\\
&= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{\left(152\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int\frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{125\sqrt{-5+2x}}\\
&\quad +\frac{\left(8953\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int\frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx,x,\sqrt{2-3x}\right)}{125\sqrt{-5+2x}}\\
&\quad -\frac{\left(18\sqrt{-5+2x}\right)\int\frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{25\sqrt{5-2x}}
\end{aligned}
$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)} + \frac{6\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}} + \frac{152\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{125\sqrt{-5+2x}} + \frac{26859\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{7750\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \frac{\sqrt{-5+2x}\left(-\frac{51150\sqrt{2-3x}\sqrt{1+4x}}{7+5x} + \frac{3\sqrt{11}\left(20460E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+9424\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-26859\,\text{EllipticPi}\left(\frac{5}{1\ldots}\right.\right.}{\sqrt{5-2x}}\right.}{255750}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((-51150*Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(7 + 5*x) + (3*Sqrt[1
1]*(20460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 9424*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 26859*EllipticPi[55/124, Ar
cSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/255750
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{5(7+5x)}+\frac{602\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{15125\sqrt{-24x^3+70x^2-21x-10}}-\frac{36\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{11\ldots}}{\ldots}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4\ldots}}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(55430\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x+18975\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\ldots\right.\right.}{\ldots}$
risch	$\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{5(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{602\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{45375\sqrt{-24x^3+70x^2-21x-10}}-\frac{12\sqrt{22-33x}\,\sqrt{-66\ldots}}{\ldots}\right.}{\ldots}$

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/
2)*(-1/5/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+602/15125*(11+44*x)^(1/2)*(
22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/
11*(11+44*x)^(1/2),3^(1/2))-36/3025*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44
*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(
1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))-17906/347875*(11
+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2
)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\left(5\,x+7\right)^2}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 + 70*x + 49), x
)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{\left(5x+7\right)^2}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\left(5\,x+7\right)^2}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^2}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^2}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2,x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2, x)
```

3.41 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3} dx$

Optimal result

Integrand size = 35, antiderivative size = 227

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3} dx &= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} \\ &- \frac{8953\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1390350\sqrt{5-2x}} \\ &+ \frac{397\sqrt{\frac{3}{22}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{89125\sqrt{-5+2x}} \\ &- \frac{14832503\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{287339000\sqrt{11}\sqrt{-5+2x}}\end{aligned}$$

```
[Out] 397/1960750*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-
2*x)^(1/2)/(-5+2*x)^(1/2)-14832503/3160729000*EllipticPi(2/11*(2-3*x)^(1/2)
*11^(1/2),55/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-8953/
1390350*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2
*x)^(1/2)/(5-2*x)^(1/2)-1/10*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+
5*x)^2+8953/556140*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {166, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx$$
$$= \frac{397\sqrt{\frac{3}{22}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{89125\sqrt{2x-5}} - \frac{8953\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1390350\sqrt{5-2x}} - \frac{14832503\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{287339000\sqrt{11}\sqrt{2x-5}} + \frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{556140(5x+7)} - \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

```
[Out] -1/10*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2 + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(556140*(7 + 5*x)) - (8953*Sqrt[11]
*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(13903
50*Sqrt[5 - 2*x]) + (397*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/1
1]*Sqrt[1 + 4*x]], 1/3])/(89125*Sqrt[-5 + 2*x]) - (14832503*Sqrt[5 - 2*x]*E
llipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(287339000*Sqr
t[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) && !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{1}{20}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\int \frac{-106729-199200x+214872x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{1112280}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\int \frac{-\frac{2500104}{25}+\frac{214872x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1112280} + \frac{14832503\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{27807000}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{1191\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{178250} + \frac{8953\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{463450} - \frac{14832503\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)}{13903500}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\left(1191\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{89125\sqrt{22}\sqrt{-5+2x}} - \frac{\left(14832503\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{4634500\sqrt{33}\sqrt{-5+2x}} + \frac{\left(8953\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{463450\sqrt{5-2x}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} - \frac{8953\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1390350\sqrt{5-2x}} + \frac{397\sqrt{\frac{3}{22}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{89125\sqrt{-5+2x}} - \frac{14832503\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{287339000\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.77 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx$$
$$=\frac{\sqrt{-5+2x}\left(\frac{17050\sqrt{2-3x}\sqrt{1+4x}(7057+44765x)}{(7+5x)^2}+\frac{\sqrt{11}\left(-61059460E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+5759676\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)\right.}{\sqrt{5-2x}}\right.}{9482187000}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7057 + 44765*x))/(7 +
5*x)^2 + (Sqrt[11]*(-61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] + 5759676*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 4449
7509*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5
- 2*x]))/9482187000
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{10(7+5x)^2}+\frac{8953\sqrt{-24x^3+70x^2-21x-10}}{556140(7+5x)}-\frac{104171\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44}}{11}\right.}{140193625\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-}}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(7057+44765x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{556140(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{104171\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i}{}\right.}{420580875\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(512860900\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2+283138625\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\right.\right.}{}$

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/
2)*(-1/10/(7+5*x)^2*(-24*x^3+70*x^2-21*x-10)^(1/2)+8953/556140/(7+5*x)*(-24
*x^3+70*x^2-21*x-10)^(1/2)-104171/140193625*(11+44*x)^(1/2)*(22-33*x)^(1/2)
*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(
1/2),3^(1/2))+8953/28038725*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2
)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(
```

```
1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))+14832503/19346720250*(11
+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2
)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\left(5\,x+7\right)^3}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3 + 525*x^2 + 73
5*x + 343), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{\left(5x+7\right)^3}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\left(5\,x+7\right)^3}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^3}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^3}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3,x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3, x)
```

3.42 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4} dx$

Optimal result

Integrand size = 35, antiderivative size = 263

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4} dx \\
&= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} \\
&\quad + \frac{16830401\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{30929169960(7+5x)} \\
&\quad - \frac{16830401\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{77322924900\sqrt{5-2x}} \\
&\quad + \frac{24957247\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{4956597750\sqrt{66}\sqrt{-5+2x}} \\
&\quad + \frac{15664616449\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{15980071146000\sqrt{11}\sqrt{-5+2x}}
\end{aligned}$$

```
[Out] 15664616449/175780782606000*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1
/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+24957247/327135451500*E
llipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-
5+2*x)^(1/2)-16830401/77322924900*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2
*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-1/15*(2-3*x)^(1/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3+8953/1668420*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)/(7+5*x)^2+16830401/30929169960*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(
1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {166, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\begin{aligned}&\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx\\&=\frac{24957247\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{4956597750\sqrt{66}\sqrt{2x-5}}\\&-\frac{16830401\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{77322924900\sqrt{5-2x}}\\&+\frac{15664616449\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{15980071146000\sqrt{11}\sqrt{2x-5}}\\&+\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{30929169960(5x+7)}\\&+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1668420(5x+7)^2}-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}\end{aligned}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

```
[Out] -1/15*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3 + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1668420*(7 + 5*x)^2) + (16830401*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(30929169960*(7 + 5*x)) - (16830
401*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(77322924900*Sqrt[5 - 2*x]) + (24957247*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(4956597750*Sqrt[66]*Sqrt[-5 + 2*x]) + (
15664616449*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[
11]], -1/2])/(15980071146000*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
```

```
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{1}{30}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{\int \frac{-401471+855020x-214872x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx}{3336840}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{30929169960(7+5x)} + \frac{\int \frac{-2850617379+1003030560x+1211788872x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{185575019760}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{30929169960(7+5x)} + \frac{\int \frac{-\frac{3467369304}{25}+\frac{1211788872x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{185575019760} - \frac{15664616449\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{1546458498000}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{30929169960(7+5x)} + \frac{16830401\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{25774308300} + \frac{24957247\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{9913195500} + \frac{15664616449\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)}{773229249000}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{30929169960(7+5x)} + \frac{\left(24957247\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{4956597750\sqrt{22}\sqrt{-5+2x}} + \frac{\left(15664616449\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{257743083000\sqrt{33}\sqrt{-5+2x}} + \frac{\left(16830401\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{25774308300\sqrt{5-2x}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{16830401\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{30929169960(7+5x)} - \frac{16830401\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{77322924900\sqrt{5-2x}} + \frac{24957247\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{4956597750\sqrt{66}\sqrt{-5+2x}} + \frac{15664616449\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{15980071146000\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \frac{\sqrt{-5+2x}\left(\frac{17050\sqrt{2-3x}\sqrt{1+4x}(-75460017+2007981640x+420760025x^2)}{(7+5x)^3} + \frac{\sqrt{11}\left(-114783334820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+12069324\right.}{}\right.}{527342347818000}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-75460017 + 2007981640
*x + 420760025*x^2))/(7 + 5*x)^3 + (Sqrt[11]*(-114783334820*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 120693246492*EllipticF[ArcSin[(2*Sqr
t[2 - 3*x])/Sqrt[11]], -1/2] - 46993849347*EllipticPi[55/124, ArcSin[(2*Sqr
t[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/527342347818000
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{15(7+5x)^3}+\frac{8953\sqrt{-24x^3+70x^2-21x-10}}{1668420(7+5x)^2}+\frac{16830401\sqrt{-24x^3+70x^2-21x-10}}{30929169960(7+5x)}-\frac{48157907\sqrt{11+}}{77967}\right.}{}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(420760025x^2+2007981640x-75460017)\sqrt{(2-3x)(-5+2x)(1+4x)}}{30929169960(7+5x)^3\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{48157907\sqrt{22-33x}\,\sqrt{-66x+165}}{23390184782250\sqrt{-}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(274048323500\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^3-2661307158125\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-}\right.}{}$

[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x,method=_RETURNVERBOSE)

[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)*(-1/15/(7+5*x)^3*(-24*x^3+70*x^2-21*x-10)^(1/2)+8953/1668420/(7+5*x)^2*(-24*x^3+70*x^2-21*x-10)^(1/2)+16830401/30929169960/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)-48157907/7796728260750*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+16830401/1559345652150*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))-15664616449/1075948499983500*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^4}\,dx$$

[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm="fricas")

[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4 + 3500*x^3 + 7350*x^2 + 6860*x + 2401), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^4}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**4,x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^4}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="maxima")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^4}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{2x-5}}{(5x+7)^4}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4,x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4, x)
```

3.43 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 570

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$
$$- \frac{2\sqrt{-de+cf}(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+ \frac{2\sqrt{-de+cf}(3a^2dfh^2-3ab(de+cf)h^2-b^2(dg(fg-eh)-ch(fg+2eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}}{3b^3d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$
$$- \frac{2(be-af)\sqrt{-de+cf}(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2/3*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b-2/3*(3*a*d*f*h-b*(c*f*h+d*e
*h+d*f*g))*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/
(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/
2)/b^2/d/h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/3*(3*a^2*d*
f*h^2-3*a*b*(c*f+d*e)*h^2-b^2*(d*g*(-e*h+f*g)-c*h*(2*e*h+f*g)))*EllipticF(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*
f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^3/
d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*f+b*e)*(-a*h+b*g)*EllipticPi(
f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e
)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h
*x+g)/(-c*h+d*g))^(1/2)/b^3/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {167, 1621, 175, 552, 551, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$$
$$= \frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(3a^2dfh^2-3abh^2(cf+de)-(b^2(dg(fg-eh)-ch(2eh+fg))))\,\text{EllipticF}\,(}{3b^3d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2(be-af)(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(3adfh-b(cfh+deh+dfg))E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*b) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], (
(d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^2*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(
g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a^2*d*f*h^2 - 3*a*b*(d*e
 + c*f)*h^2 - b^2*(d*g*(f*g - e*h) - c*h*(f*g + 2*e*h)))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqr
t[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^3*d
*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]
*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)
]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d
*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^3*Sqrt[f]*Sq
rt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} \\
&\quad + \frac{\int \frac{3bceg-a(deg+cfg+ceh)+2(b(deg+cfg+ceh)-a(dfg+deh+cfh))x-(3adfh-b(dfg+deh+cfh))x^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3b} \\
&= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} \\
&\quad + \frac{\int \frac{2deg+2cfg-\frac{3adfg}{b}+2ceh-\frac{3adeh}{b}-\frac{3acfh}{b}+\frac{3a^2dfh}{b^2}+\left(dfg+deh+cfh-\frac{3adfh}{b}\right)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3b} \\
&\quad + \frac{((bc-ad)(be-af)(bg-ah))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^3}
\end{aligned}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{(2(bc-ad)(be-af)(bg-ah))\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^3} + \frac{\left(dfg+deh+cfh-\frac{3adfh}{b}\right)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3bh} + \frac{\left(h\left(2deg+2cfg-\frac{3adfg}{b}+2ceh-\frac{3adeh}{b}-\frac{3acfh}{b}+\frac{3a^2dfh}{b^2}\right)-g\left(dfg+deh+cfh-\frac{3adfh}{b}\right)\right)\int \frac{}{\sqrt{c+}}}{3bh}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{\left(2(bc-ad)(be-af)(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+d}\right.}{b^3\sqrt{e+fx}} + \frac{\left(\left(h\left(2deg+2cfg-\frac{3adfg}{b}+2ceh-\frac{3adeh}{b}-\frac{3acfh}{b}+\frac{3a^2dfh}{b^2}\right)-g\left(dfg+deh+cfh-\frac{3adfh}{b}\right)\right)\sqrt{\frac{d(}{d}}\right.}{3bh\sqrt{e+fx}} + \frac{\left(\left(dfg+deh+cfh-\frac{3adfh}{b}\right)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{3bh\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{\left(2(bc-ad)(be-af)(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}\,dx,\right.}{b^3\sqrt{e+fx}\sqrt{g+hx}} + \frac{\left(\left(h\left(2deg+2cfg-\frac{3adfg}{b}+2ceh-\frac{3adeh}{b}-\frac{3acfh}{b}+\frac{3a^2dfh}{b^2}\right)-g\left(dfg+deh+cfh-\frac{3adfh}{b}\right)\right)\sqrt{\frac{d(}{d}}\right.}{3bh\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{2\sqrt{-de+cf}(3a^2dfh^2-3ab(de+cf)h^2-b^2(dg(fg-eh)-ch(fg+2eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}}{3b^3d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}} - \frac{2(be-af)\sqrt{-de+cf}(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.02 (sec) , antiderivative size = 1254, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$$

$$= \frac{2\sqrt{c+dx}\left(3b^2eg-3abfg+\frac{b^2fg^2}{h}-3abeh+\frac{b^2e^2h}{f}-\frac{b^2c^2fh}{d^2}+\frac{3abcfh}{d}+2b^2fgx+2b^2ehx-3abfhx+\frac{b^2cfh}{d}\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(3*b^2*e*g - 3*a*b*f*g + (b^2*f*g^2)/h - 3*a*b*e*h + (b^2*
e^2*h)/f - (b^2*c^2*f*h)/d^2 + (3*a*b*c*f*h)/d + 2*b^2*f*g*x + 2*b^2*e*h*x
- 3*a*b*f*h*x + (b^2*c*f*h*x)/d + b^2*f*h*x^2 - (b^2*c*e*g)/(c + d*x) - (3*
a*b*d*e*g)/(c + d*x) + (3*a*b*c*f*g)/(c + d*x) + (3*a*b*c*e*h)/(c + d*x) +
(b^2*c^3*f*h)/(d^2*(c + d*x)) - (3*a*b*c^2*f*h)/(d*(c + d*x)) + (b^2*d*e^2*
g)/(c*f + d*f*x) - (b^2*c*e^2*h)/(c*f + d*f*x) + (b^2*d*e*g^2)/(c*h + d*h*x
) - (b^2*c*f*g^2)/(c*h + d*h*x) - (I*b*Sqrt[-c + (d*e)/f]*(3*a*d*f*h - b*(d
*f*g + d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt
[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c
 + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/d^2 + (I*b*Sqrt[-c + (d*e)/f]*(
-2*b*f*g - b*e*h + 3*a*f*h)*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]
*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/S
qrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/d + ((3*I)*b^2*e*Sqrt[-c +
 (d*e)/f]*f*g*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g +
h*x))/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSi
nh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/(d*
e - c*f) + ((3*I)*a*b*Sqrt[-c + (d*e)/f]*f^2*g*Sqrt[c + d*x]*Sqrt[(d*(e + f
```

```
*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[-((b*c*f -
 a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f
*g - c*f*h)/(d*e*h - c*f*h)])/(-(d*e) + c*f) + ((3*I)*a^2*Sqrt[-c + (d*e)/f
]*f^2*h*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/
(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqr
t[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/(d*e - c*
f) + ((3*I)*a*b*e*Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(
f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f
)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)])/(-(d*e) + c*f)))/(3*b^3*Sqrt[e + f*x]*Sqrt[g + h*x]
)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3b}+\frac{2\left(\frac{a^2dfh-abcfh-abdeh-abdfg+b^2ceh+b^2cfg+b^2deg}{b^3}-\right.}{\sqrt{dfh\,x^3+c}}\right.}{}$
default	Expression too large to display

```
[In] int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x,method=_RETURNVERBO
SE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2/3/b*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*
g)^(1/2)+2*((a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^
2*d*e*g)/b^3-2/3/b*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(g/h-e/f)*((x+g/h)/(g/h
-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x
^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*Ellip
ticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))+2*(-1/b^2*(a*
d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)-2/3/b*(c*f*h+d*e*h+d*f*g))*(g/h-e/f)*((x+g/h
)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d
*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)
*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1
/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))
)-2*(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+
a*b^2*d*e*g-b^3*c*e*g)/b^4*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g
/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*
```

```
f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticPi(((x+g/h)
/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$$

```
[In] integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(h*x+g)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bx+a}\,dx$$

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bx+a}\,dx$$

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}{a+b\,x}\,dx$$

```
[In] int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x),x)
[Out] int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x), x)
```

3.44 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 243

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}} \, dx \\
&= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad + \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 \\
&\quad + \frac{2629157597\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{163296\sqrt{5-2x}} \\
&\quad - \frac{2161804579\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{54432\sqrt{-5+2x}}
\end{aligned}$$

```
[Out] -2161804579/326592*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1
/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+2629157597/163296*EllipticE(2/11*(2-3*x)^(
1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+46134551
/38880*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+26291/540*(7+5*x)*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1679/756*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)+1/9*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {168, 1614, 1629, 164, 115, 114, 122, 120}

$$\begin{aligned}&\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx\\&=-\frac{2161804579\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{54432\sqrt{2x-5}}\\&\quad+\frac{2629157597\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{163296\sqrt{5-2x}}\\&\quad+\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3+\frac{1679}{756}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2\\&\quad+\frac{26291}{540}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)+\frac{46134551\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{38880}\end{aligned}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

```
[Out] (46134551*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/38880 + (26291*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/540 + (1679*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/756 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/9 + (2629157597*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipt
icE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(163296*Sqrt[5 - 2*x]) - (21
61804579*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]
], 1/3])/(54432*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
```

```
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=\frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3-\frac{1}{18}\int\frac{(7+5x)^2\left(-699-565x+3358x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&=\frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad+\frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3+\frac{\int\frac{(7+5x)\left(1987250-276290x-8833776x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{3024}\\
&=\frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)+\frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad+\frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3-\frac{\int\frac{-3851232672+4914194640x+15501209136x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{362880}\\
&=\frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880}+\frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\
&\quad+\frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad+\frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3-\frac{\int\frac{-904221216360+3785986939680x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{39191040}
\end{aligned}$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 - \frac{2629157597\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}dx}{54432} - \frac{23779850369\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}dx}{108864}$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 - \frac{\left(2161804579\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int\frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}dx}{54432\sqrt{-5+2x}} - \frac{\left(2629157597\sqrt{-5+2x}\right)\int\frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}dx}{54432\sqrt{5-2x}}$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 + \frac{2629157597\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{163296\sqrt{5-2x}} - \frac{2161804579\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{54432\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.53

$$\int\frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}dx = \frac{6\sqrt{2-3x}\sqrt{1+4x}(-455686385+51484034x+21329208x^2+8614800x^3+1512000x^4)+2629157597\sqrt{6}\cdots}{326592\sqrt{\cdots}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-455686385 + 51484034*x + 21329208*x^2 + 86
14800*x^3 + 1512000*x^4) + 2629157597*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcS
in[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 2161804579*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(326592*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.61

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(108864000x^6+574905600x^5+1227098543\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-2629157597\sqrt{1+4}\right.}{7838208x^3-22861440x^2+685}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{51901x\sqrt{-24x^3+70x^2-21x-10}}{108}+\frac{13019611\sqrt{-24x^3+70x^2-21x-10}}{7776}+\frac{10873271\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F}{57024\sqrt{-24x^3+70x^2-21x-}}\right.}{}$
risch	$-\frac{(756000x^3+6197400x^2+26158104x+91137277)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{54432\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{10873271\sqrt{22-33x}\,\sqrt{-66x}}{171072\sqrt{-}}\right.}{}$

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/326592*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(108864000*x^6+57490560
0*x^5+1227098543*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ellipti
cF(1/11*(11+44*x)^(1/2),3^(1/2))-2629157597*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^
(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+1259114976*x^4+
2963596608*x^3-34609891236*x^2+13052783142*x+5468236620)/(24*x^3-70*x^2+21*
x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx$$
$$= \frac{1}{54432}\left(756000\,x^3+6197400\,x^2+26158104\,x+91137277\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}$$
$$+\frac{4958213249}{419904}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$-\frac{2629157597}{163296}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/54432*(756000*x^3 + 6197400*x^2 + 26158104*x + 91137277)*sqrt(4*x + 1)*sq
rt(2*x - 5)*sqrt(-3*x + 2) + 4958213249/419904*sqrt(-6)*weierstrassPInverse
(847/108, 6655/2916, x - 35/36) - 2629157597/163296*sqrt(-6)*weierstrassZet
a(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\,(5x+7)^3}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**3/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{(5\,x+7)}^3}{\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2), x)
```

3.45 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 205

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx &= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\ &+ \frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\ &+ \frac{8198333\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}}\\ &- \frac{1679161\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{756\sqrt{-5+2x}}\end{aligned}$$

```
[Out] -1679161/4536*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)+8198333/9072*EllipticE(2/11*(2-3*x)^(1/2)*11^(1
/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+73207/1080*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+173/60*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(
1/2)*(1+4*x)^(1/2)+1/7*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {168, 1614, 1629, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx$$
$$= -\frac{1679161\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{756\sqrt{2x-5}} + \frac{8198333\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}} + \frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2 + \frac{173}{60}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7) + \frac{73207\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1080}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

```
[Out] (73207*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1080 + (173*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/60 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/7 + (8198333*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE
[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (1679161
*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/(756*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
```

```
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
```

```
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2-\frac{1}{14}\int \frac{(7+5x)\left(-543-175x+2422x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&= \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\
&\quad +\frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{\int \frac{1054354-1137830x-4099592x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1680}\\
&= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080}+\frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\
&\quad +\frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{\int \frac{243007380-983799960x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{181440}\\
&= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080}+\frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\
&\quad +\frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad -\frac{8198333\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{3024}-\frac{18470771\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1512}\\
&= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080}+\frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\
&\quad +\frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\
&\quad -\frac{\left(1679161\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{756\sqrt{-5+2x}}\\
&\quad -\frac{\left(8198333\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{3024\sqrt{5-2x}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&+ \frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&+ \frac{8198333\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}} \\
&- \frac{1679161\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{756\sqrt{-5+2x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.61

$$
\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx
$$

$$
= \frac{12\sqrt{2-3x}\sqrt{1+4x}(-717955+102592x+46836x^2+10800x^3)+8198333\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\right.\right.}{18144\sqrt{-5+2x}}
$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-717955 + 102592*x + 46836*x^2 + 10800*x^3
) + 8198333*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3] - 6716644*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(18144*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.70

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(1555200x^5+3753266\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-8198333\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\right.}{435456x^3-1270080x^2+381024x+181440}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{293x\sqrt{-24x^3+70x^2-21x-10}}{12}+\frac{20513\sqrt{-24x^3+70x^2-21x-10}}{216}+\frac{17533\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+}}{11}\right.}{1584\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}}$
risch	$-\frac{(5400x^2+36918x+143591)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{1512\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{17533\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{2}}{}\right.}{4752\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/18144*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(1555200*x^5+3753266*(1+
4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1
/2),3^(1/2))-8198333*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ell
ipticE(1/11*(11+44*x)^(1/2),3^(1/2))+6096384*x^4+11703888*x^3-110665104*x^2
+40615092*x+17230920)/(24*x^3-70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.29

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx \\
&= \frac{1}{1512}\left(5400\,x^2+36918\,x+143591\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\
&\quad+\frac{30577063}{46656}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\
&\quad-\frac{8198333}{9072}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)
\end{aligned}$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/1512*(5400*x^2 + 36918*x + 143591)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x
+ 2) + 30577063/46656*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x -
35/36) - 8198333/9072*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstr
assPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx=\int \frac{\sqrt{2-3x}\sqrt{4x+1}\,(5x+7)^2}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**2/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{(5\,x+7)}^2}{\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2), x)
```

3.46 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 162

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{1397\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27\sqrt{5-2x}} - \frac{4543\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{36\sqrt{-5+2x}}$$

```
[Out] -4543/216*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*
x)^(1/2)/(-5+2*x)^(1/2)+1/4*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)+1397
/27*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^
(1/2)/(5-2*x)^(1/2)+95/18*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{159, 164, 115, 114, 122, 120\}$

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = -\frac{4543\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{36\sqrt{2x-5}} + \frac{1397\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27\sqrt{5-2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2} + \frac{95}{18}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

```
[Out] (95*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/18 + (Sqrt[2 - 3*x]*Sqrt[-5
 + 2*x]*(1 + 4*x)^(3/2))/4 + (1397*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin
[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(27*Sqrt[5 - 2*x]) - (4543*Sqrt[11/6]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5
 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
```

```
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}+\frac{1}{20}\int\frac{\left(\frac{1065}{2}-950x\right)\sqrt{1+4x}}{\sqrt{2-3x}\sqrt{-5+2x}}\,dx\\
&= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}\\
&\quad-\frac{1}{180}\int\frac{-\frac{29535}{2}+55880x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2}\\
&\quad-\frac{1397}{9}\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx-\frac{49973}{72}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx
\end{aligned}
$$

$$= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{\left(4543\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{36\sqrt{-5+2x}} - \frac{\left(1397\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{9\sqrt{5-2x}}$$

$$= \frac{95}{18}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} + \frac{1397\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27\sqrt{5-2x}} - \frac{4543\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{36\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{6\sqrt{2-3x}\sqrt{1+4x}(-995+218x+72x^2) + 5588\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right) - 4543\sqrt{66}\sqrt{}}{216\sqrt{-5+2x}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-995 + 218*x + 72*x^2) + 5588*Sqrt[66]*Sqrt
[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 4543*Sqrt[66]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(216*Sqrt[-
5 + 2*x])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(2453\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-5588\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+51\ldots\right.}{5184x^3-15120x^2+4536x+2160}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(x\sqrt{-24x^3+70x^2-21x-10}+\frac{199\sqrt{-24x^3+70x^2-21x-10}}{36}+\frac{179\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.}{264\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{(199+36x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\left(\frac{179\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{792\sqrt{-24x^3+70x^2-21x-10}}-\frac{254\sqrt{2}\ldots}{}\right.$

[In] int((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/216*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2453*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-5588*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+5184*x^4+13536*x^3-79044*x^2+27234*x+11940)/(24*x^3-70*x^2+21*x+10)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{1}{36}\,(36\,x+199)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} + \frac{142417}{3888}\,\sqrt{-6}\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{1397}{27}\,\sqrt{-6}\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")

[Out] 1/36*(36*x + 199)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 142417/3888*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) - 1397/27*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36))

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\cdot(5x+7)}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{(5x+7)\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
maxima")
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{(5x+7)\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
giac")
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,(5x+7)}{\sqrt{2x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2), x)
```

3.47 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{55\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{18\sqrt{5-2x}} - \frac{11\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{3\sqrt{-5+2x}}$$

```
[Out] -11/9*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)+55/18*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/
2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+1/3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {103, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = -\frac{11\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{3\sqrt{2x-5}} + \frac{55\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{18\sqrt{5-2x}} + \frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3 + (55*Sqrt[11]*Sqrt[-5 + 2*x
]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(18*Sqrt[5 - 2*x]) -
 (11*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(3*Sqrt[-5 + 2*x])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{1}{3}\int \frac{-\frac{33}{2}+55x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{55}{6}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx \\
&\quad - \frac{121}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{\left(11\sqrt{22}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{3\sqrt{-5+2x}} \\
&\quad - \frac{\left(55\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{6\sqrt{5-2x}} \\
&= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{55\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{18\sqrt{5-2x}} \\
&\quad - \frac{11\sqrt{\frac{22}{3}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{3\sqrt{-5+2x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \frac{12\sqrt{2-3x}(-5+2x)\sqrt{1+4x}+55\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)-44\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}}{36\sqrt{-5+2x}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 55*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 44*Sqrt[66]*Sqrt[5 - 2*x]*El
lipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(22\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-55\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+288x\right.}{864x^3-2520x^2+756x+360}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-24x^3+70x^2-21x-10}}{3}+\frac{\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{22\sqrt{-24x^3+70x^2-21x-10}}-\frac{5\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}}{33\sqrt{}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{3\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{66\sqrt{-24x^3+70x^2-21x-10}}-\frac{5\sqrt{22-33x}\,\sqrt{-66x+}}{}\right.}{\sqrt{2}}$

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(22*(1+4*x)^(1/2)*(2-3*x)^(
1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-55*(1+4
*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/
2),3^(1/2))+288*x^3-840*x^2+252*x+120)/(24*x^3-70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$$
$$= \frac{1}{3}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2} + \frac{1331}{648}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$-\frac{55}{18}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 1331/648*sqrt(-6)*weierstr
assPInverse(847/108, 6655/2916, x - 35/36) - 55/18*sqrt(-6)*weierstrassZeta
(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2), x)
```

3.48 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 151

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}} - \frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{25\sqrt{-5+2x}} + \frac{69\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{25\sqrt{11}\sqrt{-5+2x}}$$

```
[Out] -41/825*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+69/275*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,
1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+2/5*EllipticE(2/11*(2-
3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {169, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = -\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}} + \frac{2\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}} + \frac{69\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{25\sqrt{11}\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

```
[Out] (2*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(5*Sqrt[5 - 2*x]) - (41*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt
[3/11]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) + (69*Sqrt[5 - 2*x]*Ellipt
icPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt[11]*Sqrt[-
5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
```

```
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 169

```
Int[(Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)])/(((a_.) + (b_.)*(x_
))*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[(b*e - a*f)*((b*g - a*h)/b^
2), Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + D
ist[1/b^2, Int[Simp[b*f*g + b*e*h - a*f*h + b*f*h*x, x]/(Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{25}\int \frac{109-60x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&\quad -\frac{713}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx \\
&= -\left(\frac{6}{5}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx\right)-\frac{41}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&\quad +\frac{1426}{25}\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx,x,\sqrt{2-3x}\right) \\
&= -\frac{\left(41\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{25\sqrt{-5+2x}} \\
&\quad +\frac{\left(1426\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx,x,\sqrt{2-3x}\right)}{25\sqrt{-5+2x}} \\
&\quad -\frac{\left(6\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{5\sqrt{5-2x}} \\
&= \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}} \\
&\quad -\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{25\sqrt{-5+2x}} \\
&\quad +\frac{69\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{11}\sqrt{-5+2x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx$$
$$= \frac{\sqrt{5-2x}\Big(-110E\Big(\arcsin\Big(\frac{2\sqrt{2-3x}}{\sqrt{11}}\Big)|-\frac{1}{2}\Big)+41\,\text{EllipticF}\Big(\arcsin\Big(\frac{2\sqrt{2-3x}}{\sqrt{11}}\Big),-\frac{1}{2}\Big)+69\,\text{EllipticPi}\Big(\frac{55}{124},\text{arcsin}}{25\sqrt{-55+22x}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

```
[Out] (Sqrt[5 - 2*x]*(-110*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
41*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 69*EllipticPi[55/1
24, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(25*Sqrt[-55 + 22*x])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.44

method	result
default	$\frac{\left(69F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+55E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-124\Pi\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{275\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{109\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{3025\sqrt{-24x^3+70x^2-21x-10}}-\frac{12\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{12}\right.}{605\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/275*(69*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+55*EllipticE(1/11*(11+44*
x)^(1/2),3^(1/2))-124*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))*(5-2
*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
fricas")
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 - 11*x - 35), x
)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\cdot(5x+7)}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
maxima")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx=\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,\left(5\,x+7\right)}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)), x)
```

3.49 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 189

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx &= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} \\ &- \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}} \\ &- \frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{25\sqrt{-5+2x}} \\ &- \frac{6101\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{20150\sqrt{11}\sqrt{-5+2x}}\end{aligned}$$

```
[Out] -2/275*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^
(1/2)/(-5+2*x)^(1/2)-6101/221650*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/
124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-2/195*EllipticE(2/
11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1
/2)+1/39*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules

used = {170, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = -\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}} - \frac{2\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}} - \frac{6101\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{20150\sqrt{11}\sqrt{2x-5}} + \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*(7 + 5*x)) - (2*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(195*Sqr
t[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticPi[55/12
4, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(20150*Sqrt[11]*Sqrt[-5 + 2*x
])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
```

```
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{-29+120x-24x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{\frac{768}{25}-\frac{24x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx + \frac{6101\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{1950}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} + \frac{2}{65}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{6}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx - \frac{6101}{975}\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{25\sqrt{-5+2x}} - \frac{\left(6101\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{325\sqrt{33}\sqrt{-5+2x}} + \frac{\left(2\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{65\sqrt{5-2x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}}$$
$$- \frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{25\sqrt{-5+2x}}$$
$$- \frac{6101\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{20150\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx$$
$$= \frac{\frac{51150\sqrt{2-3x}(-5+2x)\sqrt{1+4x}}{7+5x} + 3\sqrt{55-22x}\left(6820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) + 14508\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\right.\right.}{1994850\sqrt{-5+2x}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

```
[Out] ((51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) + 3*Sqrt[55 - 22*
x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14508*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 18303*EllipticPi[55/124, Ar
cSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(1994850*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-24x^3+70x^2-21x-10}}{273+195x} - \frac{128\sqrt{11+44x}\sqrt{22-33x}\sqrt{110-44x}F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{39325\sqrt{-24x^3+70x^2-21x-10}} + \frac{4\sqrt{11+44x}\sqrt{22-33x}\sqrt{110-}}{7}\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$
default	$\frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{-5+2x}\left(39560\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x+6325\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\right.}{}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{39(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}} - \frac{\left(-\frac{128\sqrt{22-33x}\sqrt{-66x+165}\sqrt{33+132x}F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{117975\sqrt{-24x^3+70x^2-21x-10}} + \frac{4\sqrt{22-33x}\sqrt{-}}{}\right.}{}$

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/
2)*(1/39/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)-128/39325*(11+44*x)^(1/2)*(
22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/
11*(11+44*x)^(1/2),3^(1/2))+4/7865*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*
x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1
/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))+12202/2713425*(11
+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2
)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 + 15*x^2 - 252*
x - 245), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^2}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,(5\,x+7)^2}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

3.50 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 225

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2}-\frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)}+\frac{361\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1204970\sqrt{5-2x}}-\frac{6101\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{231725\sqrt{66}\sqrt{-5+2x}}-\frac{6655867\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{747081400\sqrt{11}\sqrt{-5+2x}}$$

```
[Out] -6655867/8217895400*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2^(
1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-6101/15293850*EllipticF(1/11*33
^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+361
/1204970*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+
2*x)^(1/2)/(5-2*x)^(1/2)+1/78*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7
+5*x)^2-361/481988*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {170, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = -\frac{6101\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{231725\sqrt{66}\sqrt{2x-5}} + \frac{361\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1204970\sqrt{5-2x}} - \frac{6655867\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{747081400\sqrt{11}\sqrt{2x-5}} - \frac{361\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{481988(5x+7)} + \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(78*(7 + 5*x)^2) - (361*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(481988*(7 + 5*x)) + (361*Sqrt[11]*Sq
rt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1204970*
Sqrt[5 - 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(231725*Sqrt[66]*Sqrt[-5 + 2*x]) - (6655867*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(747081400*Sqrt[11
]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{1}{156}\int \frac{-37+100x+24x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\int \frac{-272145+485280x+77976x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{8675784}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\int \frac{\frac{1880568}{25}+\frac{77976x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{8675784} + \frac{6655867\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{72298200}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{1083\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{1204970} - \frac{6101\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{463450} - \frac{6655867\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)}{36149100}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\left(6101\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{231725\sqrt{22}\sqrt{-5+2x}} - \frac{\left(6655867\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{12049700\sqrt{33}\sqrt{-5+2x}} - \frac{\left(1083\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{1204970\sqrt{5-2x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} + \frac{361\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1204970\sqrt{5-2x}} - \frac{6101\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{231725\sqrt{66}\sqrt{-5+2x}} - \frac{6655867\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{747081400\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx$$
$$= \frac{-\frac{17050\sqrt{2-3x}(-5+2x)\sqrt{1+4x}(-10957+5415x)}{(7+5x)^2} - 3\sqrt{55-22x}\left(2462020E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 9834812\,\mathrm{Ellipti}\ldots\right.}{24653686200\sqrt{-5+2x}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

```
[Out] ((-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(-10957 + 5415*x))/(7 + 5*x
)^2 - 3*Sqrt[55 - 22*x]*(2462020*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11
]], -1/2] - 9834812*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 6
655867*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(2465
3686200*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-24x^3+70x^2-21x-10}}{78(7+5x)^2}-\frac{361\sqrt{-24x^3+70x^2-21x-10}}{481988(7+5x)}-\frac{26119\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{\ldots}\right.}{364503425\sqrt{-24x^3+70x^2-21x-10}}\ldots\right.}{\sqrt{2-3x}\ldots}$
risch	$\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(-10957+5415x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{1445964(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{26119\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}\ldots}{2}\right.}{1093510275\sqrt{-24x^3+70x^2-21x-10}}\ldots\right.}{\ldots}$
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(205130100\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2-34249875\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{1\ldots}}{}\right.\right.}{\ldots}$

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/
2)*(1/78/(7+5*x)^2*(-24*x^3+70*x^2-21*x-10)^(1/2)-361/481988/(7+5*x)*(-24*x
^3+70*x^2-21*x-10)^(1/2)-26119/364503425*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(1
10-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2
),3^(1/2))-1083/72900685*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(
-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2
```

```
))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))+6655867/50301472650*(11+44*
x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*El
lipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4 + 425*x^3 - 11
55*x^2 - 2989*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^3}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,(5\,x+7)^3}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

3.51 $\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 205

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} \\ &+ \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\ &+ \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\ &+ \frac{15629623\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}} \\ &- \frac{25260049\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{6048\sqrt{-5+2x}}\end{aligned}$$

```
[Out] -25260049/36288*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)
*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+15629623/9072*EllipticE(2/11*(2-3*x)^(1/2)*11
^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+110743/864*(2-3
*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+121/24*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x
)^(1/2)*(1+4*x)^(1/2)+5/28*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {180, 1614, 1629, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25260049\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{6048\sqrt{2x-5}} + \frac{15629623\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}} + \frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2 + \frac{121}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7) + \frac{110743}{864}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (110743*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/864 + (121*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 + (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2
*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/28 + (15629623*Sqrt[11]*Sqrt[-5 + 2*x]*Ellip
ticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (252
60049*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/(6048*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
```

```
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&\quad -\frac{1}{56}\int\frac{(7+5x)\left(-7223+2667x+16940x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad +\frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{\int\frac{10251500-9171580x-31008040x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{6720} \\
&= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad +\frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{\int\frac{2083915260-7502219040x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{725760} \\
&= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad +\frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&\quad -\frac{15629623\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{3024}-\frac{277860539\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{12096}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad + \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&\quad - \frac{\left(25260049\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{6048\sqrt{-5+2x}} \\
&\quad - \frac{\left(15629623\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{3024\sqrt{5-2x}} \\
&= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad + \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&\quad + \frac{15629623\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}} \\
&\quad - \frac{25260049\sqrt{\frac{11}{6}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{6048\sqrt{-5+2x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 9.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.61

$$
\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{30\sqrt{2-3x}\sqrt{1+4x}(-1041565+188566x+64224x^2+10800x^3)+31259246\sqrt{66}\sqrt{5-2x}\,E\left(\arcsin\left(\sqrt{\frac{3}{11}}\right.\right.}{36288\sqrt{-5+2x}}
$$

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (30*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-1041565 + 188566*x + 64224*x^2 + 10800*x^
3) + 31259246*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3] - 25260049*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sq
rt[1 + 4*x]], 1/3])/(36288*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.70

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(13261655\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-31259246\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+4}}{11}\right.\right.}{870912x^3-2540160x^2+762048x+362880}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{905x\sqrt{-24x^3+70x^2-21x-10}}{24}+\frac{148795\sqrt{-24x^3+70x^2-21x-10}}{864}+\frac{1653901\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\sqrt{}\right.}{69696\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2}}$
risch	$-\frac{5(5400x^2+45612x+208313)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{6048\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{1653901\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\right.}{209088\sqrt{-24x^3+70x^2-21x-}}\right.}{}$

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/36288*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(13261655*(1+4*x)^(1/2)*
(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)
)-31259246*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/1
1*(11+44*x)^(1/2),3^(1/2))+3888000*x^5+21500640*x^4+57602160*x^3-407101740*
x^2+144920790*x+62493900)/(24*x^3-70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{6048}\left(5400\,x^2+45612\,x+208313\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} + \frac{111640903}{93312}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{15629623}{9072}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 5/6048*(5400*x^2 + 45612*x + 208313)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x
+ 2) + 111640903/93312*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x -
 35/36) - 15629623/9072*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weiers
trassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}\,(5x+7)^3}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)**3/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,{(5\,x+7)}^3}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.52 $\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 167

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} \\ &+ \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\ &+ \frac{44569\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{432\sqrt{5-2x}} \\ &- \frac{17533\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{72\sqrt{-5+2x}}\end{aligned}$$

```
[Out] -17533/432*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2
*x)^(1/2)/(-5+2*x)^(1/2)+44569/432*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/
2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+68/9*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(1/2)+1/4*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

= {180, 1629, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{17533\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{72\sqrt{2x-5}} + \frac{44569\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{432\sqrt{5-2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7) + \frac{68}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (68*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (Sqrt[2 - 3*x]*Sqrt[-5
+ 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/4 + (44569*Sqrt[11]*Sqrt[-5 + 2*x]*Elliptic
E[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(432*Sqrt[5 - 2*x]) - (17533*S
qrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
72*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&& !LtQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) && !(SimplerQ[e + f*x, a + b*x]
```

```
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{1}{40}\int \frac{-5155+3605x+10880x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{\int \frac{-899460+2674140x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{4320}$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{44569}{144}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{192863}{144}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{\left(17533\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{72\sqrt{-5+2x}} - \frac{\left(44569\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{144\sqrt{5-2x}}$$

$$= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{44569\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{432\sqrt{5-2x}} - \frac{17533\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{72\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 7.85 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{120\sqrt{2-3x}\sqrt{1+4x}(-335+89x+18x^2) + 44569\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right) - 35066\sqrt{6}}{864\sqrt{-5+2x}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
[Out] (120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-335 + 89*x + 18*x^2) + 44569*Sqrt[66]*Sq
rt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 35066*Sqrt[6
6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(864*Sqr
t[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(16060\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-44569\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+\right.}{20736x^3-60480x^2+18144x+8640}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{5x\sqrt{-24x^3+70x^2-21x-10}}{4}+\frac{335\sqrt{-24x^3+70x^2-21x-10}}{36}+\frac{4997\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.}{2904\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{5(67+9x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{4997\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{8712\sqrt{-24x^3+70x^2-21x-10}}-\frac{44569}{}\right.}{}$

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/864*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(16060*(1+4*x)^(1/2)*(2-3*
x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-445
69*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44
*x)^(1/2),3^(1/2))+25920*x^4+117360*x^3-540120*x^2+179640*x+80400)/(24*x^3-
70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.32

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= \frac{5}{36}\,(9\,x+67)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\
&\quad + \frac{1020239}{15552}\,\sqrt{-6}\,\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\
&\quad - \frac{44569}{432}\,\sqrt{-6}\,\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)
\end{aligned}$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 5/36*(9*x + 67)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 1020239/15552*
sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) - 44569/432*sqr
t(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655
/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}(5x+7)^2}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)**2/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^2\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^2\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,(5\,x+7)^2}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.53 $\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 131

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{241\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{36\sqrt{5-2x}} - \frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{12\sqrt{-5+2x}}$$

```
[Out] -179/72*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+241/36*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^
(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+5/12*(2-3*x)^(1/2)*(-5+2*x)^(1
/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {159, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{12\sqrt{2x-5}} + \frac{241\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{36\sqrt{5-2x}} + \frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/12 + (241*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(36*Sqrt[5 - 2*x
]) - (179*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(12*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{12}\int \frac{\frac{441}{2}-482x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&= \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{241}{12}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx\\
&\quad -\frac{1969}{24}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&= \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{\left(179\sqrt{\frac{11}{2}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{12\sqrt{-5+2x}}\\
&\quad -\frac{\left(241\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{12\sqrt{5-2x}}\\
&= \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{241\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{36\sqrt{5-2x}}\\
&\quad -\frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{12\sqrt{-5+2x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{30\sqrt{2-3x}(-5+2x)\sqrt{1+4x}+241\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)-179\sqrt{66}\sqrt{5-2x}\,\text{Elliptic}}{72\sqrt{-5+2x}}$$

[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]

[Out] (30*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 241*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 179*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(72*Sqrt[-5 + 2*x])

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(55\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-241\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+720x^3\right.}{1728x^3-5040x^2+1512x+720}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{5\sqrt{-24x^3+70x^2-21x-10}}{12}+\frac{147\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{968\sqrt{-24x^3+70x^2-21x-10}}-\frac{241\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110}}{7}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{5(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{12\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{49\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{968\sqrt{-24x^3+70x^2-21x-10}}-\frac{241\sqrt{22-33x}\,\sqrt{-6}}{}\right.}{\sqrt{}}$

[In] int((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/72*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(55*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-241*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+720*x^3-2100*x^2+630*x+300)/(24*x^3-70*x^2+21*x+10)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{5}{12}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2} + \frac{2233}{648}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$-\frac{241}{36}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] 5/12*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 2233/648*sqrt(-6)*weierst
rassPInverse(847/108, 6655/2916, x - 35/36) - 241/36*sqrt(-6)*weierstrassZe
ta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}(5x+7)}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,(5\,x+7)}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.54 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 47

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{-5+2x}}$$

```
[Out] 1/4*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))*22^(1/2)*(5-2*x)^(1/2)/(
-5+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {115, 114}

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{4x+1}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{2x-5}}$$

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[11/2]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[1 + 4*x]/Sqrt[11]], 3])/(2*
Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
```

```
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{5-2x}\int \frac{\sqrt{\frac{8}{11}-\frac{12x}{11}}}{\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{\sqrt{2}\sqrt{-5+2x}}\\ &= \frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\sin^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{-5+2x}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.

Time = 2.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{\frac{2(-5+2x)(-2+3x)}{\sqrt{\frac{1}{2}+2x}}+\sqrt{11}\sqrt{\frac{-5+2x}{1+4x}}\sqrt{\frac{-2+3x}{1+4x}}(1+4x)E\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right)\middle|3\right)}{2\sqrt{2-3x}\sqrt{-10+4x}}$$

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/2*((2*(-5 + 2*x)*(-2 + 3*x))/Sqrt[1/2 + 2*x] + Sqrt[11]*Sqrt[(-5 + 2*x)/
(1 + 4*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*EllipticE[ArcSin[Sqrt[11/3]
/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

method	result
default	$\frac{E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{4\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{121\sqrt{-24x^3+70x^2-21x-10}}-\frac{3\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{12}\right.}{121\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

```
[In] int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)
^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{11}{72}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{1}{2}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 11/72*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) - 1/2*sqr
t(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655
/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.55 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 103

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{5\sqrt{-5+2x}} - \frac{3\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{-5+2x}}$$

```
[Out] -1/55*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)-3/55*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*
I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {181, 122, 120, 174, 552, 551}

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}} - \frac{3\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{2x-5}}$$

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] -1/5*(Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/Sqrt[-5 + 2*x] - (3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2
 - 3*x])/Sqrt[11]], -1/2])/(5*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 181

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{3}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\right) \\
&\quad+\frac{31}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx \\
&= -\left(\frac{62}{5}\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)\right) \\
&\quad-\frac{\left(3\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{5\sqrt{-5+2x}} \\
&= -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}} \\
&\quad-\frac{\left(62\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{5\sqrt{-5+2x}} \\
&= -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}}-\frac{3\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{-5+2x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \frac{3\sqrt{5-2x}\left(\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)\right)}{5\sqrt{-55+22x}}$$

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (3*Sqrt[5 - 2*x]*(EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - Ell
ipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(5*Sqrt[-55 + 2
2*x])
```

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

method	result
default	$-\frac{\left(69F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-124\Pi\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{1265\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{3\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{605\sqrt{-24x^3+70x^2-21x-10}}+\frac{124\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\Pi\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)}{13915\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

```
[In] int((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/1265*(69*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-124*EllipticPi(1/11*(11
+44*x)^(1/2),-55/23,3^(1/2)))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx=\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 - 34*x^2 - 151*
x - 35), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx=\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\cdot(5x+7)}\,dx$$

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)}\,dx$$

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

3.56 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 189

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} + \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}} - \frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{115\sqrt{-5+2x}} - \frac{3571\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{92690\sqrt{11}\sqrt{-5+2x}}$$

```
[Out] -2/1265*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)-3571/1019590*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+2/897*EllipticE(
2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^
(1/2)-5/897*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules

used = {183, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = -\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{115\sqrt{2x-5}} + \frac{2\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}} - \frac{3571\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{92690\sqrt{11}\sqrt{2x-5}} - \frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*(7 + 5*x)) + (2*Sqrt[1
1]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(897
*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*S
qrt[1 + 4*x]], 1/3])/(115*Sqrt[-5 + 2*x]) - (3571*Sqrt[5 - 2*x]*EllipticPi[
55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(92690*Sqrt[11]*Sqrt[-5
+ 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
```

```
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 183

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h)))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{-479+336x+120x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{1794} \\
&= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{\frac{168}{5}+24x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1794} + \frac{3571\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{8970} \\
&= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{2}{299}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx \\
&\quad - \frac{6}{115}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&\quad - \frac{3571\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)}{4485} \\
&= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{115\sqrt{-5+2x}} \\
&\quad - \frac{\left(3571\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{1495\sqrt{33}\sqrt{-5+2x}} \\
&\quad - \frac{\left(2\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{299\sqrt{5-2x}}
\end{aligned}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} + \frac{2\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}} - \frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{115\sqrt{-5+2x}} - \frac{3571\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{92690\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.68 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \frac{-\frac{51150\sqrt{2-3x}(-5+2x)\sqrt{1+4x}}{7+5x} - 3\sqrt{55-22x}\left(6820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 14508\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\right.\right.}{9176310\sqrt{-5+2x}}$$

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 22
*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ellip
ticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 10713*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(9176310*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{5\sqrt{-24x^3+70x^2-21x-10}}{897(7+5x)} - \frac{28\sqrt{11+44x}\sqrt{22-33x}\sqrt{110-44x}F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{180895\sqrt{-24x^3+70x^2-21x-10}} - \frac{4\sqrt{11+44x}\sqrt{22-33x}\sqrt{110-}}{36}\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$
default	$\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\left(14260\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x - 6325\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x\right.}{}$
risch	$\frac{5(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{897(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}} - \frac{\left(-\frac{4\sqrt{22-33x}\sqrt{-66x+165}\sqrt{33+132x}\left(-\frac{11E\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{6} + \frac{5F\left(\frac{2\sqrt{22-33}}{11}\right.}{2}\right.}{108537\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$

```
[In] int((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)*(-5/897/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)-28/180895*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-4/36179*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))+7142/12481755*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{-3\,x+2}}{\left(5\,x+7\right)^2\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4 + 110*x^3 - 993*x^2 - 1232*x - 245), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\left(5x+7\right)^2}\,dx$$

[In] integrate((2-3*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)

[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{-3\,x+2}}{\left(5\,x+7\right)^2\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm ="giac")

[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^2}\,dx$$

[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)

[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)

3.57 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 225

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx \\
&= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} \\
&\quad + \frac{5365\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{16628586\sqrt{5-2x}} \\
&\quad - \frac{13243\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{1065935\sqrt{66}\sqrt{-5+2x}} \\
&\quad - \frac{16369941\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{3436574440\sqrt{11}\sqrt{-5+2x}}
\end{aligned}$$

```
[Out] -16369941/37802318840*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2
^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-13243/70351710*EllipticF(1/11
*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+
5365/16628586*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)
*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-5/1794*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)/(7+5*x)^2-26825/33257172*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7
+5*x)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {183, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$$
$$= -\frac{13243\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{1065935\sqrt{66}\sqrt{2x-5}}$$
$$+\frac{5365\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{16628586\sqrt{5-2x}}$$
$$-\frac{16369941\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{3436574440\sqrt{11}\sqrt{2x-5}}$$
$$-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{33257172(5x+7)}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1794*(7 + 5*x)^2) - (26825
*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(33257172*(7 + 5*x)) + (5365*S
qrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])
/(16628586*Sqrt[5 - 2*x]) - (13243*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11
]*Sqrt[1 + 4*x]], 1/3])/(1065935*Sqrt[66]*Sqrt[-5 + 2*x]) - (16369941*Sqrt[
5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(343
6574440*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 183

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h)))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
```

```
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{\int \frac{-1063+1372x-120x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx}{3588}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{\int \frac{-7905051+4073184x+1931400x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{199543032}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{\int \frac{\frac{1369224}{5}+386280x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{199543032} + \frac{5456647\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{110857240}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{5365\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{5542862} - \frac{13243\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{2131870} - \frac{5456647\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)}{55428620}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{\left(5456647\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{55428620\sqrt{-5+2x}} - \frac{\left(13243\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{1065935\sqrt{22}\sqrt{-5+2x}} - \frac{\left(5365\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{5542862\sqrt{5-2x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} + \frac{5365\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{16628586\sqrt{5-2x}} - \frac{13243\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{1065935\sqrt{66}\sqrt{-5+2x}} - \frac{16369941\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{3436574440\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \frac{-17050\sqrt{2-3x}(-5+2x)\sqrt{1+4x}(56093+26825x)-\sqrt{55-22x}(7+5x)^2\left(36589300E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right.\right.\right.}{113406956520\sqrt{-}}$$

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(56093 + 26825*x) - Sqrt[55
- 22*x]*(7 + 5*x)^2*(36589300*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] - 64043148*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 491
09823*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(11340
6956520*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{5\sqrt{-24x^3+70x^2-21x-10}}{1794(7+5x)^2}-\frac{26825\sqrt{-24x^3+70x^2-21x-10}}{33257172(7+5x)}-\frac{19017\sqrt{11+44x}\sqrt{22-33x}\sqrt{110-44x}F\left(\frac{\sqrt{11+44x}}{11}\right.}{1676715755\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}}$
risch	$\frac{5(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}(56093+26825x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{33257172(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}} - \frac{\left(-\frac{5365\sqrt{22-33x}\sqrt{-66x+165}\sqrt{33+132x}\left(-\frac{11E\left(\frac{2\sqrt{22-33x}}{11},\right.}{6}\right.}{1006029453\sqrt{-24x^3+70x^2-21x}}\right.}{}$
default	$\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\left(254612300\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2-169668125\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}E\left(\frac{\sqrt{11}}{1}\right.\right.}{}$

```
[In] int((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/
2)*(-5/1794/(7+5*x)^2*(-24*x^3+70*x^2-21*x-10)^(1/2)-26825/33257172/(7+5*x)
*(-24*x^3+70*x^2-21*x-10)^(1/2)-19017/1676715755*(11+44*x)^(1/2)*(22-33*x)^
(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44
*x)^(1/2),3^(1/2))-5365/335343151*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x
)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/
```

```
2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))+5456647/7712892473
0*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)
^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5 + 1950*x^4 -
4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^3}\,dx$$

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^3}\,dx$$

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

3.58 $\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 293

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g
))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g
))^(1/2)/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*EllipticPi(f^(1/2)*(d*x+c)
^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g
))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g
))^(1/2)/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used

$= \{181, 122, 121, 175, 552, 551\}$

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$- \frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[In] Int[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d
*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((
d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) - (
2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*
g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sq
rt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt
[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 181

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b}$$

$$= -\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b} + \frac{\left(d\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{b\sqrt{e+fx}}$$

$$= -\frac{\left(2(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b\sqrt{e+fx}} + \frac{\left(d\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{b\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{\left(2(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int\frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}dx,x,\sqrt{c+dx}\right)}{b\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.82 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.69

$$\int\frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}dx = -\frac{2i\sqrt{c+dx}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)-\text{EllipticPi}\left(\frac{b(-de+cf)}{(bc-ad)f},i\text{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right.\right.\right.}{b\sqrt{\frac{f(c+dx)}{d(e+fx)}}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*Sqrt[c + d*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*(EllipticF[I*ArcSinh[
Sqrt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - Ellipt
icPi[(b*(-(d*e) + c*f))/((b*c - a*d)*f), I*ArcSinh[Sqrt[(f*(c + d*x))/(d*e
- c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(b*Sqrt[(f*(c + d*x))/(d*(e +
f*x))]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.63

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2d\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{b\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg}}-\frac{2(ad-bc)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+e}{-\frac{g}{h}+}}}{b^{2}\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+ce}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\left(F\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)ade\,h^{2}-F\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)adfgh-F\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)bdegh+F\left(\sqrt{-\frac{(hx}{eh}}\right.\right.}{}$

```
[In] int((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2*d/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/
f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g
*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g
/h+c/d))^(1/2))-2*(a*d-b*c)/b^2*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d
)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x
^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticPi(((x
+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^(1/2))
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
[Out] integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
[Out] integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)}\,dx$$

```
[In] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
[Out] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

3.59 $\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 449

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$
$$+\frac{2(bc-ad)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2(bc-ad)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*(-a*d+b*c)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/
f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g
)/(-c*h+d*g))^(1/2)/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*d+b*c)*El
lipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,(
(-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1
/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)+2*
d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)
/h)^(1/2))*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/b/f/h
^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {185, 122, 121, 175, 552, 551, 115, 114}

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-c\ldots}{f(dg-\ldots}\right.}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$+\frac{2d\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{g+hx}\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

```
[In] Int[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(b*f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g +
 h*x]) + (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*
Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e +
f*x]*Sqrt[g + h*x]) - (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/(
(b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
- c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 185

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d(bc-ad)}{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} + \frac{(bc-ad)^2}{b^2(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} + \frac{d\sqrt{c+dx}}{b\sqrt{e+fx}\sqrt{g+hx}} \right) dx$$

$$= \frac{d\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(d(bc-ad))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2}$$

$$= -\frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^2} + \frac{\left(d(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{b^2\sqrt{e+fx}} + \frac{\left(d\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\right)\int \frac{\sqrt{\frac{cf}{-de+cf}+\frac{dfx}{-de+cf}}}{\sqrt{e+fx}\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}\,dx}{b\sqrt{\frac{f(c+dx)}{-de+cf}}\sqrt{g+hx}}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} - \frac{\left(2(bc-ad)^2\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^2\sqrt{e+fx}} + \frac{\left(d(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{b^2\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$
$$+ \frac{2(bc-ad)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$- \frac{\left(2(bc-ad)^2\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int\frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}dx,x,\sqrt{c+dx}\right)}{b^2\sqrt{e+fx}\sqrt{g+hx}}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$
$$+ \frac{2(bc-ad)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$- \frac{2(bc-ad)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.30 (sec) , antiderivative size = 1176, normalized size of antiderivative = 2.62

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\left(b^2d^2e^2f\sqrt{-e+\frac{cf}{d}}g - b^2cdef^2\sqrt{-e+\frac{cf}{d}}g - abd^2ef^2\sqrt{-e+\frac{cf}{d}}g + ab\right.}{}$$

```
[In] Integrate[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(b^2*d^2*e^2*f*Sqrt[-e + (c*f)/d]*g - b^2*c*d*e*f^2*Sqrt[-e + (c*f)/d]*g
 - a*b*d^2*e*f^2*Sqrt[-e + (c*f)/d]*g + a*b*c*d*f^3*Sqrt[-e + (c*f)/d]*g -
b^2*d^2*e^3*Sqrt[-e + (c*f)/d]*h + b^2*c*d*e^2*f*Sqrt[-e + (c*f)/d]*h + a*b
*d^2*e^2*f*Sqrt[-e + (c*f)/d]*h - a*b*c*d*e*f^2*Sqrt[-e + (c*f)/d]*h - b^2*
d^2*e*f*Sqrt[-e + (c*f)/d]*g*(e + f*x) + a*b*d^2*f^2*Sqrt[-e + (c*f)/d]*g*(
e + f*x) + 2*b^2*d^2*e^2*Sqrt[-e + (c*f)/d]*h*(e + f*x) - b^2*c*d*e*f*Sqrt[
-e + (c*f)/d]*h*(e + f*x) - 2*a*b*d^2*e*f*Sqrt[-e + (c*f)/d]*h*(e + f*x) +
a*b*c*d*f^2*Sqrt[-e + (c*f)/d]*h*(e + f*x) - b^2*d^2*e*Sqrt[-e + (c*f)/d]*h
*(e + f*x)^2 + a*b*d^2*f*Sqrt[-e + (c*f)/d]*h*(e + f*x)^2 + I*b*d*(b*e - a*
f)*(d*e - c*f)*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*
(g + h*x))/(h*(e + f*x))]*EllipticE[I*ArcSinh[Sqrt[-e + (c*f)/d]/Sqrt[e + f
```

```
*x]], (d*(-(f*g) + e*h))/((d*e - c*f)*h)] + I*b*(-(b*c) + a*d)*f*(d*e - c*f
)*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h
*(e + f*x))]*EllipticF[I*ArcSinh[Sqrt[-e + (c*f)/d]/Sqrt[e + f*x]], (d*(-(f
*g) + e*h))/((d*e - c*f)*h)] - I*b^2*c^2*f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f
*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*EllipticPi[(b*d*e -
 a*d*f)/(b*d*e - b*c*f), I*ArcSinh[Sqrt[-e + (c*f)/d]/Sqrt[e + f*x]], (d*(-
(f*g) + e*h))/((d*e - c*f)*h)] + (2*I)*a*b*c*d*f^2*h*Sqrt[(f*(c + d*x))/(d*
(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*EllipticPi[(b
*d*e - a*d*f)/(b*d*e - b*c*f), I*ArcSinh[Sqrt[-e + (c*f)/d]/Sqrt[e + f*x]],
 (d*(-(f*g) + e*h))/((d*e - c*f)*h)] - I*a^2*d^2*f^2*h*Sqrt[(f*(c + d*x))/(
d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*EllipticPi[
(b*d*e - a*d*f)/(b*d*e - b*c*f), I*ArcSinh[Sqrt[-e + (c*f)/d]/Sqrt[e + f*x]
], (d*(-(f*g) + e*h))/((d*e - c*f)*h)]))/(b^2*f^2*(-(b*e) + a*f)*Sqrt[-e +
(c*f)/d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(-\frac{2d(ad-2bc)\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2d^2\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x}{-}}}{b\sqrt{dfh\,x^3+}}\right.}{}$
default	Expression too large to display

```
[In] int((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
-2*d*(a*d-2*b*c)/b^2*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d
))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^
2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),
((-g/h+e/f)/(-g/h+c/d))^(1/2))+2*d^2/b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*
((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+
d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*Ellipt
icE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(
((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2)))+2*(a^2*d^2-2*a*b*
c*d+b^2*c^2)/b^3*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(
1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*
e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticPi(((x+g/h)/(g/h-e/f)
```

```
)^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(c+d\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)}\,dx$$

```
[In] int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
[Out] int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

3.60 $\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 203

$$\begin{aligned}
&\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad - \frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&\quad - \frac{5109835\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{756\sqrt{5-2x}} \\
&\quad + \frac{392989907\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{2016\sqrt{66}\sqrt{-5+2x}}
\end{aligned}$$

```
[Out] 392989907/133056*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2
)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)-5109835/756*EllipticE(2/11*(2-3*x)^(1/2)*11^
(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-120355/288*(2-3*
x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-305/24*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)
^(1/2)*(1+4*x)^(1/2)-25/84*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {173, 1614, 1629, 164, 115, 114, 122, 120}

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{392989907\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{2016\sqrt{66}\sqrt{2x-5}} - \frac{5109835\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{756\sqrt{5-2x}} - \frac{25}{84}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2 - \frac{305}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7) - \frac{120355}{288}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-120355*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/288 - (305*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 - (25*Sqrt[2 - 3*x]*Sqrt[-5 +
 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/84 - (5109835*Sqrt[11]*Sqrt[-5 + 2*x]*Elli
pticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(756*Sqrt[5 - 2*x]) + (392
989907*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(201
6*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
```

```
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
```

```
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&\quad+\frac{1}{168}\int\frac{(7+5x)\left(48949+134855x+128100x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= -\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad-\frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2-\frac{\int\frac{-9476460-227834100x-303294600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{20160} \\
&= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad-\frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2-\frac{\int\frac{8530322220-88297948800x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{2177280} \\
&= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad-\frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\
&\quad+\frac{5109835}{252}\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx+\frac{392989907\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{4032}
\end{aligned}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)$$
$$- \frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2$$
$$+ \frac{\left(392989907\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{2016\sqrt{22}\sqrt{-5+2x}}$$
$$+ \frac{\left(5109835\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{252\sqrt{5-2x}}$$
$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)$$
$$- \frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2$$
$$- \frac{5109835\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{756\sqrt{5-2x}}$$
$$+ \frac{392989907\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{2016\sqrt{66}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 22.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.62

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{-1650\sqrt{2-3x}\sqrt{1+4x}(-210245+50078x+10608x^2+1200x^3) - 449665480\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\right.\right.}{133056\sqrt{-5+2x}}$$

```
[In] Integrate[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-1650*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-210245 + 50078*x + 10608*x^2 + 1200*x^
3) - 449665480*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 +
4*x]], 1/3] + 392989907*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*
Sqrt[1 + 4*x]], 1/3])/(133056*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(449665480\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-279638761\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+4}}{11}\right.\right.}{3193344x^3-9313920x^2+2794176x+1330}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{675x\sqrt{-24x^3+70x^2-21x-10}}{8}-\frac{150175\sqrt{-24x^3+70x^2-21x-10}}{288}-\frac{752233\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\sqrt{1}\right.}{23232\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}}$
risch	$\frac{25\left(600x^2+6804x+42049\right)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{2016\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(\frac{752233\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33}}{11}\right.}{69696\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$

```
[In] int((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/133056*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(449665480*(1+4*x)^(1/2
)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/
2))-279638761*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(
1/11*(11+44*x)^(1/2),3^(1/2))-23760000*x^5-200138400*x^4-900068400*x^3+4611
000900*x^2-1569263850*x-693808500)/(24*x^3-70*x^2+21*x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.29

$$\begin{aligned}
&\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= -\frac{25}{2016}\left(600\,x^2+6804\,x+42049\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\
&\quad -\frac{184083109}{31104}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\
&\quad +\frac{5109835}{756}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)
\end{aligned}$$

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] -25/2016*(600*x^2 + 6804*x + 42049)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x +
 2) - 184083109/31104*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x -
35/36) + 5109835/756*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstra
ssPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^4}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)**4/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral((5*x + 7)**4/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^4}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^4}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^4}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int((5*x + 7)^4/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((5*x + 7)^4/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.61 $\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 165

$$
\begin{aligned}
&\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad - \frac{487585\sqrt{11}\sqrt{-5+2x}\,E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1296\sqrt{5-2x}} \\
&\quad + \frac{2474201\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{216\sqrt{66}\sqrt{-5+2x}}
\end{aligned}
$$

```
[Out] 2474201/14256*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)-487585/1296*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/
2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-2135/108*(2-3*x)^(1
/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-5/12*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{173, 1629, 164, 115, 114, 122, 120\}$

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{2474201\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{216\sqrt{66}\sqrt{2x-5}} - \frac{487585\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1296\sqrt{5-2x}} - \frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7) - \frac{2135}{108}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/108 - (5*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/12 - (487585*Sqrt[11]*Sqrt[-5 + 2*x]
*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1296*Sqrt[5 - 2*x])
+ (2474201*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/
(216*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
```

```
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = -\frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)+\frac{1}{120}\int\frac{34985+104825x+85400x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)+\frac{\int\frac{1088280+29255100x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{12960}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{487585}{432}\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx + \frac{2474201}{432}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{\left(2474201\sqrt{5-2x}\right)\int\frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{216\sqrt{22}\sqrt{-5+2x}} + \frac{\left(487585\sqrt{-5+2x}\right)\int\frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{432\sqrt{5-2x}}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{487585\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1296\sqrt{5-2x}} + \frac{2474201\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{216\sqrt{66}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 18.64 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

$$\int\frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{-6600\sqrt{2-3x}\sqrt{1+4x}(-490+151x+18x^2)-5363435\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)+494\cdots}{28512\sqrt{-5+2x}}$$

```
[In] Integrate[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-6600*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-490 + 151*x + 18*x^2) - 5363435*Sqrt[6
6]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 4948402
*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
28512*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(4118336\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-5363435\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+4\cdots}}{11}\cdots\right)\right)}{28512(24x^3-70x^2+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{25x\sqrt{-24x^3+70x^2-21x-10}}{12}-\frac{1225\sqrt{-24x^3+70x^2-21x-10}}{54}+\frac{3023\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+4\cdots}}{11}\cdots\right)}{4356\sqrt{-24x^3+70x^2-21x-10}}\cdots\right)}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\cdots}$
risch	$\frac{25(98+9x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{108\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(-\frac{3023\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{13068\sqrt{-24x^3+70x^2-21x-10}}-\frac{487\cdots}{\cdots}\right)}{\cdots}$

[In] int((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/28512*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(4118336*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-5363435*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+1425600*x^4+11365200*x^3-44028600*x^2+14176800*x+6468000)/(24*x^3-70*x^2+21*x+10)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= -\frac{25}{108}\,(9\,x+98)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} - \frac{17718443}{46656}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) + \frac{487585}{1296}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

```
[Out] -25/108*(9*x + 98)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 17718443/46
656*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 487585/12
96*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108
, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral((5*x + 7)**3/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.62 $\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 129

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{2135\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{108\sqrt{5-2x}} + \frac{24353\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{36\sqrt{66}\sqrt{-5+2x}}$$

```
[Out] 24353/2376*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2
*x)^(1/2)/(-5+2*x)^(1/2)-2135/108*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2
*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-25/36*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {173, 24, 164, 115, 114, 122, 120}

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{24353\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{36\sqrt{66}\sqrt{2x-5}} - \frac{2135\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{108\sqrt{5-2x}} - \frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

```
[In] Int[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/36 - (2135*Sqrt[11]*Sqrt[-
5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(108*Sqrt[5 -
 2*x]) + (24353*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(36*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
```

```
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{72}\int\frac{21021+74795x+42700x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx\\
&= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{\int\frac{75075+213500x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1800}\\
&= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{2135}{36}\int\frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx\\
&\quad+\frac{24353}{72}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\\
&= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{\left(24353\sqrt{5-2x}\right)\int\frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{36\sqrt{22}\sqrt{-5+2x}}\\
&\quad+\frac{\left(2135\sqrt{-5+2x}\right)\int\frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{36\sqrt{5-2x}}
\end{aligned}$$

$$= -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{2135\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{108\sqrt{5-2x}} + \frac{24353\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{36\sqrt{66}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 16.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.89

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{1650\sqrt{2-3x}(5-2x)\sqrt{1+4x} - 23485\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right) + 24353\sqrt{66}\sqrt{5-2x}\,\mathrm{F}}{2376\sqrt{-5+2x}}$$

```
[In] Integrate[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
[Out] (1650*Sqrt[2 - 3*x]*(5 - 2*x)*Sqrt[1 + 4*x] - 23485*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 24353*Sqrt[66]*Sqrt[5 -
2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2376*Sqrt[-5 + 2*x]
)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(26089\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-23485\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{\right.\right.}{2376(24x^3-70x^2+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{25\sqrt{-24x^3+70x^2-21x-10}}{36}+\frac{91\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{264\sqrt{-24x^3+70x^2-21x-10}}+\frac{2135\sqrt{11+44x}\,\sqrt{22-33x}}{}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(-\frac{91\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{792\sqrt{-24x^3+70x^2-21x-10}}-\frac{2135\sqrt{22-33x}}{}\right.}{}$

```
[In] int((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2376*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(26089*(1+4*x)^(1/2)*(2-
3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-2
3485*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+
44*x)^(1/2),3^(1/2))+39600*x^3-115500*x^2+34650*x+16500)/(24*x^3-70*x^2+21*
x+10)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.38

$$\begin{aligned}\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= -\frac{25}{36}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\ &\quad -\frac{12719}{486}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\ &\quad +\frac{2135}{108}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)\end{aligned}$$

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] -25/36*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 12719/486*sqrt(-6)*weie
rstrassPInverse(847/108, 6655/2916, x - 35/36) + 2135/108*sqrt(-6)*weierstr
assZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/
36))
```

Sympy [F]

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^2}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**2/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^2}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^2}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^2}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.63 $\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{5\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}} + \frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{\sqrt{-5+2x}}$$

```
[Out] 13/22*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)-5/6*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2)
)*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {164, 115, 114, 122, 120}

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}} - \frac{5\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}}$$

```
[In] Int[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-5*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(6*Sqrt[5 - 2*x]) + (13*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqr
t[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5}{2}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx + \frac{39}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= \frac{\left(39\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{\sqrt{22}\sqrt{-5+2x}} + \frac{\left(5\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{2\sqrt{5-2x}} \\
&= -\frac{5\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}} + \frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{\sqrt{-5+2x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.91

$$
\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{220\sqrt{1+4x}(10-19x+6x^2)+55\sqrt{66}\sqrt{\frac{-5+2x}{1+4x}}\sqrt{\frac{-2+3x}{1+4x}}(1+4x)^2E\left(\arcsin\left(\frac{\sqrt{11}}{\sqrt{1+4x}}\right)|\frac{1}{3}\right)-78\sqrt{66}\sqrt{\frac{-5+2x}{1+4x}}\,\cdots}{132\sqrt{2-3x}\sqrt{-5+2x}(1+4x)}
$$

```
[In] Integrate[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (220*Sqrt[1 + 4*x]*(10 - 19*x + 6*x^2) + 55*Sqrt[66]*Sqrt[(-5 + 2*x)/(1 + 4
*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*EllipticE[ArcSin[Sqrt[11]/Sqrt[
1 + 4*x]], 1/3] - 78*Sqrt[66]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*Sqrt[(-2 + 3*x)/(1
 + 4*x)]*(1 + 4*x)^2*EllipticF[ArcSin[Sqrt[11]/Sqrt[1 + 4*x]], 1/3])/(132*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*(1 + 4*x))
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

method	result
default	$\frac{\left(124F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-55E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{132\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{7\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{121\sqrt{-24x^3+70x^2-21x-10}}+\frac{5\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{12}+\cdots\right.}{121\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

```
[In] int((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/132*(124*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-55*EllipticE(1/11*(11+44
*x)^(1/2),3^(1/2)))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.27

$$\begin{aligned}\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= -\frac{427}{216}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\ &\quad+\frac{5}{6}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)\end{aligned}$$

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] -427/216*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 5/6*
sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6
655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5x+7}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5\,x+7}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5\,x+7}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5\,x+7}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.64 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 48

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{\sqrt{-5+2x}}$$

```
[Out] 1/33*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1
/2)/(-5+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {122, 120}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/Sqrt[-5 + 2*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
```

```
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{\sqrt{-5+2x}}$$
$$= \frac{\sqrt{\frac{2}{33}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= -\frac{\sqrt{\frac{-2+3x}{1+4x}}(1+4x)\sqrt{\frac{-10+4x}{11+44x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right),3\right)}{\sqrt{2-3x}\sqrt{-5+2x}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
[Out] -((Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*Sqrt[(-10 + 4*x)/(11 + 44*x)]*Ellip
ticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]))
```

Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{11\sqrt{-5+2x}}$	33
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{121\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{-24x^{3}+70x^{2}-21x-10}}$	94

```
[In] int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/11*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x
)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{1}{6}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas
")
[Out] -1/6*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima
")
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.65 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 51

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\frac{3\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{-5+2x}}$$

```
[Out] -3/341*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2^(1/2))*(5-2*x)
^(1/2)*11^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {174, 552, 551}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\frac{3\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{2x-5}}$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (-3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(31*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{\sqrt{-5+2x}}$$

$$= -\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$$

$$= \frac{3i(-2+3x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}}\right),-\frac{1}{2}\right)-\text{EllipticPi}\left(-\frac{62}{55},i\text{arcsinh}\left(\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}}\right),-\frac{1}{2}\right)\right)}{31\sqrt{1+4x}\sqrt{-55+22x}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

[Out] (((3*I)/31)*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(EllipticF[I*ArcSinh[Sqrt[11/2]/Sqrt[2 - 3*x]], -1/2] - EllipticPi[-62/55, I*ArcSinh[Sqrt[11/2]/Sqrt[2 - 3*x]], -1/2]))/(Sqrt[1 + 4*x]*Sqrt[-55 + 22*x])

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{4\Pi\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{253\sqrt{-5+2x}}$	34
elliptic	$\frac{4\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\Pi\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)}{2783\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{-24x^3+70x^2-21x-10}}$	95

[In] int(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 4/253*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4 - 182*x^3 - 385*x^2 + 197*x + 70), x)

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\cdot(5x+7)}\,dx$$

[In] integrate(1/(7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)

[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\left(5\,x+7\right)}\,dx$$

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

3.66 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 189

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{10\sqrt{11}\sqrt{-5+2x}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27807\sqrt{5-2x}} - \frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{713\sqrt{-5+2x}} - \frac{8953\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{574678\sqrt{11}\sqrt{-5+2x}}$$

```
[Out] -2/7843*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)-8953/6321458*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+10/27807*Ellipti
cE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*
x)^(1/2)-25/27807*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules

used = {178, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$
$$= -\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{713\sqrt{2x-5}}$$
$$+\frac{10\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27807\sqrt{5-2x}}$$
$$-\frac{8953\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{574678\sqrt{11}\sqrt{2x-5}}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)) + (10*Sq
rt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/
(27807*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3])/(713*Sqrt[-5 + 2*x]) - (8953*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(574678*Sqrt[11]*S
qrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
```

```
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 178

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b^2*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(
b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)}+\frac{\int \frac{7777-1680x-600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{55614} \\
&= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)}+\frac{\int \frac{-168-120x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{55614} \\
&\quad +\frac{8953\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{55614} \\
&= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)}-\frac{10\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{9269} \\
&\quad -\frac{6}{713}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&\quad -\frac{8953\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx,x,\sqrt{2-3x}\right)}{27807}
\end{aligned}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{713\sqrt{-5+2x}}$$
$$- \frac{\left(8953\sqrt{5-2x}\right)\mathrm{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{9269\sqrt{33}\sqrt{-5+2x}}$$
$$- \frac{\left(10\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{9269\sqrt{5-2x}}$$
$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{10\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27807\sqrt{5-2x}}$$
$$- \frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{713\sqrt{-5+2x}} - \frac{8953\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{574678\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 4.65 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$
$$= \frac{-\frac{51150\sqrt{2-3x}(-5+2x)\sqrt{1+4x}}{7+5x} - 3\sqrt{55-22x}\left(6820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 14508\,\mathrm{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right.\right.\right.}{56893122\sqrt{-5+2x}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 22
*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ellip
ticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 26859*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(56893122*Sqrt[-5 + 2*x])
```

Maple [A] (verified)

Time = 7.26 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{28\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{1121549\sqrt{-24x^3+70x^2-21x-10}}-\frac{20\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.}{12}\right.}{1121549\sqrt{-24x^3+70x^2-21x-1}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(14260\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x-6325\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x\right.}{}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{27807(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(\frac{28\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{3364647\sqrt{-24x^3+70x^2-21x-10}}+\frac{20\sqrt{22-33x}\,\sqrt{-66x}}{}\right.}{}$

[In] int(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)*(-28/1121549*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-20/1121549*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))-25/27807/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+17906/77386881*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx=\int \frac{1}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5 - 70*x^4 - 3199*x^3 - 1710*x^2 + 1729*x + 490), x)

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^2}\,dx$$

```
[In] integrate(1/(7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="maxima")
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}\,(5x+7)^2}\,dx$$

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

3.67 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 225

$$\begin{aligned}
&\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx \\
&= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} \\
&\quad + \frac{44765\sqrt{11}\sqrt{-5+2x}\,E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{515486166\sqrt{5-2x}} \\
&\quad - \frac{24007\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right),\frac{1}{3}\right)}{6608797\sqrt{66}\sqrt{-5+2x}} \\
&\quad - \frac{48493305\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{21306761528\sqrt{11}\sqrt{-5+2x}}
\end{aligned}$$

```
[Out] -48493305/234374376808*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*
2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-24007/436180602*EllipticF(1/
11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2
)+44765/515486166*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(
1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-25/55614*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+
4*x)^(1/2)/(7+5*x)^2-223825/1030972332*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)
^(1/2)/(7+5*x)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {178, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$$
$$= -\frac{24007\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{6608797\sqrt{66}\sqrt{2x-5}}$$
$$+\frac{44765\sqrt{11}\sqrt{2x-5}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{515486166\sqrt{5-2x}}$$
$$-\frac{48493305\sqrt{5-2x}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{21306761528\sqrt{11}\sqrt{2x-5}}$$
$$-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1030972332(5x+7)}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(55614*(7 + 5*x)^2) - (223
825*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1030972332*(7 + 5*x)) + (4
4765*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
-1/2])/(515486166*Sqrt[5 - 2*x]) - (24007*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sq
rt[3/11]*Sqrt[1 + 4*x]], 1/3])/(6608797*Sqrt[66]*Sqrt[-5 + 2*x]) - (4849330
5*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2
])/(21306761528*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 178

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b^2*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(
b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
```

```
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} + \frac{\int \frac{16079-6860x+600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx}{111228}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} + \frac{\int \frac{123236883-38449440x-16115400x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{6185833992}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} + \frac{\int \frac{-3177576-3223080x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{6185833992} + \frac{16164435\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{687314888}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} - \frac{44765\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx}{171828722} - \frac{24007\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{13217594} - \frac{16164435\,\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)}{343657444}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} - \frac{\left(16164435\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3x}\right)}{343657444\sqrt{-5+2x}} - \frac{\left(24007\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{6608797\sqrt{22}\sqrt{-5+2x}} - \frac{\left(44765\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,dx}{171828722\sqrt{5-2x}}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} + \frac{44765\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{515486166\sqrt{5-2x}} - \frac{24007\sqrt{5-2x}F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{6608797\sqrt{66}\sqrt{-5+2x}} - \frac{48493305\sqrt{5-2x}\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{21306761528\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$$
$$= \frac{-17050\sqrt{2-3x}(-5+2x)\sqrt{1+4x}(81209+44765x)-\sqrt{55-22x}(7+5x)^2\left(61059460E\left(\arcsin\left(\frac{2\sqrt{2-3}}{\sqrt{11}}\right.\right.\right.}{703123130424\sqrt{}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(81209 + 44765*x) - Sqrt[55
- 22*x]*(7 + 5*x)^2*(61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] - 116097852*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14
5479915*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(703
123130424*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{25\sqrt{-24x^3+70x^2-21x-10}}{55614(7+5x)^2}-\frac{223825\sqrt{-24x^3+70x^2-21x-10}}{1030972332(7+5x)}-\frac{44133\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,F\left(\frac{\sqrt{11}}{1}\right.}{10395637681\sqrt{-24x^3+70x^2-21x-1}}\right.}{\sqrt{2}}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(81209+44765x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{1030972332(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(\frac{14711\sqrt{22-33x}\,\sqrt{-66x+165}\,\sqrt{33+132x}\,F\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right.}{10395637681\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(510436700\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,F\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2-283138625\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,E\left(\right.\right.}{}$

```
[In] int(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] (-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/
2)*(-25/55614/(7+5*x)^2*(-24*x^3+70*x^2-21*x-10)^(1/2)-223825/1030972332/(7
+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)-44133/10395637681*(11+44*x)^(1/2)*(22-
33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*
(11+44*x)^(1/2),3^(1/2))-44765/10395637681*(11+44*x)^(1/2)*(22-33*x)^(1/2)*
(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+
```

44*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))+16164435/478199333326*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^6 + 3850*x^5 - 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^3}\,dx$$

[In] integrate(1/(7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)

[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**3), x)

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^3}\,dx$$

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

3.68 $\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 137

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{-fg+eh}i\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$

```
[Out] 2*i*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*
e)/h)^(1/2))*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/f/h
^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 115, 114}

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2i\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{g+hx}\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

```
[In] Int[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(f*g) + e*h]*i*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
```

```
((d*e - c*f)*h))])/(f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h
*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx \\
&= \frac{\left(i\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\right)\int \frac{\sqrt{\frac{cf}{-de+cf}+\frac{dfx}{-de+cf}}}{\sqrt{e+fx}\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}\,dx}{\sqrt{\frac{f(c+dx)}{-de+cf}}\sqrt{g+hx}} \\
&= \frac{2\sqrt{-fg+eh}i\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.74 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2ii\sqrt{c+dx}\sqrt{g+hx}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)\right)}{h\sqrt{\frac{f(c+dx)}{d(e+fx)}}\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

```
[In] Integrate[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*i*Sqrt[c + d*x]*Sqrt[g + h*x]*(EllipticE[I*ArcSinh[Sqrt[(f*(c + d*x
))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - EllipticF[I*ArcSinh[Sq
rt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(h*Sqrt[
(f*(c + d*x))/(d*(e + f*x))]*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2i\left(ce\,h^{2}-cfgh-degh+df\,g^{2}\right)E\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)\sqrt{\frac{(fx+e)h}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}{h^{2}f\left(dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg\right)}$
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2ci\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\,\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\,\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg}}+\frac{2di\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\,\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\,\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\left(\left(- \cdots\right.\right.}{\sqrt{dfh\,x^{3}+cfh\,x^{2}+de \cdots}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$

```
[In] int((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2*i*(c*e*h^2-c*f*g*h-d*e*g*h+d*f*g^2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/
2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*((f*x+e)*h/(e*h-f*g))^(1/2)*((d*x+c)*h/
(c*h-d*g))^(1/2)*(-(h*x+g)*f/(e*h-f*g))^(1/2)/h^2/f*(d*x+c)^(1/2)*(f*x+e)^(
1/2)*(h*x+g)^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x
+d*e*g*x+c*e*g)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.85

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$
$$-\frac{2\left(3\sqrt{dfh}dfhi\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-(d^2ef+cdf^2)gh+(d^2e^2-cdef+c^2f^2)h^2\right)}{3\,d^2f^2h^2},-\frac{4\left(2\,d^3f^3g^3-3\,(d^3ef^2+cd^2f^3)g^2h-3\,(c\right.}{}\right.\right.}{}$$

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="fricas")
```

```
[Out] -2/3*(3*sqrt(d*f*h)*d*f*h*i*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c
*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^
3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
+ c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3
)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d
*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*
f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 +
c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*
h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))) + (d*
f*g + (d*e - 2*c*f)*h)*sqrt(d*f*h)*i*weierstrassPInverse(4/3*(d^2*f^2*g^2 -
 (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2)
, -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4
*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^
2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/
(d*f*h)))/(d*f^2*h^2)
```

Sympy [F]

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = i\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((d*i*x+c*i)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] i*Integral(sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}} dx = \int \frac{dix + ci}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}} dx$$

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}} dx = \int \frac{dix + ci}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}} dx$$

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}} dx = \int \frac{c\,i + d\,i\,x}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,\sqrt{c + d\,x}} dx$$

```
[In] int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.69 $\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*b*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d
*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(-a*h+b*g)*EllipticF(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*
f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used

$= \{164, 115, 114, 122, 121\}$

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

```
[In] Int[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ellip
ticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f
*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
 - (2*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*S
qrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
```

```
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{h}+\frac{(-bg+ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{h}\\
&= \frac{\left((-bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{h\sqrt{e+fx}}\\
&\quad+\frac{\left(b\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\\
&= \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\\
&\quad+\frac{\left((-bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{h\sqrt{e+fx}\sqrt{g+hx}}\\
&= \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\\
&\quad-\frac{2\sqrt{-de+cf}(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.43 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(-bd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx)-ib(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right)\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c+dx}}$$

```
[In] Integrate[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(-(b*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*b*(d*e - c*f)*h*(c
 + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c +
d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*
h)/(d*e*h - c*f*h)] + I*d*(b*e - a*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/
(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-
c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*Sqrt[-
c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2a\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}F\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2b\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\left(\left(-\frac{g}{h}\right.\right.}{\sqrt{dfh\,x^3+cfh\,x^2+deh}}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$-\frac{2\left(F\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)ade\,h^2-F\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)adfgh-F\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)bce\,h^2+F\left(\sqrt{-\frac{(hx+g}{eh-f}}\right.\right.}{}$

```
[In] int((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
2*a*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)
/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x
+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h
```

```
+c/d))^(1/2))+2*b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^
(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c
*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f
))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(
1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.36

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(3\sqrt{dfh}bdfh\,\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-(d^2ef+cdf^2)gh+(d^2e^2-cdef+c^2f^2)h^2\right)}{3d^2f^2h^2},-\frac{4\left(2d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h-3\left(\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}$$

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(3*sqrt(d*f*h)*b*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c
*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^
3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
+ c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3
)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d
*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*
f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 +
c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*
h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))) + (b*
d*f*g + (b*d*e + (b*c - 3*a*d)*f)*h)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d
^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(
d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d
^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*
e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

Sympy [F]

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral((a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{bx+a}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{bx+a}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{a+b\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.70 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 165

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] -2*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c
)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e
))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {175, 552, 551}

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)}$$

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*
```

```
Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c
 - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{e+fx}}$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{e+fx}\sqrt{g+hx}}$$

$$= -\frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)-\text{EllipticPi}\left(-\frac{bcf-adf}{bde-bcf},i\text{arcsinh}\left(\right.\right.\right.}{(-bc+ad)\sqrt{-c+\frac{de}{f}}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g -
c*f*h)/(d*e*h - c*f*h)] - EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*
ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]
))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{2\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{\frac{(fx+e)h}{eh-fg}}\,\Pi\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-gb)},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)(eh-fg)}{f(ah-gb)(dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg)}$	222
elliptic	$\frac{2\sqrt{(dx+c)(fx+e)(hx+g)}\,\left(\frac{g}{h}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}}\,\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\,\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,\Pi\left(\sqrt{\frac{x+\frac{g}{h}}{\frac{g}{h}-\frac{e}{f}}},\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{a}{b}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}\,\left(-\frac{g}{h}+\frac{a}{b}\right)}$	274

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f*(-(h*x+g)*f/(e*h-f*g))^(1/2)
*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+
g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/
2))*(e*h-f*g)/(a*h-b*g)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*
f*g*x+d*e*g*x+c*e*g)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)\,\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.71 $\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 393

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2d\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} - \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^
(1/2)-2*b*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-
a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-
c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^2/f^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2)-2*d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*
(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))*h^(1/2)*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h
*x+g)/(-e*h+f*g))^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(-f*(d*x+c)/(-c*f+
d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {185, 106, 21, 115, 114, 175, 552, 551}

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2d\sqrt{h}\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{\sqrt{g+hx}(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}} - \frac{2b\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2} + \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{\sqrt{c+dx}(bc-ad)(de-cf)(dg-ch)}$$

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*Sq
rt[c + d*x]) - (2*d*Sqrt[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h
*x))/(f*g - e*h)]*EllipticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*
h]], -((d*(f*g - e*h))/((d*e - c*f)*h))])/((b*c - a*d)*(d*e - c*f)*(d*g - c
*h)*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*b*Sqrt[-(d*e) +
c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*Ellipt
icPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^2*Sqrt[f]*
Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 185

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{d}{(bc-ad)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}} + \frac{b}{(bc-ad)(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\right) dx$$

$$= \frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} - \frac{d\int \frac{1}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{bc-ad} + \frac{(2d)\int \frac{-\frac{1}{2}cfh-\frac{1}{2}dfhx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(de-cf)(dg-ch)}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(dfh)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(de-cf)(dg-ch)} - \frac{\left(2b\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{(bc-ad)\sqrt{e+fx}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{\left(2b\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}\,dx, x, \sqrt{c+dx}\right)}{(bc-ad)\sqrt{e+fx}\sqrt{g+hx}} - \frac{\left(dfh\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\right)\int \frac{\sqrt{\frac{cf}{-de+cf}+\frac{dfx}{-de+cf}}}{\sqrt{e+fx}\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}\,dx}{(bc-ad)(de-cf)(dg-ch)\sqrt{\frac{f(c+dx)}{-de+cf}}\sqrt{g+hx}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2d\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} - \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.22 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left((bc-ad)hE\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right.\right.\right.}{}$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*((b*c - a*d)*h*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*
x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + (b*d*g - 2*b*c*h + a*d*h)*EllipticF
[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*
h)] + b*(-(d*g) + c*h)*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*Arc
Sinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/
((b*c - a*d)^2*Sqrt[-c + (d*e)/f]*(-(d*g) + c*h)*Sqrt[e + f*x]*Sqrt[g + h*x
])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(353) = 706$.

Time = 2.06 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.48

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(-\frac{2\left(dfh\,x^2+dehx+dfgx+deg\right)d}{\left(c^2fh-cdeh-cdfg+d^2eg\right)(ad-bc)\sqrt{\left(x+\frac{c}{d}\right)\left(dfh\,x^2+dehx+dfgx+deg\right)}}+\frac{2\left(\frac{d(cfh-deh-dfg)}{\left(c^2fh-cdeh-cdfg+d^2eg\right)(ad-bc)}+\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
-2*(d*f*h*x^2+d*e*h*x+d*f*g*x+d*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)*d/(a
*d-b*c)/((x+c/d)*(d*f*h*x^2+d*e*h*x+d*f*g*x+d*e*g))^(1/2)+2*(1/(c^2*f*h-c*d
*e*h-c*d*f*g+d^2*e*g)*d*(c*f*h-d*e*h-d*f*g)/(a*d-b*c)+(d*e*h+d*f*g)/(c^2*f*
h-c*d*e*h-c*d*f*g+d^2*e*g)*d/(a*d-b*c))*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)
*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2
+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+g/h
)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))+2/(c^2*f*h-c*d*e*h-c*d*f*
g+d^2*e*g)*d^2*f*h/(a*d-b*c)*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(
-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+
d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/
h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))-c/d*EllipticF(((x+g/h)/(
g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2)))-2/(a*d-b*c)*(g/h-e/f)*((x+g
/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/
(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/
2)/(-g/h+a/b)*EllipticPi(((x+g/h)/(g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((
-g/h+e/f)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)\,(c+dx)^{\frac{3}{2}}\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(1/((a + b*x)*(c + d*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
[Out] integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
[Out] integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)),x)
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)), x)
```

3.72 $\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 875

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}} + \frac{4d\sqrt{f}(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(-de+cf)^{3/2}(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2bd\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} - \frac{2\sqrt{f}(2dfg+deh-3cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(-de+cf)^{3/2}(dg-ch)\sqrt{e+fx}\sqrt{g+hx}} - \frac{2b^2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

```
[Out] 2/3*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c
)^(3/2)+2*b*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h+d
*g)/(d*x+c)^(1/2)-4/3*d^2*(-2*c*f*h+d*e*h+d*f*g)*(f*x+e)^(1/2)*(h*x+g)^(1/2
)/(-a*d+b*c)/(-c*f+d*e)^2/(-c*h+d*g)^2/(d*x+c)^(1/2)+4/3*d*(-2*c*f*h+d*e*h+
d*f*g)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*
h+d*g))^(1/2))*f^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c
)/(c*f-d*e)^(3/2)/(-c*h+d*g)^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2
/3*(-3*c*f*h+d*e*h+2*d*f*g)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2)
,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d
```

```
*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/(c*f-d*e)^(3/2)/(-c*h+d*g)/(f*x+e)^(1
/2)/(h*x+g)^(1/2)-2*b^2*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b
*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)
*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^3/f^(
1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*b*d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h
-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))*h^(1/2)*(e*h-f*g)^(1/2)*(d*
x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h+d*g)/
(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {185, 106, 157, 164, 115, 114, 122, 121, 21, 175, 552, 551}

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)b^2}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2d\sqrt{h}\sqrt{eh-fg}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)b}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} + \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}b}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} + \frac{4d\sqrt{f}(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(cf-de)^{3/2}(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{f}(2dfg+deh-3cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(cf-de)^{3/2}(dg-ch)\sqrt{e+fx}\sqrt{g+hx}} - \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}} + \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}}$$

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)*(d*g - c*h)*
(c + d*x)^(3/2)) + (2*b*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(d*
e - c*f)*(d*g - c*h)*Sqrt[c + d*x]) - (4*d^2*(d*f*g + d*e*h - 2*c*f*h)*Sqrt
[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)^2*(d*g - c*h)^2*Sqrt[c
+ d*x]) + (4*d*Sqrt[f]*(d*f*g + d*e*h - 2*c*f*h)*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) +
c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2
```

```
)*(d*g - c*h)^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*b*d*Sqr
t[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/((b*c - a*d)^2*(d*e - c*f)*(d*g - c*h)*Sqrt[-((f*(c + d*
x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*Sqrt[f]*(2*d*f*g + d*e*h - 3*c*f*h)*S
qrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[Ar
cSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
 c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*Sqrt[e + f*x]*Sqrt
[g + h*x]) - (2*b^2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)),
ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g
 - c*h))])/((b*c - a*d)^3*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
```

```
*x]
```

Rule 185

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{d}{(bc-ad)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}} - \frac{bd}{(bc-ad)^2(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}} + \frac{b^2}{(bc-ad)^2(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\right) dx \\
&= \frac{b^2 \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2} - \frac{(bd)\int \frac{1}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2} - \frac{d\int \frac{1}{(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} \\
&= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} \\
&\quad - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{(bc-ad)^2} \\
&\quad + \frac{(2bd)\int \frac{-\frac{1}{2}cfh-\frac{1}{2}dfhx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2(de-cf)(dg-ch)} + \frac{(2d)\int \frac{\frac{1}{2}(2dfg+2deh-3cfh)+\frac{1}{2}dfhx}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3(bc-ad)(de-cf)(dg-ch)}
\end{aligned}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}}$$
$$- \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}}$$
$$- \frac{(4d)\int \frac{-\frac{1}{4}fh(d^2eg-3c^2fh+cd(fg+eh))-\frac{1}{2}dfh(dfg+deh-2cfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3(bc-ad)(de-cf)^2(dg-ch)^2}$$
$$- \frac{(bdfh)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2(de-cf)(dg-ch)}$$
$$- \frac{\left(2b^2\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{(bc-ad)^2\sqrt{e+fx}}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}}$$
$$- \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}}$$
$$- \frac{(df(2dfg+deh-3cfh))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{3(bc-ad)(de-cf)^2(dg-ch)}$$
$$+ \frac{(2d^2f(dfg+deh-2cfh))\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3(bc-ad)(de-cf)^2(dg-ch)^2}$$
$$- \frac{\left(2b^2\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}\,dx, x, \sqrt{c+dx}\right)}{(bc-ad)^2\sqrt{e+fx}\sqrt{g+hx}}$$
$$- \frac{\left(bdfh\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\right)\int \frac{\sqrt{\frac{cf}{-de+cf}+\frac{dfx}{-de+cf}}}{\sqrt{e+fx}\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}\,dx}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{\frac{f(c+dx)}{-de+cf}}\sqrt{g+hx}}$$

$$
\begin{aligned}
&= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} \\
&- \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}} \\
&- \frac{2bd\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} \\
&- \frac{2b^2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} \\
&- \frac{\left(df(2dfg+deh-3cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int\frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{3(bc-ad)(de-cf)^2(dg-ch)\sqrt{e+fx}} \\
&+ \frac{\left(2d^2f(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)\int\frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}\,dx}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} \\
&- \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}} \\
&+ \frac{4d\sqrt{f}(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(-de+cf)^{3/2}(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} \\
&- \frac{2bd\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} \\
&- \frac{2b^2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} \\
&- \frac{\left(df(2dfg+deh-3cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\int\frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}\,dx}{3(bc-ad)(de-cf)^2(dg-ch)\sqrt{e+fx}\sqrt{g+hx}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} \\
&- \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}} \\
&+ \frac{4d\sqrt{f}(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(-de+cf)^{3/2}(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} \\
&- \frac{2bd\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} \\
&+ \frac{2\sqrt{f}(3cfh-d(2fg+eh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(-de+cf)^{3/2}(dg-ch)\sqrt{e+fx}\sqrt{g+hx}} \\
&- \frac{2b^2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.51 (sec) , antiderivative size = 4180, normalized size of antiderivative = 4.78

$$
\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*((2*d^2)/(3*(b*c - a*d)*(-(d*e) +
 c*f)*(-(d*g) + c*h)*(c + d*x)^2) + (2*d^2*(3*b*d^2*e*g - 5*b*c*d*f*g + 2*a
*d^2*f*g - 5*b*c*d*e*h + 2*a*d^2*e*h + 7*b*c^2*f*h - 4*a*c*d*f*h))/(3*(b*c
- a*d)^2*(-(d*e) + c*f)^2*(-(d*g) + c*h)^2*(c + d*x))) + (2*(c + d*x)^(3/2)
*(-3*b^2*c*d^2*e*Sqrt[-c + (d*e)/f]*f*g*h + 3*a*b*d^3*e*Sqrt[-c + (d*e)/f]*
f*g*h + 5*b^2*c^2*d*Sqrt[-c + (d*e)/f]*f^2*g*h - 7*a*b*c*d^2*Sqrt[-c + (d*e
)/f]*f^2*g*h + 2*a^2*d^3*Sqrt[-c + (d*e)/f]*f^2*g*h + 5*b^2*c^2*d*e*Sqrt[-c
 + (d*e)/f]*f*h^2 - 7*a*b*c*d^2*e*Sqrt[-c + (d*e)/f]*f*h^2 + 2*a^2*d^3*e*Sq
rt[-c + (d*e)/f]*f*h^2 - 7*b^2*c^3*Sqrt[-c + (d*e)/f]*f^2*h^2 + 11*a*b*c^2*
d*Sqrt[-c + (d*e)/f]*f^2*h^2 - 4*a^2*c*d^2*Sqrt[-c + (d*e)/f]*f^2*h^2 - (3*
b^2*c*d^4*e^2*Sqrt[-c + (d*e)/f]*g^2)/(c + d*x)^2 + (3*a*b*d^5*e^2*Sqrt[-c
+ (d*e)/f]*g^2)/(c + d*x)^2 + (8*b^2*c^2*d^3*e*Sqrt[-c + (d*e)/f]*f*g^2)/(c
 + d*x)^2 - (10*a*b*c*d^4*e*Sqrt[-c + (d*e)/f]*f*g^2)/(c + d*x)^2 + (2*a^2*
d^5*e*Sqrt[-c + (d*e)/f]*f*g^2)/(c + d*x)^2 - (5*b^2*c^3*d^2*Sqrt[-c + (d*e
)/f]*f^2*g^2)/(c + d*x)^2 + (7*a*b*c^2*d^3*Sqrt[-c + (d*e)/f]*f^2*g^2)/(c +
 d*x)^2 - (2*a^2*c*d^4*Sqrt[-c + (d*e)/f]*f^2*g^2)/(c + d*x)^2 + (8*b^2*c^2
```

*d^3*e^2*Sqrt[-c + (d*e)/f]*g*h)/(c + d*x)^2 - (10*a*b*c*d^4*e^2*Sqrt[-c +
(d*e)/f]*g*h)/(c + d*x)^2 + (2*a^2*d^5*e^2*Sqrt[-c + (d*e)/f]*g*h)/(c + d*x
)^2 - (20*b^2*c^3*d^2*e*Sqrt[-c + (d*e)/f]*f*g*h)/(c + d*x)^2 + (28*a*b*c^2
*d^3*e*Sqrt[-c + (d*e)/f]*f*g*h)/(c + d*x)^2 - (8*a^2*c*d^4*e*Sqrt[-c + (d*
e)/f]*f*g*h)/(c + d*x)^2 + (12*b^2*c^4*d*Sqrt[-c + (d*e)/f]*f^2*g*h)/(c + d
*x)^2 - (18*a*b*c^3*d^2*Sqrt[-c + (d*e)/f]*f^2*g*h)/(c + d*x)^2 + (6*a^2*c^
2*d^3*Sqrt[-c + (d*e)/f]*f^2*g*h)/(c + d*x)^2 - (5*b^2*c^3*d^2*e^2*Sqrt[-c
+ (d*e)/f]*h^2)/(c + d*x)^2 + (7*a*b*c^2*d^3*e^2*Sqrt[-c + (d*e)/f]*h^2)/(c
 + d*x)^2 - (2*a^2*c*d^4*e^2*Sqrt[-c + (d*e)/f]*h^2)/(c + d*x)^2 + (12*b^2*
c^4*d*e*Sqrt[-c + (d*e)/f]*f*h^2)/(c + d*x)^2 - (18*a*b*c^3*d^2*e*Sqrt[-c +
 (d*e)/f]*f*h^2)/(c + d*x)^2 + (6*a^2*c^2*d^3*e*Sqrt[-c + (d*e)/f]*f*h^2)/(
c + d*x)^2 - (7*b^2*c^5*Sqrt[-c + (d*e)/f]*f^2*h^2)/(c + d*x)^2 + (11*a*b*c
^4*d*Sqrt[-c + (d*e)/f]*f^2*h^2)/(c + d*x)^2 - (4*a^2*c^3*d^2*Sqrt[-c + (d*
e)/f]*f^2*h^2)/(c + d*x)^2 - (3*b^2*c*d^3*e*Sqrt[-c + (d*e)/f]*f*g^2)/(c +
d*x) + (3*a*b*d^4*e*Sqrt[-c + (d*e)/f]*f*g^2)/(c + d*x) + (5*b^2*c^2*d^2*Sq
rt[-c + (d*e)/f]*f^2*g^2)/(c + d*x) - (7*a*b*c*d^3*Sqrt[-c + (d*e)/f]*f^2*g
^2)/(c + d*x) + (2*a^2*d^4*Sqrt[-c + (d*e)/f]*f^2*g^2)/(c + d*x) - (3*b^2*c
*d^3*e^2*Sqrt[-c + (d*e)/f]*g*h)/(c + d*x) + (3*a*b*d^4*e^2*Sqrt[-c + (d*e)
/f]*g*h)/(c + d*x) + (16*b^2*c^2*d^2*e*Sqrt[-c + (d*e)/f]*f*g*h)/(c + d*x)
- (20*a*b*c*d^3*e*Sqrt[-c + (d*e)/f]*f*g*h)/(c + d*x) + (4*a^2*d^4*e*Sqrt[-
c + (d*e)/f]*f*g*h)/(c + d*x) - (17*b^2*c^3*d*Sqrt[-c + (d*e)/f]*f^2*g*h)/(
c + d*x) + (25*a*b*c^2*d^2*Sqrt[-c + (d*e)/f]*f^2*g*h)/(c + d*x) - (8*a^2*c
*d^3*Sqrt[-c + (d*e)/f]*f^2*g*h)/(c + d*x) + (5*b^2*c^2*d^2*e^2*Sqrt[-c + (
d*e)/f]*h^2)/(c + d*x) - (7*a*b*c*d^3*e^2*Sqrt[-c + (d*e)/f]*h^2)/(c + d*x)
 + (2*a^2*d^4*e^2*Sqrt[-c + (d*e)/f]*h^2)/(c + d*x) - (17*b^2*c^3*d*e*Sqrt[
-c + (d*e)/f]*f*h^2)/(c + d*x) + (25*a*b*c^2*d^2*e*Sqrt[-c + (d*e)/f]*f*h^2
)/(c + d*x) - (8*a^2*c*d^3*e*Sqrt[-c + (d*e)/f]*f*h^2)/(c + d*x) + (14*b^2*
c^4*Sqrt[-c + (d*e)/f]*f^2*h^2)/(c + d*x) - (22*a*b*c^3*d*Sqrt[-c + (d*e)/f
]*f^2*h^2)/(c + d*x) + (8*a^2*c^2*d^2*Sqrt[-c + (d*e)/f]*f^2*h^2)/(c + d*x)
 - (I*(-(b*c) + a*d)*(-(d*e) + c*f)*h*(2*a*d*(d*f*g + d*e*h - 2*c*f*h) + b*
(3*d^2*e*g + 7*c^2*f*h - 5*c*d*(f*g + e*h)))*Sqrt[1 - c/(c + d*x) + (d*e)/(
f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticE[I*ArcSi
nh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqr
t[c + d*x] + (I*(-(d*e) + c*f)*(a^2*d^2*h*(d*f*g + 2*d*e*h - 3*c*f*h) + b^2
*(3*d^3*e*g^2 - 9*c^3*f*h^2 - 3*c*d^2*g*(f*g + 3*e*h) + 2*c^2*d*h*(5*f*g +
4*e*h)) + a*b*d*h*(3*d^2*e*g + 9*c^2*f*h - c*d*(5*f*g + 7*e*h)))*Sqrt[1 - c
/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x)
)]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(
d*e*h - c*f*h)])/Sqrt[c + d*x] + ((3*I)*b^2*d^4*e^2*g^2*Sqrt[1 - c/(c + d*x
) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*Ellipt
icPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[
c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - ((6*I)*b^2*c*d
^3*e*f*g^2*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x)
 + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*Ar
cSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/

```
Sqrt[c + d*x] + ((3*I)*b^2*c^2*d^2*f^2*g^2*Sqrt[1 - c/(c + d*x) + (d*e)/(f*
(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f
 - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d
*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - ((6*I)*b^2*c*d^3*e^2*g*h*Sq
rt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(
c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c
 + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x]
 + ((12*I)*b^2*c^2*d^2*e*f*g*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*
Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b
*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h
)/(d*e*h - c*f*h)])/Sqrt[c + d*x] - ((6*I)*b^2*c^3*d*f^2*g*h*Sqrt[1 - c/(c
+ d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*E
llipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/
Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((3*I)*b^
2*c^2*d^2*e^2*h^2*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c
 + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)
), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*
f*h)])/Sqrt[c + d*x] - ((6*I)*b^2*c^3*d*e*f*h^2*Sqrt[1 - c/(c + d*x) + (d*e
)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((
b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]
], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d*x] + ((3*I)*b^2*c^4*f^2*h^2
*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(
h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt
[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[c + d
*x]))/(3*(b*c - a*d)*(-(b*c) + a*d)^2*Sqrt[-c + (d*e)/f]*(-(d*e) + c*f)^2*(
-(d*g) + c*h)^2*Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))/d]*Sqrt[g + ((c
+ d*x)*(h - (c*h)/(c + d*x)))/d])
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 1335, normalized size of antiderivative = 1.53

method	result	size
elliptic	Expression too large to display	1335
default	Expression too large to display	16647

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] ((d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(
-2/3/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)/(a*d-b*c)*(d*f*h*x^3+c*f*h*x^2+d*e*h
*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(x+c/d)^2-2/3*(d*f*h*x^
2+d*e*h*x+d*f*g*x+d*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2*d*(4*a*c*d*f*h
-2*a*d^2*e*h-2*a*d^2*f*g-7*b*c^2*f*h+5*b*c*d*e*h+5*b*c*d*f*g-3*b*d^2*e*g)/(
a*d-b*c)^2/((x+c/d)*(d*f*h*x^2+d*e*h*x+d*f*g*x+d*e*g))^(1/2)+2*(-1/3*d*f*h/
```

```
(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)/(a*d-b*c)+1/3*d*(c*f*h-d*e*h-d*f*g)*(4*a*
c*d*f*h-2*a*d^2*e*h-2*a*d^2*f*g-7*b*c^2*f*h+5*b*c*d*e*h+5*b*c*d*f*g-3*b*d^2
*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2/(a*d-b*c)^2+1/3*(d*e*h+d*f*g)/(c^
2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2*d*(4*a*c*d*f*h-2*a*d^2*e*h-2*a*d^2*f*g-7*b
*c^2*f*h+5*b*c*d*e*h+5*b*c*d*f*g-3*b*d^2*e*g)/(a*d-b*c)^2)*(g/h-e/f)*((x+g/
h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(
d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2
)*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))^(1/2))+2/3*f*
h*d^2*(4*a*c*d*f*h-2*a*d^2*e*h-2*a*d^2*f*g-7*b*c^2*f*h+5*b*c*d*e*h+5*b*c*d*
f*g-3*b*d^2*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2/(a*d-b*c)^2*(g/h-e/f)*
((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(
1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g
)^(1/2)*((-g/h+c/d)*EllipticE(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c
/d))^(1/2))-c/d*EllipticF(((x+g/h)/(g/h-e/f))^(1/2),((-g/h+e/f)/(-g/h+c/d))
^(1/2)))+2/(a*d-b*c)^2*b*(g/h-e/f)*((x+g/h)/(g/h-e/f))^(1/2)*((x+c/d)/(-g/h
+c/d))^(1/2)*((x+e/f)/(-g/h+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*
g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-g/h+a/b)*EllipticPi(((x+g/h)/(
g/h-e/f))^(1/2),(-g/h+e/f)/(-g/h+a/b),((-g/h+e/f)/(-g/h+c/d))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{(a+bx)\,(c+dx)^{\frac{5}{2}}\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(5/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{5}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{5}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)), x)
```

3.73 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*f+b),2^(1/2)*(d/(c*f+d))^(1
/2))*(f*(d*x+c)/(c*f+d))^(1/2)/(a*f+b)/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {174, 552, 551}

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = -\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af-bx^2)\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx, x, \sqrt{1-fx}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}(b+af-bx^2)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f}\right)f}}}\,dx, x, \sqrt{1-fx}\right)}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx$$
$$= \frac{2i(c+dx)\sqrt{\frac{d(-1+fx)}{f(c+dx)}}\sqrt{\frac{d+dfx}{cf+dfx}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right), \frac{-d+cf}{d+cf}\right) - \text{EllipticPi}\left(\frac{bcf-adf}{bd+bcf}, i\text{arcsinh}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.\right.}{(-bc+ad)\sqrt{-\frac{d+cf}{f}}\sqrt{1-f^2x^2}}$$

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(71) = 142$.

Time $= 3.41$ (sec) , antiderivative size $= 184$, normalized size of antiderivative $= 2.49$

method	result	size
default	$-\frac{2(cf-d)\Pi\left(\sqrt{\frac{(dx+c)f}{cf-d}}, -\frac{(cf-d)b}{f(ad-bc)}, \sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{(fx+1)d}{cf-d}}\,\sqrt{-\frac{(fx-1)d}{cf+d}}\,\sqrt{\frac{(dx+c)f}{cf-d}}\,\sqrt{fx+1}\,\sqrt{-fx+1}\,\sqrt{dx+c}}{f(ad-bc)(d\,f^2x^3+c\,f^2x^2-dx-c)}$	184
elliptic	$\frac{2\sqrt{-(f^2x^2-1)(dx+c)}\left(\frac{c}{d}-\frac{1}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x-\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x+\frac{1}{f}}{-\frac{c}{d}+\frac{1}{f}}}\,\Pi\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}}, \frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}+\frac{a}{b}}, \sqrt{\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\right)}{\sqrt{dx+c}\,\sqrt{-fx+1}\,\sqrt{fx+1}\,b\sqrt{-d\,f^2x^3-c\,f^2x^2+dx+c}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}$	239

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)*f/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)*d/(c*f-d))^(1/2)*(-(f*x-1)*d/(c*f+d))^(1/2)*
((d*x+c)*f/(c*f-d))^(1/2)*(f*x+1)^(1/2)*(-f*x+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d
-b*c)/(d*f^2*x^3+c*f^2*x^2-d*x-c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx=\int \frac{1}{(a+bx)\,\sqrt{c+dx}\sqrt{-fx+1}\sqrt{fx+1}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f*x+1)**(1/2)/(f*x+1)**(1/2),x)
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f*x + 1)*sqrt(f*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx=\int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+1}\sqrt{-fx+1}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="maxima")
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx=\int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+1}\sqrt{-fx+1}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="giac")
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx=\int \frac{1}{\sqrt{1-f\,x}\,\sqrt{f\,x+1}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
[Out] int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.74 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx=-\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*f+b),2^(1/2)*(d/(c*f+d))^(1
/2))*(f*(d*x+c)/(c*f+d))^(1/2)/(a*f+b)/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {946, 174, 552, 551}

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx=-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx \\
&= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af-bx^2)\,\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx, x, \sqrt{1-fx}\right)\right) \\
&= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}(b+af-bx^2)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f}\right)f}}}\,dx, x, \sqrt{1-fx}\right)}{\sqrt{c+dx}} \\
&= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af}; \sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)\middle|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(-1+fx)}{f(c+dx)}}\sqrt{\frac{d+dfx}{cf+dfx}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right),\frac{-d+cf}{d+cf}\right)-\text{EllipticPi}\left(\frac{bcf-adf}{bd+bcf},i\text{arcsinh}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right),\frac{-d+cf}{d+cf}\right)\right)}{(-bc+ad)\sqrt{-\frac{d+cf}{f}}\sqrt{1-f^2x^2}}$$

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(71) = 142.

Time = 2.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.45

method	result	size
default	$-\frac{2(cf-d)\Pi\left(\sqrt{\frac{(dx+c)f}{cf-d}},-\frac{(cf-d)b}{f(ad-bc)},\sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{(fx+1)d}{cf-d}}\sqrt{-\frac{(fx-1)d}{cf+d}}\sqrt{\frac{(dx+c)f}{cf-d}}\sqrt{-f^2x^2+1}\sqrt{dx+c}}{f(ad-bc)(d f^2x^3+c f^2x^2-dx-c)}$	181
elliptic	$\frac{2\sqrt{-(f^2x^2-1)(dx+c)}\left(\frac{c}{d}-\frac{1}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}}\sqrt{\frac{x-\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\sqrt{\frac{x+\frac{1}{f}}{-\frac{c}{d}+\frac{1}{f}}}\Pi\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}},\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\right)}{\sqrt{-f^2x^2+1}\sqrt{dx+c}\,b\sqrt{-d f^2x^3-c f^2x^2+dx+c}\left(-\frac{c}{d}+\frac{a}{b}\right)}$	236

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)*f/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)*d/(c*f-d))^(1/2)*(-(f*x-1)*d/(c*f+d))^(1/2)*
((d*x+c)*f/(c*f-d))^(1/2)*(-f^2*x^2+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d-b*c)/(d*f
^2*x^3+c*f^2*x^2-d*x-c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{-(fx-1)(fx+1)}\,(a+bx)\sqrt{c+dx}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(f*x - 1)*(f*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{-f^2x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{-f^2x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{1-f^2\,x^2}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
[Out] int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.75 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 86

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\text{EllipticPi}\left(\frac{2b}{b+af^2},\arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right),\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+
d))^(1/2))*(f^2*(d*x+c)/(c*f^2+d))^(1/2)/(a*f^2+b)/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {174, 552, 551}

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = -\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\text{EllipticPi}\left(\frac{2b}{af^2+b},\arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right),\frac{2d}{cf^2+d}\right)}{(af^2+b)\sqrt{c+dx}}$$

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af^2-bx^2)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\,dx, x, \sqrt{1-f^2x}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}(b+af^2-bx^2)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f^2}\right)f^2}}}\,dx, x, \sqrt{1-f^2x}\right)}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.94 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx$$

$$= \frac{2i(c+dx)\sqrt{\frac{d(-1+f^2x)}{f^2(c+dx)}}\sqrt{\frac{d(1+f^2x)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right), \frac{-d+cf^2}{d+cf^2}\right) - \text{EllipticPi}\left(\frac{(bc-ad)f^2}{b(d+cf^2)}, i\text{arcsinh}\right.\right.}{(-bc+ad)\sqrt{-c-\frac{d}{f^2}}\sqrt{1-f^4x^2}}$$

[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]

[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x))/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(83) = 166$.

Time = 3.43 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.47

method	result	size
default	$-\frac{2(c f^2-d)\Pi\left(\sqrt{\frac{(dx+c)f^2}{c f^2-d}},-\frac{\left(c f^2-d\right)b}{f^2(ad-bc)},\sqrt{\frac{c f^2-d}{c f^2+d}}\right)\sqrt{-\frac{\left(f^2x+1\right)d}{c f^2-d}}\sqrt{-\frac{\left(f^2x-1\right)d}{c f^2+d}}\sqrt{\frac{(dx+c)f^2}{c f^2-d}}\sqrt{f^2x+1}\sqrt{-f^2x+1}\sqrt{dx+c}}{f^2(ad-bc)(d f^4x^3+c f^4x^2-dx-c)}$	212
elliptic	$\frac{2\sqrt{-(f^4x^2-1)(dx+c)}\left(\frac{c}{d}-\frac{1}{f^2}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}}\sqrt{\frac{x-\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}}\sqrt{\frac{x+\frac{1}{f^2}}{-\frac{c}{d}+\frac{1}{f^2}}}\Pi\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}},\frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}}\right)}{\sqrt{dx+c}\sqrt{-f^2x+1}\sqrt{f^2x+1}\,b\sqrt{-d f^4x^3-c f^4x^2+dx+c}\left(-\frac{c}{d}+\frac{a}{b}\right)}$	243

[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*d-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)*d/(c*f^2-d))^(1/2)*(-(f^2*x-1)*d/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(f^2*x+1)^(1/2)*(-f^2*x+1)^(1/2)*(d*x+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \text{Timed out}$$

[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{-f^2x+1}\sqrt{f^2x+1}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x+1)**(1/2)/(f**2*x+1)**(1/2),x)
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f**2*x + 1)*sqrt(f**2*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{\sqrt{f^2x+1}\sqrt{-f^2x+1}(bx+a)\sqrt{dx+c}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="maxima")
[Out] integrate(1/(sqrt(f^2*x + 1)*sqrt(-f^2*x + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{\sqrt{f^2x+1}\sqrt{-f^2x+1}(bx+a)\sqrt{dx+c}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{(a+b\,x)\,\sqrt{1-f^2\,x}\,\sqrt{x\,f^2+1}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)), x)
```

3.76 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}} \, dx = -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}} \text{EllipticPi}\left(\frac{2b}{b+af^2}, \arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right), \frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+
d))^(1/2))*(f^2*(d*x+c)/(c*f^2+d))^(1/2)/(a*f^2+b)/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {946, 174, 552, 551}

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}} \, dx = -\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}} \text{EllipticPi}\left(\frac{2b}{af^2+b}, \arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right), \frac{2d}{cf^2+d}\right)}{(af^2+b)\sqrt{c+dx}}$$

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx \\
&= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af^2-bx^2)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\,dx, x, \sqrt{1-f^2x}\right)\right) \\
&= -\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}(b+af^2-bx^2)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f^2}\right)f^2}}}\,dx, x, \sqrt{1-f^2x}\right)}{\sqrt{c+dx}} \\
&= -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(-1+f^2x)}{f^2(c+dx)}}\sqrt{\frac{d(1+f^2x)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right),\frac{-d+cf^2}{d+cf^2}\right)-\text{EllipticPi}\left(\frac{(bc-ad)f^2}{b(d+cf^2)},i\text{arcsin}\right.\right.}{(-bc+ad)\sqrt{-c-\frac{d}{f^2}}\sqrt{1-f^4x^2}}$$

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x)
)/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-
d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*A
rcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c
) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(83) = 166.

Time = 2.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.38

method	result	size
default	$-\frac{2(c f^2-d)\Pi\left(\sqrt{\frac{(dx+c)f^2}{c f^2-d}},-\frac{(c f^2-d)b}{f^2(ad-bc)},\sqrt{\frac{c f^2-d}{c f^2+d}}\right)\sqrt{-\frac{(f^2x+1)d}{c f^2-d}}\sqrt{-\frac{(f^2x-1)d}{c f^2+d}}\sqrt{\frac{(dx+c)f^2}{c f^2-d}}\sqrt{-f^4x^2+1}\sqrt{dx+c}}{f^2(ad-bc)(d f^4x^3+c f^4x^2-dx-c)}$	205
elliptic	$\frac{2\sqrt{-(f^4x^2-1)(dx+c)}\left(\frac{c}{d}-\frac{1}{f^2}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}}\sqrt{\frac{x-\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}}\sqrt{\frac{x+\frac{1}{f^2}}{-\frac{c}{d}+\frac{1}{f^2}}}\Pi\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}},\frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}}\right)}{\sqrt{-f^4x^2+1}\sqrt{dx+c}\,b\sqrt{-d f^4x^3-c f^4x^2+dx+c}\left(-\frac{c}{d}+\frac{a}{b}\right)}$	236

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*d
-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)*d/(c*f^2-d))^(1/2)*(-(f^2*x-
1)*d/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(-f^4*x^2+1)^(1/2)*(d*x
+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{-(f^2x-1)(f^2x+1)}\,(a+bx)\,\sqrt{c+dx}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**4*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(f**2*x - 1)*(f**2*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{-f^4x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{-f^4x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{1-f^4\,x^2}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
[Out] int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.77 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 471

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}}$$
$$-\frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1658880}$$
$$-\frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560}-\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400}$$
$$+\frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2}+\frac{1450582567\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{2457600\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$

```
[Out] -83363/34560*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-427/2
400*(7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/25*(7+5*x)^(
7/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-57691792727443/213497856000
*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*
I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5
+2*x)^(1/2)/(1+4*x)^(1/2)-1450582567/3686400*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7
+5*x)^(1/2)/(-5+2*x)^(1/2)-70489981/1658880*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1
+4*x)^(1/2)*(7+5*x)^(1/2)-245264762213/2289254400*(1/(4+2*(1+4*x)/(2-3*x)))
^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^
(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)
/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+1450582567/7372800*EllipticE(1/23*8
97^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(
1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {167, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \frac{1450582567\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{2457600\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{57691792727443(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{497664000\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}} - \frac{245264762213\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{99532800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{1}{25}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2} - \frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}{2400} - \frac{83363\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}{34560} - \frac{70489981\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}{1658880} - \frac{1450582567\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{3686400\sqrt{2x-5}}$$

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```

```
[Out] (-1450582567*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3686400*Sqrt[-5 +
2*x]) - (70489981*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
/1658880 - (83363*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2
))/34560 - (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))
/2400 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(7/2))/25 + (
1450582567*Sqrt[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[Ar
cSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(2457600*Sqrt[(2
 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (245264762213*Sqrt[11/23]*Sqrt[7 + 5*x]
*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(9953280
0*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (57691792727443*(2 - 3*x)*Sqr
t[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcS
in[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(497664000*Sqrt[429
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
```

```
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} \\
&\quad + \frac{1}{50}\int \frac{(7+5x)^{5/2}\left(-3-1190x+854x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} \\
&\quad + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} - \frac{\int \frac{(7+5x)^{3/2}\left(343070+1303340x-1667260x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{9600} \\
&= -\frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560} \\
&\quad - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} \\
&\quad + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} \\
&\quad + \frac{\int \frac{\sqrt{7+5x}\left(-840990780-627111520x+2819599240x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1382400} \\
&= -\frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1658880} \\
&\quad - \frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560} \\
&\quad - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} \\
&\quad + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} \\
&\quad - \frac{\int \frac{1120606854440-1345996898960x-3133258344720x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{132710400} \\
&= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} \\
&\quad - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1658880} \\
&\quad - \frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560} \\
&\quad - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} \\
&\quad + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} \\
&\quad + \frac{\int \frac{-1027194164487840+893292274489440x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{31850496000} - \frac{207433307081\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{2457600}
\end{aligned}$$

$$= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1658880} - \frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} - \frac{1861025571853\int\frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{199065600} - \frac{2697912384343\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{199065600} + \frac{\left(18857573371\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{2457600\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1658880} - \frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} + \frac{1450582567\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{2457600\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{\left(57691792727443(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{99532800\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} - \frac{\left(245264762213\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{99532800\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{1450582567\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1658880} - \frac{83363\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{34560} - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}{2400} + \frac{1}{25}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2} + \frac{1450582567\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{2457600\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{245264762213\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{99532800\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} - \frac{57691792727443(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{497664000\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.03 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.20

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \frac{\frac{868108390133985\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{2-3x}} + 886600\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}(-90202093+810}{}$$

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```

```
[Out] ((868108390133985*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]
 + 886600*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-902020
93 + 8103984*x + 27457920*x^2 + 8294400*x^3) - ((289369463377995*I)*Sqrt[25
3]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticE[I*ArcSinh[(Sqrt[11/3
9]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(
-2 + 3*x)]) - (34625405874290*Sqrt[429]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1
+ 4*x]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39]
)/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) - (499055525185546*Sqrt[429]*
Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticPi[-69/55, ArcSin[(Sqrt[1
```

```
1/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x
)/(-2 + 3*x)]) + ((58133423485995*I)*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)
/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[
-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (2966521710
99570*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55,
ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*
Sqrt[(-2 + 3*x)/(1 + 4*x)]))/1470763008000
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.06

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{168833x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{34560}-\frac{90202093\sqrt{-120x^4+182x^3+385x^2-197x-70}}{1658880}-\frac{280151713}{\ldots}\right.}{\ldots}$
risch	$-\frac{(8294400x^3+27457920x^2+8103984x-90202093)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{1658880\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{28015\ldots}{\ldots}\right.}{\ldots}$
default	$\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(242812114590870\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+10\ldots\right.}{\ldots}$

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(168833/34560*x*(-120*x^4+182*x^3+385*x^2-197*x-70
)^(1/2)-90202093/1658880*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-28015171
361/507413237760*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/
2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x
)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39
*I*897^(1/2))+16824961237/253706618880*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3
+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2
)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(
x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/
5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+1450582567/122880*((x+7/5)*(x-
5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((
x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*Ellipti
cF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1
/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*
(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/
3+x)*(x-5/2)*(x+1/4))^(1/2)+1589/96*x^2*(-120*x^4+182*x^3+385*x^2-197*x-70)
^(1/2)+5*x^3*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2))
```

Fricas [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \int (5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \int (5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \int (5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^{5/2}\,dx$$

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2), x)
```

3.78 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 429

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120}$$
$$-\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} + \frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}$$
$$+\frac{1471781\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{102400\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$-\frac{982275517\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{4147200\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$-\frac{145131624827(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{20736000\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

```
[Out] -427/1440*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/20*(7+
5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-145131624827/88957440
00*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/3
9*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(
-5+2*x)^(1/2)/(1+4*x)^(1/2)-1471781/51200*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*
x)^(1/2)/(-5+2*x)^(1/2)-267029/69120*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)*(7+5*x)^(1/2)-982275517/95385600*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*
(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(
1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1
```

```
/2)/((7+5*x)/(5-2*x))^(1/2)+1471781/102400*EllipticE(1/23*897^(1/2)*(1+4*x)
^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5
-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {167, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \frac{1471781\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{102400\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{145131624827(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{20736000\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}} - \frac{982275517\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{4147200\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2} - \frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}{1440} - \frac{267029\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}{69120} - \frac{1471781\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{51200\sqrt{2x-5}}$$

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

```
[Out] (-1471781*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(51200*Sqrt[-5 + 2*x])
 - (267029*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/69120
- (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/1440 + (
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/20 + (1471781*S
qrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(102400*Sqrt[(2 - 3*x)/(5 - 2*
x)]*Sqrt[7 + 5*x]) - (982275517*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[
Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(4147200*Sqrt[-5 + 2*x]*Sq
rt[(7 + 5*x)/(5 - 2*x)]) - (145131624827*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)
]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[
7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(20736000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[
1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
```

```
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} \\
&\quad+\frac{1}{40}\int \frac{(7+5x)^{3/2}\left(-3-1190x+854x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} \\
&\quad+\frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}-\frac{\int \frac{\sqrt{7+5x}\left(256662+723440x-1068116x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{5760} \\
&= -\frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120}-\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} \\
&\quad+\frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}+\frac{\int \frac{-382895716+213099880x+953714088x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{552960} \\
&= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}}-\frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120} \\
&\quad-\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} \\
&\quad+\frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}-\frac{\int \frac{322693781136-224719935216x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{132710400} \\
&\quad-\frac{631394049\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{102400} \\
&= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}}-\frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120} \\
&\quad-\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} \\
&\quad+\frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}-\frac{4681665317\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{8294400} \\
&\quad-\frac{10805030687\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{8294400} \\
&\quad+\frac{\left(57399459\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{102400\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120} \\
&\quad - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} \\
&\quad + \frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} \\
&\quad + \frac{1471781\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{102400\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad - \frac{\left(145131624827(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{4147200\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} \\
&\quad - \frac{\left(982275517\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{4147200\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1471781\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120} \\
&\quad - \frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440} \\
&\quad + \frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} \\
&\quad + \frac{1471781\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{102400\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad - \frac{982275517\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{4147200\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&\quad - \frac{145131624827(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{20736000\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.34 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.32

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}(-241157+139440x+86400x^2)}{69120}$$

$$+\frac{\frac{880794698355\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{2-3x}} - \frac{293598232785i\sqrt{253}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{1+4x}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{11}{39}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}} - \frac{35131412470\sqrt{429}\sqrt{\frac{-5\cdots}{-2\cdots}}}{\cdots}}{\cdots}$$

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-241157 + 139440
*x + 86400*x^2))/69120 + ((880794698355*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7
 + 5*x])/Sqrt[2 - 3*x] - ((293598232785*I)*Sqrt[253]*Sqrt[(-5 + 2*x)/(-2 +
3*x)]*Sqrt[1 + 4*x]*EllipticE[I*ArcSinh[(Sqrt[11/39]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) - (3513141247
0*Sqrt[429]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticF[ArcSin[(Sqr
t[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[-5 + 2*x]*Sqrt[(1 +
4*x)/(-2 + 3*x)]) - (506348591678*Sqrt[429]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqr
t[1 + 4*x]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3
*x]], -23/39])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) + ((57853855345*
I)*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*
ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x
)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (276827203510*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt[(-
5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sq
rt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))/20427264
000
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.10

method	result
elliptic	$\dfrac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{581x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{288}-\frac{241157\sqrt{-120x^4+182x^3+385x^2-197x-70}}{69120}-\frac{95723929\sqrt{-\ldots}}{\ldots}\right.}{\left(-\frac{95723929\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(\ldots\right.}{211\ldots}\right.}$
risch	$-\frac{(86400x^2+139440x-241157)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{69120\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\ldots\right.}{\ldots}$
default	$\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(972452761830\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+2612\ldots\right.}{\ldots}$

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(581/288*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/
2)-241157/69120*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-95723929/21142218
240*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^
(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+
1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))
+5327497/2114221824*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x
-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/
3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1
/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),
-69/55,1/39*I*897^(1/2)))+4415343/5120*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3
795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2
139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(
-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3
+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))
^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1
/2)+5/4*x^2*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2))
```

Fricas [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx=\int (5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int (5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int (5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{3/2}\,dx$$

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2), x)
```

3.79 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 391

$$
\begin{aligned}
&\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx \\
&= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad - \frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad + \frac{13027\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{3200\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad - \frac{1368371\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{43200\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&\quad - \frac{65750101(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{216000\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

```
[Out] -65750101/92664000*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^
(1/2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(
1/2)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-13027/4800*(2-3*x)^(1/2)*(1+4*x
)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-1/9*(2-3*x)^(3/2)*(-5+2*x)^(1/2)*(1+4*
x)^(1/2)*(7+5*x)^(1/2)+23/240*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7
+5*x)^(1/2)-1368371/993600*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-
3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3
*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x
)/(5-2*x))^(1/2)+13027/9600*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
```

```
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {167, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \frac{13027\sqrt{\frac{143}{3}}\sqrt{\frac{5x+7}{5-2x}}\sqrt{2-3x}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{3200\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{65750101\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}(2-3x)\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{216000\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}} - \frac{1368371\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{43200\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} - \frac{1}{9}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}(2-3x)^{3/2} + \frac{23}{240}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x} - \frac{13027\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x}}{4800\sqrt{2x-5}}$$

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

```
[Out] (-13027*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4800*Sqrt[-5 + 2*x]) +
(23*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/240 - ((2 - 3
*x)^(3/2)*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/9 + (13027*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(3200*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1368371*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(43200*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) - (65750101*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(216000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
```

```
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{1}{18}\int\frac{\sqrt{2-3x}(617+1042x-138x^2)}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}-\frac{\int\frac{170254+143540x-468972x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{2880} \\
&= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}}+\frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad +\frac{\int\frac{-154352184+50903304x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{691200}-\frac{1862861\int\frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{3200} \\
&= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}}+\frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{2120971\int\frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{86400}-\frac{15052081\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{86400} \\
&\quad +\frac{\left(169351\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{3200\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}} \\
&= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}}+\frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad +\frac{13027\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{3200\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad -\frac{\left(65750101(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx,x,\frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{43200\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} \\
&\quad -\frac{\left(1368371\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{43200\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}
\end{aligned}
$$

$$= -\frac{13027\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}$$
$$- \frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}$$
$$+ \frac{13027\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{3200\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$- \frac{1368371\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{43200\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$- \frac{65750101(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{216000\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.20 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.43

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx$$
$$= \frac{1}{720}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}(-91+240x)$$
$$+ \frac{\frac{7796073285\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{2-3x}} - \frac{2598691095i\sqrt{253}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{1+4x}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{11}{39}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}} - \frac{310954490\sqrt{429}\sqrt{\frac{-5+2}{-2+3}}}{}}{}$$

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-91 + 240*x))/72
0 + ((7796073285*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]
- ((2598691095*I)*Sqrt[253]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*Ellip
ticE[I*ArcSinh[(Sqrt[11/39]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -39/23])/(Sqrt[-
5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) - (310954490*Sqrt[429]*Sqrt[(-5 + 2*x)
/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqr
t[2 - 3*x]], -23/39])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) - (448178
3026*Sqrt[429]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticPi[-69/55,
 ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[-5 + 2*x
]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) + ((290533815*I)*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt
[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5
*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (
1958698170*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78
/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 +
2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))/1915056000
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{3}-\frac{91\sqrt{-120x^4+182x^3+385x^2-197x-70}}{720}-\frac{85127\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{}\right.}{}$
risch	$-\frac{(-91+240x)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{720\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\left(-\frac{85127\sqrt{1705}\,\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\,\left(x+\frac{1}{4}\right)^2\sqrt{1794}\,\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}}{220231440\sqrt{-30\left(x+\frac{7}{5}\right)}}\right.$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(2263376115\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2E\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-135\right.}{}$

[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(1/3*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-91/720*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-85127/220231440*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-7177/22023144*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+13027/160*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))

Fricas [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\,dx$$

[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2),x)

[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7), x)

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}\,dx$$

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2),x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2), x)
```

3.80 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 351

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx \\
&= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad + \frac{427\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{400\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad - \frac{20057\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1800\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&\quad + \frac{1008833(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{9000\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}$$

```
[Out] 1008833/3861000*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-427/600*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+1/10*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)*(7+5*x)^(1/2)-20057/41400*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)+427/1200*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x
)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/(
(2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {167, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx$$
$$= \frac{427\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{400\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}$$
$$+\frac{1008833(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{9000\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$
$$-\frac{20057\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$
$$+\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{427\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{600\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

```
[Out] (-427*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(600*Sqrt[-5 + 2*x]) + (Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + (427*Sqrt[143/
3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(400*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (20057*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]
/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1800*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (1008833*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(9000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad + \frac{1}{20}\int \frac{-3-1190x+854x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad - \frac{\int \frac{207388+130172x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{4800} - \frac{61061}{400}\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad + \frac{32543\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{3600} - \frac{220627\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{3600} \\
&\quad + \frac{\left(5551\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{400\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{427\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{400\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(1008833(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{1800\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} - \frac{\left(20057\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{1800\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{427\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{427\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{400\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{20057\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1800\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{1008833(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{9000\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.32 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.58

$$\int\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{\frac{85180095\sqrt{1+4x}\sqrt{7+5x}\sqrt{-75+30x}}{\sqrt{2-3x}} - \frac{85180095\sqrt{715}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}E\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}} + \frac{125222020\sqrt{715}\sqrt{-5+2x}\sqrt{\frac{1+4}{-2+}}}{\sqrt{}}}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + ((85180095*
Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x])/Sqrt[2 - 3*x] - (85180095*Sqr
```

```
t[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/
23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt
[1 + 4*x]) + (125222020*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]
*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqr
t[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - (146904226*Sqrt[715]*Sqrt[-5 + 2*x]
*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 +
 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) -
 ((5772195*I)*Sqrt[10230]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*Elliptic
Pi[-23/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(
Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (11544390*Sqrt[10230]*Sqrt[2 - 3
*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7
 + 5*x])/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]
))/(79794000*Sqrt[15])
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{\sqrt{-120x^4+182x^3+385x^2-197x-70}}{10}-\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\,\left(-\frac{2}{3}+x\right)^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,F\left(\cdots\right.}{1019590\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)\cdots}}\cdots\right.}{\cdots}$
risch	$-\frac{\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{10\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{\sqrt{1705}\,\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\,\left(x+\frac{1}{4}\right)^2\sqrt{1794}\,\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\,\sqrt{2139}\,\sqrt{\frac{-\frac{2}{3}+x}{x+\frac{1}{4}}}\,F\left(\cdots\right.}{1019590\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x\cdots\right.}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\left(19856430\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-18158994\sqrt{\cdots}\right.}{\cdots}$

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(1/10*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-1/
1019590*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+
x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)
*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1
/2))-119/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2
)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)
*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),
1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/
55,1/39*I*897^(1/2)))+427/20*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5
)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*
((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(
1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2)
,1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69
/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx=\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{\sqrt{5x+7}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/sqrt(5*x + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{\sqrt{5\,x+7}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2), x)
```

3.81 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 349

$$
\begin{aligned}
\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx &= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} \\
&+ \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{3\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{25\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&+ \frac{296\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{75\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&- \frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{375\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

```
[Out] -26474/160875*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2)
,-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*
429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-2/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+
4*x)^(1/2)/(7+5*x)^(1/2)+6/25*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5
+2*x)^(1/2)+296/1725*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^
(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(
1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2
*x))^(1/2)-3/25*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*
I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*
x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {166, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = -\frac{3\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{25\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{375\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}} + \frac{296\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{75\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{6\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{25\sqrt{2x-5}} - \frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) + (6*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(25*Sqrt[-5 + 2*x]) - (3*Sqrt[429]*S
qrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1
 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(25*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5
*x]) + (296*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(75*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
- (26474*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*E
llipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39
])/(375*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
```

```
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}}+\frac{1}{5}\int\frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx\\
&= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}}+\frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}}\\
&\quad -\frac{\int\frac{-12384-20496x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1200}+\frac{1287}{25}\int\frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx\\
&= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}}+\frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}}\\
&\quad -\frac{427}{75}\int\frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx\\
&\quad +\frac{1628}{75}\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx\\
&\quad -\frac{\left(117\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{25\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}
$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{3\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{25\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{\left(26474(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}dx,x,\frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{75\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} + \frac{\left(148\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}dx,x,\frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{75\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{3\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{25\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{296\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{75\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} - \frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{375\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.88 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.62

$$\int\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} - \frac{2\left(\frac{9\sqrt{1+4x}\sqrt{7+5x}\sqrt{-75+30x}}{2\sqrt{2-3x}} - \frac{9\sqrt{715}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}E\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{2\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}} + \frac{86\sqrt{\frac{55}{13}}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}}\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) - (2*((9*
Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x])/(2*Sqrt[2 - 3*x]) - (9*Sqrt[7
```

```
15]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/23]
*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(2*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[
1 + 4*x]) + (86*Sqrt[55/13]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Ellip
ticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 -
 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - (5549*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 +
 3*x)]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]]
, -23/39])/(Sqrt[715]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - ((39*I)*Sq
rt[165/62]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*Ar
cSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/
(5 - 2*x)]*Sqrt[1 + 4*x]) - (23*Sqrt[165/62]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/
(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*
x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)])))/(25*Sqrt[15])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2(-120x^3+350x^2-105x-50)}{25\sqrt{\left(x+\frac{7}{5}\right)(-120x^3+350x^2-105x-50)}}-\frac{14\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x}{-}}}{509795\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)(x-}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(146520\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-238266\sqrt{}\right.}{}$

[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(-2/25*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2)-14/509795*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+56/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-36/5*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)

```
/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/6
9*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-
3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795
*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(
x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 +
 70*x + 49), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{3/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2), x)
```

3.82 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 391

$$
\begin{aligned}
\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx &= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} \\
&+ \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2085525\sqrt{-5+2x}} \\
&+ \frac{17906\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{53475\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&- \frac{496\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1725\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&+ \frac{496(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{125\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

```
[Out] -2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+496/53625*(2
-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*I*8
97^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5+2*
x)^(1/2)/(1+4*x)^(1/2)+17906/417105*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)/(7+5*x)^(1/2)-35812/2085525*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(
-5+2*x)^(1/2)-496/39675*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x
))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x)
)^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(
5-2*x))^(1/2)+17906/2085525*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {166, 1618, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \frac{17906\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{53475\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}$$
$$+\frac{496(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{125\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$
$$-\frac{496\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1725\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{35812\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2085525\sqrt{2x-5}}$$
$$+\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{417105\sqrt{5x+7}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^(3/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(417105*Sqrt[7 + 5*x]) - (35
812*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2085525*Sqrt[-5 + 2*x]) + (
17906*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[
(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(53475*Sqrt[(2 - 3*x)
/(5 - 2*x)]*Sqrt[7 + 5*x]) - (496*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTa
n[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1725*Sqrt[-5 + 2*x]*Sqr
t[(7 + 5*x)/(5 - 2*x)]) + (496*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((
1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/
Sqrt[2 - 3*x]], -23/39])/(125*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
```

```
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{1}{15}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} + \frac{\int \frac{40642-726310x+429744x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{417105}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2085525\sqrt{-5+2x}} - \frac{\int \frac{94243968+96100992x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{100105200} - \frac{196966\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{53475}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2085525\sqrt{-5+2x}} + \frac{8}{25}\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx - \frac{2728\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1725} + \frac{\left(17906\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{53475\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}} - \frac{35812\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2085525\sqrt{-5+2x}} + \frac{17906\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{53475\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(496(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{25\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} - \frac{\left(248\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{1725\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}}$$
$$- \frac{35812\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2085525\sqrt{-5+2x}}$$
$$+ \frac{17906\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{53475\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$- \frac{496\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1725\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$+ \frac{496(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{125\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.09 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(34864+44765x)}{417105(7+5x)^{3/2}}$$
$$+ \frac{\frac{3571978410\sqrt{1+4x}\sqrt{7+5x}\sqrt{-75+30x}}{\sqrt{2-3x}} - \frac{3571978410\sqrt{715}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}E\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}} + \frac{5251113560\sqrt{715}\sqrt{-5+2x}\sqrt{\cdots}}{\cdots}}{\cdots}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(34864 + 44765*x))/(417105*(7
 + 5*x)^(3/2)) + ((3571978410*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x])
/Sqrt[2 - 3*x] - (3571978410*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 +
3*x)]*EllipticE[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])
/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + (5251113560*Sqrt[715]*Sqrt[-5
+ 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*
x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - (6
160344428*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticPi[-6
9/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5
- 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - ((344407635*I)*Sqrt[10230]*Sqrt[2 - 3*x]
*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[
7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]
) - (371344545*Sqrt[10230]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*Ellipti
cPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]], 39/62])/(Sqrt
[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))/(138676984875*Sqrt[15])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{375\left(x+\frac{7}{5}\right)^2}+\frac{-\frac{143248}{139035}x^3+\frac{250684}{83421}x^2-\frac{125342}{139035}x-\frac{35812}{83421}}{\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}+\frac{81284\sqrt{-\frac{3795\left(x\cdots\right.}{-\frac{2}{3}+\cdots}}\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(-2/375*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/
(x+7/5)^2+17906/2085525*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*
x^2-105*x-50))^(1/2)+81284/127582826085*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/
3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/
2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/
5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-22348/1962812709*(-3795*(x+7/5)/(-2/3+
x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)
/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*Elliptic
F(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/
69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+71624/139035*((
x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*80
6^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/3
41*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*E
llipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*Ellipt
icPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x
+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}}\,dx$$

[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3 + 525*x^2 + 735*x + 343), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{\frac{5}{2}}}\,dx$$

[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2),x)

[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**(5/2), x)

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}}\,dx$$

[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}}\,dx$$

[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx=\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^{5/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2), x)
```

3.83 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 330

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx &= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} \\ &+ \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} \\ &- \frac{2852696\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{11598438735\sqrt{-5+2x}} \\ &+ \frac{1426348\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{297395865\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\ &- \frac{48884\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{9593415\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}\end{aligned}$$

```
[Out] -2/25*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+17906/208552
5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+1426348/23196877
47*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)-2852696/1159843
8735*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-48884/2206485
45*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1
+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1
/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+1426348
/11598438735*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*8
97^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))
^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {166, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \frac{1426348\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{297395865\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{48884\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{9593415\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} - \frac{2852696\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{11598438735\sqrt{2x-5}} + \frac{1426348\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2319687747\sqrt{5x+7}} + \frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2085525(5x+7)^{3/2}} - \frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(25*(7 + 5*x)^(5/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2085525*(7 + 5*x)^(3/2)) +
(1426348*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5
*x]) - (2852696*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(11598438735*Sqr
t[-5 + 2*x]) + (1426348*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]
*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(29
7395865*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (48884*Sqrt[11/23]*Sqrt[
7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/
(9593415*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}}+\frac{1}{25}\int\frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx\\
&= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}}\\
&\quad+\frac{\int\frac{-254100+327910x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{2085525}\\
&= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}}\\
&\quad+\frac{1426348\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}+\frac{\int\frac{-762330250-648988340x+855808800x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{57992193675}
\end{aligned}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} - \frac{2852696\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{11598438735\sqrt{-5+2x}} - \frac{\int \frac{390064989600}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{13918126482000} - \frac{15689828\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{297395865}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} - \frac{2852696\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{11598438735\sqrt{-5+2x}} - \frac{268862\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{9593415} + \frac{\left(1426348\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{297395865\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} - \frac{2852696\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{11598438735\sqrt{-5+2x}} + \frac{1426348\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{297395865\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{\left(24442\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{9593415\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{1426348\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} - \frac{2852696\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{11598438735\sqrt{-5+2x}} + \frac{1426348\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{297395865\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{48884\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{9593415\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.89 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \frac{2\left(\frac{15\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(59328580+498566971x+89146750x^2)}{(7+5x)^{5/2}}+242\sqrt{15}\left(\frac{8841\sqrt{1+4}}{}\right.\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

```
[Out] (2*((15*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(59328580 + 498566971*x
+ 89146750*x^2))/(7 + 5*x)^(5/2) + 242*Sqrt[15]*((8841*Sqrt[1 + 4*x]*Sqrt[7
 + 5*x]*Sqrt[-75 + 30*x])/Sqrt[2 - 3*x] - (8841*Sqrt[715]*Sqrt[-5 + 2*x]*Sq
rt[(1 + 4*x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[
2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + (506884*Sqr
t[55/13]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticF[ArcSin[(Sqrt[1
1/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(3*Sqrt[(5 - 2*x)/(2 - 3*x)]*
Sqrt[1 + 4*x]) - (32705806*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Ellipt
icPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(3
*Sqrt[715]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + ((3203187*I)*Sqrt[3/3
410]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[
(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2
*x)]*Sqrt[1 + 4*x]) - (512187*Sqrt[30/341]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1
 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]
], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))))/173976581025
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.49

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{3125\left(x+\frac{7}{5}\right)^3}+\frac{17906\sqrt{-120x^4+182x^3+385x^2-197x-70}}{52138125\left(x+\frac{7}{5}\right)^2}+\frac{-\frac{11410784}{773229249}x^3+}{\sqrt{\left(x+\right.}}\right.}{}$
default	$-\frac{2\left(160464150\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}E\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^4-170482950\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\right.}{}$

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(-2/3125*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^3+17906/52138125*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^2+1426348/11598438735*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2)-5544220/64503557180829*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-1815352/24809060454165*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+5705392/773229249*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4
+ 3500*x^3 + 7350*x^2 + 6860*x + 2401), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{(5\,x+7)}^{7/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2), x)
```

3.84 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 370

$$
\begin{aligned}
\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx &= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} \\
&+ \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}} \\
&+ \frac{32843987836\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{451524900265803\sqrt{7+5x}} - \frac{65687975672\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2257624501329015\sqrt{-5+2x}} \\
&+ \frac{32843987836\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{57887807726385\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&- \frac{1212290288\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1867348636335\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}
\end{aligned}
$$

```
[Out] -2/35*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+2558/695175*
(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+23758016/579921936
75*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+32843987836/451
524900265803*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)-65687
975672/2257624501329015*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^
(1/2)-1212290288/42949018635705*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x
)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)
/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((
7+5*x)/(5-2*x))^(1/2)+32843987836/2257624501329015*EllipticE(1/23*897^(1/2)
*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7
+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {166, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \frac{32843987836\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{57887807726385\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{1212290288\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1867348636335\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} - \frac{65687975672\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2257624501329015\sqrt{2x-5}} + \frac{32843987836\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{451524900265803\sqrt{5x+7}} + \frac{23758016\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{57992193675(5x+7)^{3/2}} + \frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{695175(5x+7)^{5/2}} - \frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(35*(7 + 5*x)^(7/2)) + (255
8*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(695175*(7 + 5*x)^(5/2)) + (2
3758016*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(57992193675*(7 + 5*x)^
(3/2)) + (32843987836*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(45152490
0265803*Sqrt[7 + 5*x]) - (65687975672*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 +
5*x])/(2257624501329015*Sqrt[-5 + 2*x]) + (32843987836*Sqrt[11/39]*Sqrt[2 -
 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x]
)/Sqrt[-5 + 2*x]], -23/39])/(57887807726385*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1212290288*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1867348636335*Sqrt[-5 + 2*x]*Sqrt
[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{1}{35}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2}}\,dx \\
&= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} \\
&\quad + \frac{\int \frac{-548842+1382130x-429744x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx}{4866225} \\
&= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} \\
&\quad + \frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}} + \frac{\int \frac{-6576343950+7032607120x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{405945355725}
\end{aligned}$$

$$
\begin{aligned}
&=-\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}}+\frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}}\\
&\quad+\frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}}\\
&\quad+\frac{32843987836\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{451524900265803\sqrt{7+5x}}\\
&\quad+\frac{\int\frac{-20435008709500-14944014465380x+19706392701600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{11288122506645075}\\
&=-\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}}+\frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}}\\
&\quad+\frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}}\\
&\quad+\frac{32843987836\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{451524900265803\sqrt{7+5x}}\\
&\quad-\frac{65687975672\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2257624501329015\sqrt{-5+2x}}-\frac{\int\frac{9673349124067200}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{2709149401594818000}\\
&\quad-\frac{361283866196\int\frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{57887807726385}\\
&=-\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}}+\frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}}\\
&\quad+\frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}}\\
&\quad+\frac{32843987836\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{451524900265803\sqrt{7+5x}}\\
&\quad-\frac{65687975672\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2257624501329015\sqrt{-5+2x}}\\
&\quad-\frac{6667596584\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1867348636335}\\
&\quad+\frac{\left(32843987836\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{57887807726385\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}
$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}} + \frac{32843987836\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{451524900265803\sqrt{7+5x}} - \frac{65687975672\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2257624501329015\sqrt{-5+2x}} + \frac{32843987836\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{57887807726385\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{\left(606145144\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{1867348636335\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}} + \frac{32843987836\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{451524900265803\sqrt{7+5x}} - \frac{65687975672\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2257624501329015\sqrt{-5+2x}} + \frac{32843987836\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{57887807726385\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{1212290288\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)\middle|-\frac{39}{23}\right)}{1867348636335\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.71 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.54

$$\int\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \frac{2\left(\frac{90675\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(15395515423270+113490310442229x+54668919175710x^2+1}{(7+5x)^{7/2}}\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

```
[Out] (2*((90675*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(15395515423270 + 113
490310442229*x + 54668919175710*x^2 + 10263746198750*x^3))/(7 + 5*x)^(7/2)
+ 11*Sqrt[15]*((27073896336630*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x]
)/Sqrt[2 - 3*x] - (27073896336630*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(
-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23
/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + (39800941623080*Sqrt[715]
*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticF[ArcSin[(Sqrt[11/23]*Sq
rt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4
*x]) - (46692478872404*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*
EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/3
9])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + ((3535063529751*I)*Sqrt[102
30]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[(
Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*
x)]*Sqrt[1 + 4*x]) - (4405470235335*Sqrt[10230]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*
x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 +
 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))))/204710101658
008435125
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.41

method	result
elliptic	$\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{21875\left(x+\frac{7}{5}\right)^4}+\frac{2558\sqrt{-120x^4+182x^3+385x^2-197x-70}}{86896875\left(x+\frac{7}{5}\right)^3}+\frac{23758016\sqrt{-120x^4+}}{14498048}\right.$
default	Expression too large to display

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(-2/21875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2
```

```
)/(x+7/5)^4+2558/86896875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)
^3+23758016/1449804841875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)
^2+32843987836/2257624501329015*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*
x^3+350*x^2-105*x-50))^(1/2)-21231177880/1793650414527312003*(-3795*(x+7/5)
/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*(
(x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*Ellipt
icF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-5971634152/689865
544048966155*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(
-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x
-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/3
9*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,
1/39*I*897^(1/2)))+131375951344/150508300088601*((x+7/5)*(x-5/2)*(x+1/4)-1/
80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x)
)^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*
(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7
/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/
(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x
+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3125*x^5
 + 21875*x^4 + 61250*x^3 + 85750*x^2 + 60025*x + 16807), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^{9/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2), x)
```

3.85 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 429

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}}$$
$$+\frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648}$$
$$+\frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}+\frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}$$
$$-\frac{2466927\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8192\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$+\frac{861015607\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{331776\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$+\frac{331574321009(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{1658880\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

```
[Out] 1445/576*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/8*(7+5*
x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+331574321009/711659520*
(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*I
*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5+
2*x)^(1/2)/(1+4*x)^(1/2)+2466927/4096*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(
1/2)/(-5+2*x)^(1/2)+1561915/27648*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)+861015607/7630848*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4
*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*
x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/
```

```
((7+5*x)/(5-2*x))^(1/2)-2466927/8192*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)
/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))
^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {168, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx =$$

$$-\frac{2466927\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8192\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}$$
$$+\frac{331574321009(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{1658880\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$
$$+\frac{861015607\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{331776\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$
$$+\frac{1}{8}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$
$$+\frac{1445}{576}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}+\frac{1561915\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}{27648}+\frac{2466927\sqrt{2-3x}\sqrt{}}{4096\sqrt{2}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] (2466927*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4096*Sqrt[-5 + 2*x]) +
 (1561915*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/27648 +
 (1445*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/576 + (S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/8 - (2466927*Sqr
t[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23
]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8192*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (861015607*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(331776*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)]) + (331574321009*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqr
t[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5
*x])/Sqrt[2 - 3*x]], -23/39])/(1658880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*
x])
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
```

```
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} - \frac{1}{16}\int \frac{(7+5x)^{3/2}\left(-621-370x+2890x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} + \frac{\int \frac{\sqrt{7+5x}\left(1484298-798320x-6247660x^2\right)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{2304}$$

$$= \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} + \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} - \frac{\int \frac{-2228237276+4634706200x+7992843480x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{221184}$$

$$= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} + \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} + \frac{\int \frac{2469045068400-2567027001360x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{53084160} + \frac{1058311683\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{8192}$$

$$= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} + \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} + \frac{9471171677\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{663552} + \frac{10695945839\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{663552} - \frac{\left(96210153\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{8192\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$
\begin{aligned}
&= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} \\
&\quad + \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} \\
&\quad + \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} \\
&\quad - \frac{2466927\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8192\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad + \frac{\left(331574321009(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{331776\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} \\
&\quad + \frac{\left(861015607\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{331776\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2466927\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{27648} \\
&\quad + \frac{1445}{576}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} \\
&\quad + \frac{1}{8}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2} \\
&\quad - \frac{2466927\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8192\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad + \frac{861015607\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{331776\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&\quad + \frac{331574321009(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{1658880\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 39.98 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx =$$
$$-\frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-12388907394\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -1/41140224*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-12388907394*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 10666876180*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186
*(-5752341805 - 26349657233*x - 12645389558*x^2 + 3088122056*x^3 + 10048195
20*x^4 + 439372800*x^5 + 82944000*x^6) + 10695945839*Sqrt[682]*(2 - 3*x)^2*
Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Elliptic
Pi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt
[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{12265x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{576}+\frac{2216779\sqrt{-120x^4+182x^3+385x^2-197x-70}}{27648}+\frac{557059319\sqrt{}}{}\right.}{}$
risch	$-\frac{(86400x^2+588720x+2216779)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{27648\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{557059319\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x\right.}{84568}\right.}{}$
default	$-\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(852405450930\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+596\right.}{}$

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(12265/576*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(
1/2)+2216779/27648*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+557059319/8456
887296*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x
))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*
(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/
2))-579338275/4228443648*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2
)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)
*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x
))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(
1/2),-69/55,1/39*I*897^(1/2)))-37003905/2048*((x+7/5)*(x-5/2)*(x+1/4)-1/807
30*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(
1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+
7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)
/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2
/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/
4))^(1/2)+25/8*x^2*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sq
rt(2*x - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \text{Timed out}$$

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{(5\,x+7)}^{5/2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(5/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(5/2))/(2*x - 5)^(1/2), x)
```

3.86 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 391

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx &= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}}\\ &+\frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\\ &+\frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\\ &-\frac{66377\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1280\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}\\ &+\frac{2824441\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{17280\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}\\ &+\frac{963142751(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{86400\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}\end{aligned}$$

```
[Out] 1/6*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+963142751/3706
5600*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1
/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)
/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+66377/1920*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x
)^(1/2)/(-5+2*x)^(1/2)+977/288*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(
7+5*x)^(1/2)+2824441/397440*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2
-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-
3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*
x)/(5-2*x))^(1/2)-66377/3840*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x
```

```
)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/(
(2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {168, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = -\frac{66377\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{1280\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}$$
$$+\frac{963142751(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{86400\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$
$$+\frac{2824441\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{17280\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$
$$+\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$
$$+\frac{977}{288}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{66377\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{1920\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] (66377*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1920*Sqrt[-5 + 2*x]) + (
977*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/288 + (Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/6 - (66377*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1280*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) + (2824441*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(17280*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) + (963142751*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*
x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
 - 3*x]], -23/39])/(86400*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
```

```
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

integral

$$= \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}-\frac{1}{12}\int\frac{\sqrt{7+5x}(-465+20x+1954x^2)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{\int\frac{697418-1294820x-2389572x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1152}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} - \frac{\int\frac{-745656744+745658904x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{276480} + \frac{9491911\int\frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1280}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{31068851\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{34560} + \frac{31069121\int\frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{34560} - \frac{\left(862901\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{1280\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} - \frac{66377\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1280\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(963142751(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{17280\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} + \frac{\left(2824441\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{17280\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{66377\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}$$
$$+ \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}$$
$$- \frac{66377\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1280\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$+ \frac{2824441\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{17280\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$+ \frac{963142751(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{86400\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [A] (warning: unable to verify)

Time = 36.53 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx =$$
$$-\frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-37038366\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+31\ldots\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -1/2142720*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-37038366*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]
*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 31389484*Sqrt[682]*Sqrt[(-5 - 18*x
+ 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-1723
2355 - 79187903*x - 38640362*x^2 + 10641080*x^3 + 4555200*x^4 + 1152000*x^5
) + 31069121*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 1
1*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(
-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{5x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{6}+\frac{1313\sqrt{-120x^4+182x^3+385x^2-197x-70}}{288}+\frac{348709\sqrt{-\frac{3795\left(x+\right.}{-\frac{2}{3}+x}}}{}\right.}{}$
risch	$-\frac{(1313+240x)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{288\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{348709\sqrt{1705}\,\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\,\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\,\sqrt{}}{88092576\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\right.}}\right.}{}$
default	$-\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(2796196590\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+17336\right.}{}$

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(5/6*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+1
313/288*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+348709/88092576*(-3795*(x
+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1
/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*E
llipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-323705/44046
288*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^
(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+
1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(
1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*89
7^(1/2)))-66377/64*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x)
)^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(
-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*
I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*89
7^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I
*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx=\int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx=\int \frac{\sqrt{2-3x}\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**(3/2)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{(5\,x+7)}^{3/2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2), x)
```

3.87 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 351

$$
\begin{aligned}
\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx
&= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad - \frac{509\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{160\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&\quad + \frac{8959\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{720\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&\quad + \frac{2198489(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{3600\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

```
[Out] 2198489/1544400*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+509/240*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+1/4*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/
2)*(7+5*x)^(1/2)+8959/16560*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2
-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-
3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*
x)/(5-2*x))^(1/2)-509/480*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(
1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-
3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {167, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx$$
$$= -\frac{509\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{160\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}$$
$$+\frac{2198489(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{3600\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$
$$+\frac{8959\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{720\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$
$$+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{509\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{240\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (509*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(240*Sqrt[-5 + 2*x]) + (Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/4 - (509*Sqrt[143/3]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(160*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (8959*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(S
qrt[2]*Sqrt[2 - 3*x])], -39/23])/(720*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*
x)]) + (2198489*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3
*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]],
 -23/39])/(3600*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\begin{aligned}
&\text{integral} \\
&= \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{8}\int \frac{309-410x-1018x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad - \frac{\int \frac{-320516+283676x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1920} + \frac{72787}{160}\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad + \frac{70919\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1440} + \frac{98549\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1440} \\
&\quad - \frac{\left(6617\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{160\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{509\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{160\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(2198489(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{720\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} + \frac{\left(8959\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{720\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{509\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{160\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{8959\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{720\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{2198489(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{3600\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [A] (warning: unable to verify)

Time = 28.58 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \frac{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(66960(2-3x) - \frac{3\left(94674\sqrt{682}(2-3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+76\right.}{}\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) - (3*(94674*Sq
rt[682]*(2 - 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE
[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 76756*Sqrt[682]*
Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSi
```

```
n[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3
*x)]*(284022*(-35 - 151*x - 34*x^2 + 40*x^3) + 70919*Sqrt[682]*(2 - 3*x)^2*
Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Elliptic
Pi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/((2 -
 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(267840*Sqrt[2 - 3
*x])
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-120x^4+182x^3+385x^2-197x-70}}{4}+\frac{103\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}}{407836\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x\right.}}\right.}{}$
risch	$-\frac{\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{4\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{103\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\sqrt{2139}\sqrt{\frac{-\frac{2}{3}+x}{x+\frac{1}{4}}}F}{407836\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x\right.}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\sqrt{-5+2x}\left(8869410\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+3957280\right.}{}$

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(1/4*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+103
/407836*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+
x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)
*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1
/2))-205/611754*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2
)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)
*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),
1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/
55,1/39*I*897^(1/2)))-509/8*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)
/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*(
(x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1
/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),
1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/
55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*sqrt(5*x + 7)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{5\,x+7}}{\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2), x)
```

3.88 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 365

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{7\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{41\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right),\frac{39}{62}\right)}{20\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}} + \frac{943\sqrt{2-3x}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right),\frac{39}{62}\right)}{100\sqrt{682}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$

```
[Out] 41/1240*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(1/
2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+2*
x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/2)
/(1+4*x)^(1/2)+943/68200*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5
*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+
506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/2)/
((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+1/5*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5
*x)^(1/2)/(-5+2*x)^(1/2)+7/230*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
```

```
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)-1/10*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(
1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-
3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {179, 182, 435, 171, 550, 429, 553, 176}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = -\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{7\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{41\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{20\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}} + \frac{943\sqrt{2-3x}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{100\sqrt{682}\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}} + \frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(10*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) + (7*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(10*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
+ (41*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(20*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (943*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/S
qrt[-5 + 2*x]], 39/62])/(100*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 550

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[-f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{41}{20}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&\quad + \frac{77}{20}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&\quad + \frac{429}{10}\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} \\
&\quad - \frac{\left(1599\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int \frac{1}{(5-2x^2)\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\frac{22x^2}{23}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{10\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}} \\
&\quad + \frac{\left(7\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{10\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}} \\
&\quad - \frac{\left(39\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{10\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$+ \frac{7\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{10\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$- \frac{\left(41\sqrt{\frac{23}{31}}\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{22x^2}{23}}}{(5-2x^2)\sqrt{1+\frac{11x^2}{31}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{20\sqrt{2-3x}\sqrt{1+4x}}$$
$$- \frac{\left(451\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\frac{22x^2}{23}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{20\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$+ \frac{7\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{10\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$+ \frac{41\sqrt{\frac{11}{62}}\sqrt{2-3x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right)|\frac{39}{62}\right)}{20\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$
$$+ \frac{943\sqrt{2-3x}\Pi\left(\frac{78}{55};\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right)|\frac{39}{62}\right)}{100\sqrt{682}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$

Mathematica [A] (warning: unable to verify)

Time = 5.51 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx$$
$$= \frac{\sqrt{2-3x}\left(-3410\sqrt{682}\sqrt{\frac{5-2x}{7+5x}}\sqrt{\frac{1+4x}{7+5x}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\frac{155-62x}{77+55x}}\right)|\frac{23}{62}\right) + 1984\sqrt{682}\sqrt{\frac{5-2x}{7+5x}}\sqrt{\cdots}\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*(-3410*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7
 + 5*x)]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*
```

```
x)]], 23/62] + 1984*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7 +
 5*x)]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)
]], 23/62] + Sqrt[(-2 + 3*x)/(7 + 5*x)]*(17050*(10 + 21*x - 70*x^2 + 24*x^3
) - 1599*Sqrt[682]*Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^2*Sqrt[(-10 + 19*x -
 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x
)]], 23/62])))/(34100*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*((-2 + 3*x)/(7 + 5*x))^(
3/2)*(7 + 5*x)^(3/2))
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{4\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}F\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}+\frac{10\sqrt{-\cdots}}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}\sqrt{-5+2x}\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+22878\sqrt{-\cdots}\cdots\right)}{\cdots}$

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(4/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+10/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-6*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{2\,x-5}}\,dx$$

[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 - 11*x - 35), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\sqrt{5x+7}}\,dx$$

[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)

[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*sqrt(5*x + 7)), x)

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

3.89 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 279

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{195\sqrt{-5+2x}} + \frac{2\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{69\sqrt{\frac{2}{341}}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{-\frac{5-2x}{1+4x}}(1+4x)\,\text{EllipticPi}\left(\frac{78}{55},\arcsin\left(\frac{\sqrt{\frac{22}{39}}\sqrt{7+5x}}{\sqrt{1+4x}}\right),\frac{39}{62}\right)}{25\sqrt{2-3x}\sqrt{-5+2x}}$$

```
[Out] -69/8525*(1+4*x)*EllipticPi(1/39*858^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2),78/5
5,1/62*2418^(1/2))*682^(1/2)*((-2+3*x)/(1+4*x))^(1/2)*((-5+2*x)/(1+4*x))^(1
/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)+2/39*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)/(7+5*x)^(1/2)-4/195*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x
)^(1/2)+2/195*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*
897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x)
)^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {170, 1600, 1609, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \frac{2\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{69\sqrt{\frac{2}{341}}\sqrt{-\frac{2-3x}{4x+1}}\sqrt{-\frac{5-2x}{4x+1}}(4x+1)\,\text{EllipticPi}\left(\frac{78}{55},\arcsin\left(\frac{\sqrt{\frac{22}{39}}\sqrt{5x+7}}{\sqrt{4x+1}}\right),\frac{39}{62}\right)}{25\sqrt{2-3x}\sqrt{2x-5}} - \frac{4\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{195\sqrt{2x-5}} + \frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*Sqrt[7 + 5*x]) - (4*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(195*Sqrt[-5 + 2*x]) + (2*Sqrt[11/39
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) - (69*Sqrt[2/341]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[-((5 - 2*x)/(1 +
 4*x))]*(1 + 4*x)*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt
[1 + 4*x]], 39/62])/(25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1609

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*
B*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*Sqrt[a + b*x])), x] + (
-Dist[B*((b*g - a*h)/(2*f*h)), Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[g + h*x]), x], x] + Dist[B*(b*e - a*f)*((b*g - a*h)/(2*d*f*h)), Int
[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{-25+130x-48x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{(5-24x)\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx
\end{aligned}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{3}{5}\int \frac{\sqrt{1+4x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}\,dx - \frac{22}{5}\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{\left(46\sqrt{\frac{3}{403}}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{\frac{-5+2x}{1+4x}}(1+4x)\right)\text{Subst}\left(\int \frac{1}{(5-4x^2)\sqrt{1-\frac{22x^2}{39}}\sqrt{1-\frac{11x^2}{31}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{1+4x}}\right)}{5\sqrt{2-3x}\sqrt{-5+2x}} + \frac{\left(2\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{5\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{195\sqrt{-5+2x}} + \frac{2\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{69\sqrt{\frac{2}{341}}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{-\frac{5-2x}{1+4x}}(1+4x)\Pi\left(\frac{78}{55};\sin^{-1}\left(\frac{\sqrt{\frac{22}{39}}\sqrt{7+5x}}{\sqrt{1+4x}}\right)|\frac{39}{62}\right)}{25\sqrt{2-3x}\sqrt{-5+2x}}$$

Mathematica [A] (warning: unable to verify)

Time = 20.01 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-62\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\right.\right.\right.}}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/
(-2 + 3*x)]], 39/62] + 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(
-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x
)]], 39/62] - 2*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-961*(-5 - 18*x + 8*x^2) + 39*S
qrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/
(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*
x)]], 39/62])))/(6045*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)
]*(-5 - 18*x + 8*x^2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(216) = 432$.

Time = 1.60 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{-\frac{16}{13}x^3+\frac{140}{39}x^2-\frac{14}{13}x-\frac{20}{39}}{\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}+\frac{50\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}F}{11929203\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)(}}\right.}{}$
default	$\frac{2\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}\sqrt{-5+2x}\left(495\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-1116\sqrt{-\frac{253(7+}{-2+3}}\right.}{}$

[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x,method=_RETU RNVERBOSE)

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(2/195*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2)+50/11929203*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-20/917631*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+8/13*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 +
 15*x^2 - 252*x - 245), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

3.90 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 290

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx &= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} \\ &- \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3253419\sqrt{-5+2x}} \\ &- \frac{1870\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{83421\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\ &+ \frac{44\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{2691\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}\end{aligned}$$

```
[Out] 2/117*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-9350/3253419
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+3740/3253419*(2-3
*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+44/61893*(1/(4+2*(1+4*
x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1
/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(
7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-1870/3253419*EllipticE(
1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-
3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {170, 1613, 1616, 12, 176, 429, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = -\frac{1870\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{83421\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{44\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{2691\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{3740\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{3253419\sqrt{2x-5}} - \frac{9350\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{3253419\sqrt{5x+7}} + \frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(117*(7 + 5*x)^(3/2)) - (935
0*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(3253419*Sqrt[7 + 5*x]) + (37
40*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3253419*Sqrt[-5 + 2*x]) - (1
870*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(S
qrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(83421*Sqrt[(2 - 3*x)/(
5 - 2*x)]*Sqrt[7 + 5*x]) + (44*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[S
qrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(2691*Sqrt[-5 + 2*x]*Sqrt[(
7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{1}{117}\int \frac{-33+110x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx \\
&= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} \\
&\quad - \frac{\int \frac{-66308-170170x+224400x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{3253419} \\
&= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} \\
&\quad + \frac{3740\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3253419\sqrt{-5+2x}} + \frac{\int \frac{70218720}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{780820560} \\
&\quad + \frac{20570\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{83421} \\
&= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} \\
&\quad + \frac{3740\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3253419\sqrt{-5+2x}} + \frac{242\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{2691} \\
&\quad - \frac{\left(1870\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{83421\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3253419\sqrt{-5+2x}} - \frac{1870\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{83421\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(22\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{2691\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{3253419\sqrt{-5+2x}} - \frac{1870\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{83421\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{44\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{2691\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [A] (verified)

Time = 26.61 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = -\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(31\sqrt{\frac{7+5x}{-2+3x}}(-23755-122348x-94580x^2+58928x^3)-935\sqrt{682}(-2+3x)(7+5x\right.}{3253419\sqrt{2-3}}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-23755 - 1
22348*x - 94580*x^2 + 58928*x^3) - 935*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqr
t[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
2*x)/(-2 + 3*x)]], 39/62] + 506*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[(-5 -
 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-
2 + 3*x)]], 39/62]))/(3253419*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7 + 5*x)/
(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{2925\left(x+\frac{7}{5}\right)^2}-\frac{1870\left(-120x^3+350x^2-105x-50\right)}{3253419\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}+\frac{12056\sqrt{-\frac{3795(\ldots}{-\frac{2}{3}\ldots}}\ldots}{\ldots}\right.}{\ldots}$
default	$-\frac{2\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^3-42075\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\ldots\right.}{\ldots}$

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(2/2925*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/
(x+7/5)^2-1870/3253419*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x
^2-105*x-50))^(1/2)+12056/90467822133*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+
x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)
/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)
/(-2/3+x))^(1/2),1/39*I*897^(1/2))+2380/6959063241*(-3795*(x+7/5)/(-2/3+x))
^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-
```

```
2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1
/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*
(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-37400/1084473*((x+
7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^
(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341
*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*Ell
ipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*Elliptic
Pi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7
/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4
+ 425*x^3 - 1155*x^2 - 2989*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx=\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,(5\,x+7)^{5/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

3.91 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 330

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx &= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} \\ &- \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} \\ &- \frac{20464840\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{90467822133\sqrt{7+5x}} + \frac{8185936\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{90467822133\sqrt{-5+2x}} \\ &- \frac{4092968\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{2319687747\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\ &+ \frac{111628\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{74828637\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}\end{aligned}$$

```
[Out] 2/195*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3646/1626709
5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-20464840/9046782
2133*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+8185936/90467
822133*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+111628/1721
058651*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*Elliptic
F((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*89
7^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-409
2968/90467822133*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39
*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2
*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {170, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = -\frac{4092968\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{2319687747\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{111628\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{74828637\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{8185936\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{90467822133\sqrt{2x-5}} - \frac{20464840\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{90467822133\sqrt{5x+7}} - \frac{3646\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{16267095(5x+7)^{3/2}} + \frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(195*(7 + 5*x)^(5/2)) - (364
6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(16267095*(7 + 5*x)^(3/2)) -
(20464840*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(90467822133*Sqrt[7 +
 5*x]) + (8185936*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(90467822133*S
qrt[-5 + 2*x]) - (4092968*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x
)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(
2319687747*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (111628*Sqrt[11/23]*S
qrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/2
3])/(74828637*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{1}{195}\int \frac{-41+90x+48x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx \\
&= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} \\
&\quad - \frac{\int \frac{-489390+1112210x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{16267095} \\
&= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} \\
&\quad - \frac{20464840\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{90467822133\sqrt{7+5x}} - \frac{\int \frac{-1235106290-1862300440x+2455780800x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{452339110665}
\end{aligned}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{90467822133\sqrt{7+5x}} + \frac{8185936\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{90467822133\sqrt{-5+2x}} + \frac{\int \frac{890724463200}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{108561386559600} + \frac{45022648\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{2319687747}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{90467822133\sqrt{7+5x}} + \frac{8185936\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{90467822133\sqrt{-5+2x}} + \frac{613954\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{74828637} - \frac{\left(4092968\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{2319687747\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{90467822133\sqrt{7+5x}} + \frac{8185936\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{90467822133\sqrt{-5+2x}} - \frac{4092968\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{2319687747\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(55814\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{74828637\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{90467822133\sqrt{7+5x}} + \frac{8185936\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{90467822133\sqrt{-5+2x}} - \frac{4092968\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{2319687747\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{111628\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{74828637\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [A] (verified)

Time = 30.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx =$$
$$-\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(31\sqrt{\frac{7+5x}{-2+3x}}(-374624540-2271416114x-2953846743x^2+643813106x^3+37005125\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-374624540
 - 2271416114*x - 2953846743*x^2 + 643813106*x^3 + 370051256*x^4) - 2046484
*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ell
ipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 958111*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
cF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(90467822133*S
qrt[2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2)
)
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.49

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{24375\left(x+\frac{7}{5}\right)^3}-\frac{3646\sqrt{-120x^4+182x^3+385x^2-197x-70}}{406677375\left(x+\frac{7}{5}\right)^2}-\frac{4092968\left(-12\right.}{90467822133\sqrt{\left(x+\frac{7}{5}\right.}}\right.}{}$
default	$\frac{2\left(460458900\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,E\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^4-389302650\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+}{-2+}}\right.}{}$

[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(2/24375*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^3-3646/406677375*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^2-4092968/90467822133*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2)+44912956/2515638730052331*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+5209232/193510671542487*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*

```
EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-81
859360/30155940711*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x)
)^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(
-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*
I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*89
7^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I
*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1250*x^5
 + 3875*x^4 - 2800*x^3 - 23030*x^2 - 29498*x - 12005), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,(5\,x+7)^{7/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)), x)
```

3.92 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 370

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} - \frac{40944441340\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1956607901151813\sqrt{7+5x}} + \frac{16377776536\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1956607901151813\sqrt{-5+2x}} - \frac{8188888268\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{50169433362867\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{258506776\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1618368818157\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

```
[Out] 2/273*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+98/1807455*(
2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3217468/50259901185
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-40944441340/19566
07901151813*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+163777
76536/1956607901151813*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(
1/2)+258506776/37222482817611*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)-8188888268/1956607901151813*EllipticE(1/23*897^(1/2)*(1
+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*
x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {170, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = -\frac{8188888268\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{50169433362867\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{258506776\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1618368818157\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{16377776536\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{1956607901151813\sqrt{2x-5}} - \frac{40944441340\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1956607901151813\sqrt{5x+7}} - \frac{3217468\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{50259901185(5x+7)^{3/2}} + \frac{98\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1807455(5x+7)^{5/2}} + \frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(273*(7 + 5*x)^(7/2)) + (98*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1807455*(7 + 5*x)^(5/2)) - (32
17468*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(50259901185*(7 + 5*x)^(3
/2)) - (40944441340*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1956607901
151813*Sqrt[7 + 5*x]) + (16377776536*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5
*x])/(1956607901151813*Sqrt[-5 + 2*x]) - (8188888268*Sqrt[11/39]*Sqrt[2 - 3
*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/
Sqrt[-5 + 2*x]], -23/39])/(50169433362867*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (258506776*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1618368818157*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
```

```
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
```

```
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} - \frac{1}{273}\int \frac{-49+70x+96x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{7/2}}\,dx$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{\int \frac{-958104+2280510x+49392x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx}{37956555}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} - \frac{\int \frac{-11461434930+18134687340x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{3166373774655}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} - \frac{40944441340\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1956607901151813\sqrt{7+5x}} - \frac{\int \frac{-32763839696280-33533497457460x+44219996647200x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{88047355551831585}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} - \frac{40944441340\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1956607901151813\sqrt{7+5x}} + \frac{16377776536\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1956607901151813\sqrt{-5+2x}} + \frac{\int \frac{18564560715729600}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{21131365332439580400} + \frac{90077770948\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{50169433362867}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} - \frac{40944441340\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1956607901151813\sqrt{7+5x}} + \frac{16377776536\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1956607901151813\sqrt{-5+2x}} + \frac{1421787268\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1618368818157} - \frac{\left(8188888268\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{50169433362867\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} - \frac{40944441340\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1956607901151813\sqrt{7+5x}} + \frac{16377776536\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1956607901151813\sqrt{-5+2x}} - \frac{8188888268\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{50169433362867\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(129253388\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{1618368818157\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} - \frac{40944441340\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1956607901151813\sqrt{7+5x}} + \frac{16377776536\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1956607901151813\sqrt{-5+2x}} - \frac{8188888268\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{50169433362867\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{258506776\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1618368818157\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [A] (verified)

Time = 27.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \frac{2\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(\frac{(-2+3x)(2552362046246+19165803061167x+1231360817358}{(7+5x)^4}\right.}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(((-2 + 3*x)*(2552362046246 +
 19165803061167*x + 12313608173580*x^2 + 2559027583750*x^3))/(7 + 5*x)^4 -
(22*(558333291*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 186111097*S
```

```
qrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[
Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 71545594*Sqrt[682]*(-2 +
 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
 + 8*x^2))))/(1956607901151813*Sqrt[2 - 3*x])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.41

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{170625\left(x+\frac{7}{5}\right)^4}+\frac{98\sqrt{-120x^4+182x^3+385x^2-197x-70}}{225931875\left(x+\frac{7}{5}\right)^3}-\frac{3217468\sqrt{-120x^4+182x}}{1256497529625}\right.}{}$
default	Expression too large to display

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(2/170625*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2
)/(x+7/5)^4+98/225931875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^
3-3217468/1256497529625*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^2
```

```
-8188888268/1956607901151813*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3
+350*x^2-105*x-50))^(1/2)+18911307184/7772485129618352013*(-3795*(x+7/5)/(-
2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+
1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF
(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+1488888776/597883471
509104001*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/
3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/
2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I
*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/3
9*I*897^(1/2)))-163777765360/652202633717271*((x+7/5)*(x-5/2)*(x+1/4)-1/807
30*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(
1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+
7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)
/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2
/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/
4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(6250*x^6
 + 28125*x^5 + 13125*x^4 - 134750*x^3 - 308700*x^2 - 266511*x - 84035), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{9/2}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)), x)
```

3.93 $\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 391

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} \\ &+ \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} \\ &+ \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} \\ &- \frac{102487\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1024\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\ &+ \frac{5241511\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{13824\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\ &+ \frac{295576909(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{13824\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}\end{aligned}$$

```
[Out] 5/24*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+295576909/593
0496*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1
/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)
/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+102487/1536*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*
x)^(1/2)/(-5+2*x)^(1/2)+6955/1152*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)+5241511/317952*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)-102487/3072*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5
```

```
+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/
2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {180, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{102487\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{1024\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{295576909(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{13824\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}} + \frac{5241511\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{13824\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2} + \frac{6955\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}{1152} + \frac{102487\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{1536\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (102487*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1536*Sqrt[-5 + 2*x]) +
(6955*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/1152 + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/24 - (102487*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1024*Sqrt[(2 - 3*x)/(5 - 2*x)
]*Sqrt[7 + 5*x]) + (5241511*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(13824*Sqrt[-5 + 2*x]*Sqrt[(7
+ 5*x)/(5 - 2*x)]) + (295576909*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-(
(1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])
/Sqrt[2 - 3*x]], -23/39])/(13824*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
```

```
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\text{integral} = \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} - \frac{1}{48}\int \frac{\sqrt{7+5x}(-6189+3136x+13910x^2)}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{\int \frac{6899278-9847372x-18447660x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{4608}$$

$$= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} - \frac{\int \frac{-6120160440+5720843400x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1105920} + \frac{14655641\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1024}$$

$$= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{47673695\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{27648} + \frac{57656621\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{27648} - \frac{\left(1332331\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{1024\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} - \frac{102487\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1024\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(1477884545(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{13824\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} + \frac{\left(5241511\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{13824\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$
\begin{aligned}
&= \frac{102487\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} \\
&+ \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} \\
&- \frac{102487\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1024\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&+ \frac{5241511\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{13824\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&+ \frac{295576909(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{13824\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 29.93 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.87

$$
\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx =
$$

$$
-\frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-57187746\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+46
$$

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/1714176*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-57187746*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]
*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 46704724*Sqrt[682]*Sqrt[(-5 - 18*x
+ 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-2744
7805 - 124999073*x - 56065622*x^2 + 20626760*x^3 + 6542400*x^4 + 1152000*x^
5) + 47673695*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 -
11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5
+ 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/
(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{25x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{24}+\frac{8635\sqrt{-120x^4+182x^3+385x^2-197x-70}}{1152}+\frac{3449639\sqrt{-\frac{3795\left(x\right.}{-\frac{2}{3}+}}}{}\right.}{}$
risch	$-\frac{5(1727+240x)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{1152\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{3449639\sqrt{1705}\,\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\,\left(x+\frac{1}{4}\right)^2\sqrt{1794}\,\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}}{352370304\sqrt{-30\left(x+\frac{7}{5}\right)(}}\right.}{}$
default	$-\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(1037819178\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+53203\right.}{}$

[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(25/24*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)
+8635/1152*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+3449639/352370304*(-37
95*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*21
39^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1
/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-2461843
/176185152*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2
/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5
/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*
I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/
39*I*897^(1/2)))-512435/256*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)
/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*(
(x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1
/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),
1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/
55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))

Fricas [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")

[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2)/(8*x^2 - 18*x - 5), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \text{Timed out}$$

[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,(5\,x+7)^{5/2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.94 $\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 351

$$
\begin{aligned}
\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} \\
&+ \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&- \frac{785\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&+ \frac{17515\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&+ \frac{3730013(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{2880\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

```
[Out] 3730013/1235520*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+785/192*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+5/16*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)*(7+5*x)^(1/2)+17515/13248*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)-785/384*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {180, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{785\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{3730013(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{2880\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}} + \frac{17515\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7} + \frac{785\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{192\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (785*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/16 - (785*Sqrt[143
/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*S
qrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt
[7 + 5*x]) + (17515*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (3730013*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(2880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{1}{32}\int\frac{-4121+4074x+7850x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}+\frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad +\frac{\int\frac{2888740-2406460x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{7680}+\frac{112255}{128}\int\frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}+\frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad +\frac{120323\int\frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1152}+\frac{192665\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1152} \\
&\quad -\frac{\left(10205\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{128\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&- \frac{785\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&+ \frac{\left(3730013(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{576\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} \\
&+ \frac{\left(17515\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{576\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{785\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&- \frac{785\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&+ \frac{17515\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{576\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&+ \frac{3730013(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{2880\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 23.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.99

$$
\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(200880 + \frac{(2-3x)\left(-\frac{1314090\sqrt{682}(7+5x)\sqrt{\frac{-5-18x+\cdots}{(2-3x)^{\cdots}}}}{(2\cdots}\right.}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(200880 + ((2 - 3
*x)*((-1314090*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*El
lipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^
```

```
2 + (998820*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellip
ticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^2 +
 Sqrt[(7 + 5*x)/(-2 + 3*x)]*((3942270*(-35 - 151*x - 34*x^2 + 40*x^3))/(-2
+ 3*x)^3 + (1082907*Sqrt[682]*((1 + 4*x)/(-2 + 3*x))^(3/2)*Sqrt[(-35 - 11*x
 + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])/(1 + 4*x))))/(((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18
*x - 8*x^2))))/642816
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{5\sqrt{-120x^4+182x^3+385x^2-197x-70}}{16}+\frac{317\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}}{376464\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)(}}\right.}{}$
risch	$-\frac{5\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{16\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{317\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\sqrt{2139}\sqrt{\frac{-\frac{2}{3}+x}{x+\frac{1}{4}}}}{376464\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)}}\right.}{}$
default	$\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(45463275\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,x^2E\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-17339850\right.}{}$

[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(5/16*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+31
7/376464*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3
+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2
)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(
1/2))-679/815672*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/
2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x
)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2)
,1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69
/55,1/39*I*897^(1/2)))-3925/32*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7
/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2
)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))
^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/
2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-
69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))

Fricas [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")

[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2
- 18*x - 5), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}(5x+7)^{\frac{3}{2}}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)

[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)**(3/2)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,{\left(5\,x+7\right)}^{3/2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.95 $\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 365

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{39\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{179\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right),\frac{39}{62}\right)}{16\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}} + \frac{4117\sqrt{2-3x}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right),\frac{39}{62}\right)}{80\sqrt{682}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$

```
[Out] 179/992*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(1/
2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+2*
x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/2)
/(1+4*x)^(1/2)+4117/54560*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+
5*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529
+506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/2)
/((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+1/4*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+
5*x)^(1/2)/(-5+2*x)^(1/2)-39/184*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*
```

```
x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x
)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(
(7+5*x)/(5-2*x))^(1/2)-1/8*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^
(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2
-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {179, 182, 435, 171, 550, 429, 553, 176}

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} - \frac{39\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{179\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{16\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}} + \frac{4117\sqrt{2-3x}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{80\sqrt{682}\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}} + \frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
5*x]) - (39*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) +
 (179*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(16*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (4117*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/
Sqrt[-5 + 2*x]], 39/62])/(80*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 550

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[-f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{179}{16}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&\quad - \frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&\quad + \frac{429}{8}\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} \\
&\quad - \frac{\left(6981\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int \frac{1}{(5-2x^2)\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\frac{22x^2}{23}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{8\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}} \\
&\quad - \frac{\left(39\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}} \\
&\quad - \frac{\left(39\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{8\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$- \frac{39\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$- \frac{\left(179\sqrt{\frac{23}{31}}\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{22x^2}{23}}}{(5-2x^2)\sqrt{1+\frac{11x^2}{31}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{16\sqrt{2-3x}\sqrt{1+4x}}$$
$$- \frac{\left(1969\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\frac{22x^2}{23}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{16\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$- \frac{39\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$+ \frac{179\sqrt{\frac{11}{62}}\sqrt{2-3x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right)|\frac{39}{62}\right)}{16\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$
$$+ \frac{4117\sqrt{2-3x}\Pi\left(\frac{78}{55};\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right)|\frac{39}{62}\right)}{80\sqrt{682}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$

Mathematica [A] (warning: unable to verify)

Time = 6.99 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.95

$$\int\frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx =$$
$$-\frac{6820\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}(-5-18x+8x^2)\,E\left(\arcsin\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right) - 1265\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}(-5}{}$$

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/27280*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 + 4*x
)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 + 4*x
```

```
)]], 39/62] - 1265*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 +
 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 +
 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83*x - 53
*x^2 + 30*x^3) + 4117*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*Sqrt
[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22/39]*Sq
rt[(7 + 5*x)/(1 + 4*x)]], 39/62]))/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x]*Sqrt[(-5
+ 2*x)/(1 + 4*x)]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{28\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}F\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}-\frac{2\sqrt{-\frac{37}{}}}{}\right.}{}$
default	$-\frac{\sqrt{7+5x}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^{2}F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+99882\sqrt{-}\right.}{}$

```
[In] int((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(28/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-2/27807*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-15/2*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 - 18*x - 5), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{5x+7}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

[In] integrate((7+5*x)**(1/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)

[Out] Integral(sqrt(2 - 3*x)*sqrt(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{5\,x+7}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.96 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 101

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

```
[Out] 62/2145*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/5
5,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1
/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {171, 551}

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (62*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*Ellipt
icPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(5
*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\text{integral} = \frac{\left(62(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}}$$

$$= \frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [A] (warning: unable to verify)

Time = 5.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{\sqrt{\frac{1+4x}{7+5x}}(7+5x)^{3/2}\left(-62\sqrt{\frac{5-2x}{7+5x}}\sqrt{\frac{-2+3x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{155-62x}{77+55x}}\right),\frac{23}{62}\right)+117\sqrt{\frac{-10+19x-6x^2}{(7+5x)^2}}\,\text{EllipticPi}\right.}{5\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$$

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]

[Out] (Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^(3/2)*(-62*Sqrt[(5 - 2*x)/(7 + 5*x)]*S
qrt[(-2 + 3*x)/(7 + 5*x)]*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)]],
 23/62] + 117*Sqrt[(-10 + 19*x - 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, Arc
Sin[Sqrt[(155 - 62*x)/(77 + 55*x)]], 23/62]))/(5*Sqrt[682]*Sqrt[2 - 3*x]*Sq
rt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

method	result
default	$-\frac{62\Pi\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},-\frac{69}{55},\frac{i\sqrt{897}}{39}\right)\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{3}\,\sqrt{13}\,(-2+3x)\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\sqrt{7+5x}\,\sqrt{2-3x}}{49335(40x^3-34x^2-151x-35)}$
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{4\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}F\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}-\frac{2\sqrt{-\frac{3795}{-}}}{}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

[In] int((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -62/49335*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))*((1+4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*13^(1/2)*(-2+3*x)*(-253*(7+5*x)/(-2+3*x))^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)*(2-3*x)^(1/2)/(40*x^3-34*x^2-151*x-35)

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 - 34*x^2 - 151*x - 35), x)

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

3.97 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 60

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{2\sqrt{\frac{11}{39}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{22}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{62}{39}\right)}{23\sqrt{-5+2x}}$$

```
[Out] 2/897*EllipticE(1/22*858^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/39*2418^(1/2))
*429^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(60) = 120.

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {182, 433, 429, 506, 422}

$$\begin{aligned}\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = & -\frac{\sqrt{\frac{22}{31}}\sqrt{4x+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{39\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}} \\ & +\frac{2\sqrt{682}\sqrt{4x+1}E\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}}-\frac{62\sqrt{2x-5}\sqrt{4x+1}}{897\sqrt{2-3x}\sqrt{5x+7}}\end{aligned}$$

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-62*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (2*S
qrt[682]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7
 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) - (Sqrt[
22/31]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 +
 5*x]], 39/62])/(39*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{39\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= \frac{\left(\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{23x^2}{22}}\sqrt{1+\frac{31x^2}{11}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{39\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}} + \frac{\left(31\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1+\frac{23x^2}{22}}\sqrt{1+\frac{31x^2}{11}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{429\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= -\frac{62\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} - \frac{\sqrt{\frac{22}{31}}\sqrt{1+4x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)\middle|\frac{39}{62}\right)}{39\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} - \frac{\left(62\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{23x^2}{22}}}{\left(1+\frac{31x^2}{11}\right)^{3/2}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{897\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= -\frac{62\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{2\sqrt{682}\sqrt{1+4x}E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)\middle|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} - \frac{\sqrt{\frac{22}{31}}\sqrt{1+4x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)\middle|\frac{39}{62}\right)}{39\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. $2(60) = 120$.

Time = 28.52 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.95

$$\int\frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-1922\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)+62\sqrt{682}\sqrt{\frac{-5\cdots}{(\cdots}}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-1922*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
+ 8*x^2) + 62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x +
 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]
- 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*
EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(27807*
Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 435, normalized size of antiderivative = 7.25

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\left(-120x^3+350x^2-105x-50\right)}{897\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}+\frac{34\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\cdots}}{24942879\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(\cdots\right.}}\right.}{\cdots}$
default	$-\frac{2\sqrt{2-3x}\sqrt{7+5x}\sqrt{-5+2x}\sqrt{1+4x}\left(9\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-9\sqrt{-\frac{253(7+5\cdots}{-2+3x}}\cdots\right.}{\cdots}$

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(-2/897*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120
*x^3+350*x^2-105*x-50))^(1/2)+34/24942879*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-
2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(
1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+
7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+28/21105513*(-3795*(x+7/5)/(-2/3+x))
^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-
2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1
/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*
(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-40/299*((x+7/5)*(x
-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*(
(x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*Ellipt
icF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(
1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69
*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2
/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4
+ 110*x^3 - 993*x^2 - 1232*x - 245), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2}}\,dx$$

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

3.98 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 290

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39332\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{74828637\sqrt{-5+2x}} - \frac{19666\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1918683\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{716\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{61893\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

```
[Out] -10/2691*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-98330/748
28637*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+39332/748286
37*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+716/1423539*(1/
(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^
(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*2
53^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-19666/7482863
7*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*4
29^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5
*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {183, 1613, 1616, 12, 176, 429, 182, 435}

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{19666\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{1918683\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{716\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{61893\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{39332\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{74828637\sqrt{2x-5}} - \frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{74828637\sqrt{5x+7}} - \frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-10*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2691*(7 + 5*x)^(3/2)) - (
98330*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(74828637*Sqrt[7 + 5*x])
+ (39332*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(74828637*Sqrt[-5 + 2*x
]) - (19666*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[A
rcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1918683*Sqrt[(
2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (716*Sqrt[11/23]*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(61893*Sqrt[-5 +
 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 183

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h)))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
```

```
&& LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{\int \frac{-771+854x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{2691} \\
&= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} \\
&\quad - \frac{\int \frac{-2381456-1789606x+2359920x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{74828637} \\
&= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} \\
&\quad + \frac{39332\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{74828637\sqrt{-5+2x}} + \frac{\int \frac{1142650080}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{17958872880} \\
&\quad + \frac{216326\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1918683} \\
&= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} \\
&\quad + \frac{39332\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{74828637\sqrt{-5+2x}} + \frac{3938\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{61893} \\
&\quad - \frac{\left(19666\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{1918683\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}
$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39332\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{74828637\sqrt{-5+2x}} - \frac{19666\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1918683\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(358\sqrt{\frac{22}{23}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{61893\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39332\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{74828637\sqrt{-5+2x}} - \frac{19666\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{1918683\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{716\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{61893\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [A] (verified)

Time = 30.43 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(-9833\sqrt{682}(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+31\left(\sqrt{-\right.\right.}{74828637\sqrt{2}}$$

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-9833*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sq
rt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]], 39/62] + 31*(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-389005 - 1578
968*x - 20372*x^2 + 285680*x^3) + 92*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[
(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])))/(74828637*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7
+ 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.60

method	result
elliptic	$\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{13455\left(x+\frac{7}{5}\right)^2}-\frac{19666\left(-120x^3+350x^2-105x-50\right)}{74828637\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}+\frac{432992\sqrt{-}}{}\right.$
default	$-\frac{2\left(499410\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^3-442485\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\right.}{}$

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(-2/13455*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2
)/(x+7/5)^2-19666/74828637*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+3
50*x^2-105*x-50))^(1/2)+432992/2080759909059*(-3795*(x+7/5)/(-2/3+x))^(1/2)
*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x)
)^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*
(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+275324/1760642999973*(-3795*(x+7/
5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)
```

```
*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3
*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*Elli
pticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-393320
/24942879*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(
-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^
(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/
2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+
91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2
))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5
 + 1950*x^4 - 4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^{5/2}}\,dx$$

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

3.99 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 721

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx &= \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} \\
&- \frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\,E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}} \\
&+ \frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{f^2h\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} \\
&+ \frac{\sqrt{bg-ah}(adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\right.\right.}{f^2\sqrt{be-af}h^2\sqrt{a+bx}\sqrt{c+dx}}
\end{aligned}
$$

```
[Out] (a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x+g
)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d*e
)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-a*h+b*g)^(1/2)*((-e*h+f*g)*(b*
x+a)/(-a*h+b*g)/(f*x+e))^(1/2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1/2
)/f^2/h^2/(-a*f+b*e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)+(b*x+a)^(1/2)*(d*x+c
)^(1/2)*(h*x+g)^(1/2)/h/(f*x+e)^(1/2)+(-c*f+d*e)*(-2*a*f*h+b*e*h+b*f*g)*Ell
ipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*
d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d
*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/f^2/h/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d
*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-EllipticE((-e*h+
f*g)^(1/2)*(d*x+c)^(1/2)/(-c*h+d*g)^(1/2)/(f*x+e)^(1/2),((-a*f+b*e)*(-c*h+d
*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)
^(1/2)*((-c*f+d*e)*(h*x+g)/(-c*h+d*g)/(f*x+e))^(1/2)/f/h/(-(-c*f+d*e)*(b*x+
a)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {179, 182, 435, 171, 551, 176, 430}

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{(e+fx)\sqrt{bg-ah}\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\text{arcsin}\right.}{f^2h^2\sqrt{a+bx}\sqrt{c+dx}\sqrt{be-af}}$$
$$+\frac{\sqrt{g+hx}(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right.}{f^2h\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$
$$-\frac{\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{\frac{(g+hx)(de-cf)}{(e+fx)(dg-ch)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{g+hx}\sqrt{-\frac{(a+bx)(de-cf)}{(e+fx)(bc-ad)}}}$$
$$+\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

```
[In] Int[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]) - (Sqrt[d*g -
 c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*
h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt[d*g -
 c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))]
)/(f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqrt[g + h*
x]) + ((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqr
t[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/(
(d*e - c*f)*(b*g - a*h)))])/(f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c +
 d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[b*g
 - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*h)*(a + b*x))/
((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x
))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e
- a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g -
 a*h))/((b*e - a*f)*(d*g - c*h))])/(f^2*Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*S
qrt[c + d*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
```

```
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{((de-cf)(fg-eh))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx}{2fh} + \frac{((de-cf)(bfg+beh-2afh))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2f^2h} + \frac{(adfh-b(dfg+deh-cfh))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}\,dx}{2f^2h}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} + \frac{\left((adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\right)\text{Subst}\left(\int \frac{}{(h-fx^2)\sqrt{1+\frac{(-be+}{bg-}}}\right.}{f^2h\sqrt{a+bx}\sqrt{c+dx}} + \frac{\left((de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x,\right.}{f^2h(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{\left((de-cf)(fg-eh)\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-be+af)x^2}{bc-ad}}}{\sqrt{1-\frac{(fg-eh)x^2}{dg-ch}}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{f(-de+cf)h\sqrt{\frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}} + \frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{f^2h\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{\sqrt{bg-ah}(adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\Pi\left(\frac{f(bg-ah)}{(be-af)h};\sin^{-1}\left(\cdots\right.\right.}{f^2\sqrt{be-af}h^2\sqrt{a+bx}\sqrt{c+dx}}$$

Mathematica [A] (warning: unable to verify)

Time = 48.04 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{\sqrt{a+bx}\sqrt{c+dx}\left(-f^2h(g+hx) + \frac{\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}(g+hx)\left(-f(-de+cf)h(-bg+ah)E\left(\arcsin\left(\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\right)|\frac{(be-af)(dg-\cdots}{(de-cf)(bg-\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] -((Sqrt[a + b*x]*Sqrt[c + d*x]*(-(f^2*h*(g + h*x)) + (Sqrt[((f*g - e*h)*(a
+ b*x))/((b*g - a*h)*(e + f*x))]*(g + h*x)*(-(f*(-(d*e) + c*f)*h*(-(b*g) +
a*h)*EllipticE[ArcSin[Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*h)*(e + f*x))]
], ((b*e - a*f)*(d*g - c*h))/((d*e - c*f)*(b*g - a*h))]) + (d*e - c*f)*h*(b
*f*g + b*e*h - 2*a*f*h)*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(g + h*x))/((d*g
 - c*h)*(e + f*x))]], ((b*e - a*f)*(d*g - c*h))/((d*e - c*f)*(b*g - a*h))]
+ (f*g - e*h)*(-(a*d*f*h) + b*(d*f*g + d*e*h - c*f*h))*EllipticPi[(d*f*g -
c*f*h)/(d*e*h - c*f*h), ArcSin[Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*h)*(e
 + f*x))]], ((b*e - a*f)*(d*g - c*h))/((d*e - c*f)*(b*g - a*h))]))/((d*g -
c*h)*(a + b*x)*Sqrt[((-(d*e) + c*f)*(-(f*g) + e*h)*(c + d*x)*(g + h*x))/((d
*g - c*h)^2*(e + f*x)^2)])))/(f^2*h^2*Sqrt[e + f*x]*Sqrt[g + h*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(656) = 1312$.

Time = 4.03 (sec) , antiderivative size = 1544, normalized size of antiderivative = 2.14

method	result	size
elliptic	Expression too large to display	1544
default	Expression too large to display	15274

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*a*c*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d
))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b
)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*
(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(a*d+b*c)*
(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/
b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d)
)^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/
2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)
*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(
x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+b*d*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g
/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e
/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/
b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x
+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))
^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),
((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*
f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/
h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/
(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}}\,dx$$

```
[In] int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)),x)
[Out] int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)), x)
```

3.100 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 161

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{-be+af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|\frac{(-bc+ad)(fg-eh)}{(-be+af)(dg-ch)}\right)}{\sqrt{-be+af}\sqrt{bg-ah}\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}}$$

```
[Out] -2*(1/(1+(a*f-b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e)))^(1/2)*(1+(a*f-b*e)*(h*x+g)/
(-a*h+b*g)/(f*x+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*(h*x+g)^(1/2)/(-a*h+b*g
)^(1/2)/(f*x+e)^(1/2)/(1+(a*f-b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1/2),((a*d-
b*c)*(-e*h+f*g)/(a*f-b*e)/(-c*h+d*g))^(1/2))*(d*x+c)^(1/2)/(a*f-b*e)^(1/2)/
(-a*h+b*g)^(1/2)/(b*x+a)^(1/2)/((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {182, 435}

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\,E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

```
[In] Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*
h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g
```

```
- e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h
)))])/((b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]*Sqrt[g + h*x])
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}}{\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(be-af)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$= -\frac{2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

Mathematica [A] (verified)

Time = 23.63 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.39

$$\int\frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(fg-eh)\sqrt{a+bx}\sqrt{c+dx}\sqrt{\frac{(-be+af)(bg-ah)(e+fx)(g+hx)}{(fg-eh)^2(a+bx)^2}}E\left(\arcsin\left(\sqrt{\frac{(-}{(f}}\right.\right.}{(be-af)(bg-ah)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{e+fx}\sqrt{g+}}$$

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(f*g - e*h)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[((-(b*e) + a*f)*(b*g - a*h)
*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)]*EllipticE[ArcSin[Sqrt[((
-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h
))/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*(b*g - a*h)*Sqrt[((b*g - a*h)*(
c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1947 vs. 2(251) = 502.

Time = 3.96 (sec) , antiderivative size = 1948, normalized size of antiderivative = 12.10

method	result	size
elliptic	Expression too large to display	1948
default	Expression too large to display	4561

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(-2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b
*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*g)/((x
+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+
b*d*e*g*x+b*c*e*g))^(1/2)+2*(d/b-1/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f
*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*g)+(b*c*e*
h+b*c*f*g+b*d*e*g)/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*g))*(g/h-a/b)*((-g/h+c/d)
*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)
/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-
c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c
/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d
+g/h))^(1/2))+2*((a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)/(a^2*f*h-a*b*e*h-a*b*f*g
+b^2*e*g)+(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*g)
)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+
a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/
d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(
1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/
d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)
*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/
b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))+2*b*d*f*h/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*
g)*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d
))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b
)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/
h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(
(e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g
/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(
-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/
h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(
-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+
a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*f*h*x
^4 + a^2*e*g + (b^2*f*g + (b^2*e + 2*a*b*f)*h)*x^3 + ((b^2*e + 2*a*b*f)*g +
 (2*a*b*e + a^2*f)*h)*x^2 + (a^2*e*h + (2*a*b*e + a^2*f)*g)*x), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/((a + b*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^{3/2}}\,dx$$

```
[In] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)),x)
[Out] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)), x)
```

3.101 $\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 351

$$\begin{aligned}\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} \\ &- \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\ &+ \frac{2135\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\ &+ \frac{29047\sqrt{\frac{23}{11}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\ &- \frac{3431855(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{576\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}\end{aligned}$$

```
[Out] -3431855/247104*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-2135/192*(2-3*x)^(1/2)*(1+4*x)^(1/
2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-25/48*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)*(7+5*x)^(1/2)+29047/6336*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)+2135/384*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*
x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/
((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {173, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{2135\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}$$
$$-\frac{3431855(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{576\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$
$$+\frac{29047\sqrt{\frac{23}{11}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$
$$-\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{2135\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{192\sqrt{2x-5}}$$

```
[In] Int[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) - (2
5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/48 + (2135*Sqrt
[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/2
3]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (29047*Sqrt[23/11]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)
/(5 - 2*x)]) - (3431855*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(576*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
```

```
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad +\frac{1}{96}\int \frac{28003+89810x+64050x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}-\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{\int \frac{8779380-33211500x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{23040}-\frac{305305}{128}\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx \\
&= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}-\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad -\frac{553525\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1152}+\frac{668081\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1152} \\
&\quad +\frac{\left(27755\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{128\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{2135\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{\left(17159275(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{11x^2}{23}}\sqrt{1+\frac{11x^2}{39}}(5+3x^2)}dx, x, \frac{\sqrt{7+5x}}{\sqrt{2-3x}}\right)}{576\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}} + \frac{\left(29047\sqrt{\frac{23}{22}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{576\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{2135\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{29047\sqrt{\frac{23}{11}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{576\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} - \frac{3431855(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{576\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [A] (warning: unable to verify)

Time = 24.32 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(1227600(-2+3x) + \frac{-13104630\sqrt{682}(-2+3x)(7}{}\right.}{}$$

```
[In] Integrate[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(1227600*(-2 + 3*x) + (-1310463
0*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elli
pticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 17113116*Sq
rt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elliptic
F[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 385*Sqrt[(7 + 5
*x)/(-2 + 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) - 47445*Sqrt[682]*
```

```
(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)
^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/
62]))/((2 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(235699
2*Sqrt[2 - 3*x])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{25\sqrt{-120x^4+182x^3+385x^2-197x-70}}{48}+\frac{28003\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\ldots}}{14682096\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x\ldots\right.}}\right.}{\ldots}$
risch	$\frac{25\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(7+5x)(2-3x)(-5+2x)(1+4x)}}{48\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(-\frac{28003\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\sqrt{2139}\sqrt{\frac{-\frac{2}{3}+\ldots}{x+\ldots}}}{14682096\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\ldots\right.}}\right.}{\ldots}$
default	$-\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(12025458\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-61773\ldots\right.}{\ldots}$

```
[In] int((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(-25/48*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+
28003/14682096*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)
/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*
(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I
*897^(1/2))+44905/7341048*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/
2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5
)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+
x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^
(1/2),-69/55,1/39*I*897^(1/2)))+10675/32*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(
-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)
*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)
/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2
/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x
))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^
(1/2))
```

Fricas [F]

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sq
rt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \text{Timed out}$$

```
[In] integrate((7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{5/2}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.102 $\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 469

$$\begin{aligned}\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} \\
&+ \frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} \\
&+ \frac{65\sqrt{\frac{11}{23}}\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} \\
&- \frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right),\frac{39}{62}\right)}{48\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}} \\
&+ \frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{6\sqrt{2-3x}\sqrt{-5+2x}} \\
&- \frac{4117\sqrt{2-3x}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right),\frac{39}{62}\right)}{48\sqrt{682}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}\end{aligned}$$

```
[Out] -895/2976*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(
1/2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+
2*x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/
2)/(1+4*x)^(1/2)-4117/32736*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(
7+5*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(5
```

```
29+506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/
2)/((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+23/132*(7+5*x)*EllipticPi(1/11*34
1^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/124,1/62*2418^(1/2))*682^(1/2)*((2-3
*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)-5/1
2*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+65/184*(1/(4+2*(
1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*
2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/
2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+5/24*EllipticE(1/23
*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)
^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {172, 179, 182, 435, 171, 550, 429, 553, 176, 551}

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}$$
$$+\frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{4x+1}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{6\sqrt{2-3x}\sqrt{2x-5}}$$
$$+\frac{65\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$
$$-\frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{48\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}$$
$$-\frac{4117\sqrt{2-3x}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{48\sqrt{682}\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}-\frac{5\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{12\sqrt{2x-5}}$$

```
[In] Int[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(12*Sqrt[-5 + 2*x]) + (5*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*S
qrt[7 + 5*x]) + (65*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 -
2*x)]) - (895*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[
```

```
7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(48*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x]) + (23*Sqrt[31/22]*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x
)]*(7 + 5*x)*EllipticPi[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 +
 5*x]], 39/62])/(6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]) - (4117*Sqrt[2 - 3*x]*Elli
pticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(
48*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*x]*
(Sqrt[c + d*x]/(Sqrt[e + f*x]*Sqrt[g + h*x])), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 550

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[-f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\text{integral} = -\left(\frac{5}{3}\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx\right) + \frac{31}{3}\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} + \frac{895}{48}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$+ \frac{715}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$- \frac{715}{8}\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx$$

$$+ \frac{\left(713\sqrt{2}\sqrt{\frac{2-3x}{7+5x}}\sqrt{-\frac{-5+2x}{7+5x}}(7+5x)\right)\text{Subst}\left(\int \frac{1}{(4-5x^2)\sqrt{1-\frac{31x^2}{11}}\sqrt{1-\frac{39x^2}{22}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{7+5x}}\right)}{33\sqrt{2-3x}\sqrt{-5+2x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}}$$

$$+ \frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{6\sqrt{2-3x}\sqrt{-5+2x}}$$

$$+ \frac{\left(11635\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int \frac{1}{(5-2x^2)\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\frac{22x^2}{23}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{8\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}}$$

$$+ \frac{\left(65\sqrt{\frac{11}{46}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$+ \frac{\left(65\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{8\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} + \frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$+ \frac{65\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$+ \frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{6\sqrt{2-3x}\sqrt{-5+2x}}$$
$$+ \frac{\left(895\sqrt{\frac{23}{31}}\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{22x^2}{23}}}{(5-2x^2)\sqrt{1+\frac{11x^2}{31}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{48\sqrt{2-3x}\sqrt{1+4x}}$$
$$+ \frac{\left(9845\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{11x^2}{31}}\sqrt{1+\frac{22x^2}{23}}}\,dx, x, \frac{\sqrt{7+5x}}{\sqrt{-5+2x}}\right)}{48\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} + \frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$
$$+ \frac{65\sqrt{\frac{11}{23}}\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$
$$- \frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right)|\frac{39}{62}\right)}{48\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$
$$+ \frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{6\sqrt{2-3x}\sqrt{-5+2x}}$$
$$- \frac{4117\sqrt{2-3x}\Pi\left(\frac{78}{55};\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{7+5x}}{\sqrt{-5+2x}}\right)|\frac{39}{62}\right)}{48\sqrt{682}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{1+4x}}$$

Mathematica [A] (warning: unable to verify)

Time = 9.72 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.74

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{-5+2x}\left(6820\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}(-5-18x+8x^2)\,E\left(\arcsin\left(\sqrt{\frac{22}{39}}\right.\right.\right.}{}$$

```
[In] Integrate[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(
1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(
1 + 4*x)]], 39/62] - 6969*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*
x)/(1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*
x)/(1 + 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83
*x - 53*x^2 + 30*x^3) + 9821*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)
^2*Sqrt[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22
/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]], 39/62])))/(16368*Sqrt[4 - 6*x]*((-5 + 2*x)
/(1 + 4*x))^(3/2)*(1 + 4*x)^(3/2)*Sqrt[7 + 5*x])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.85

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{98\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}F\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}+\frac{140\sqrt{-\ldots}}{\ldots}\right.}{\ldots}$
default	$-\frac{\sqrt{7+5x}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\left(107694\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-238266\sqrt{-\ldots}\right.}{\ldots}$

```
[In] int((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(98/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)
^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(
-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(
-2/3+x))^(1/2),1/39*I*897^(1/2))+140/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*
(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))
^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3
795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*
(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+25/2*((x+7/5)*(x-5/2)*(x+
1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(
-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*
(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-37
95*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(
x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-
5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^
3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^{\frac{3}{2}}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**(3/2)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{3/2}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

```
[In] int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.103 $\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 100

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{23\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\operatorname{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{2\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}}$$

```
[Out] 23/1364*(7+5*x)*EllipticPi(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/12
4,1/62*2418^(1/2))*682^(1/2)*((2-3*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2
)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {171, 551}

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{23\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\operatorname{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{4x+1}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{2\sqrt{682}\sqrt{2-3x}\sqrt{2x-5}}$$

```
[In] Int[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (23*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x)]*(7 + 5*x)*EllipticP
i[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 39/62])/(2*Sqr
t[682]*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\text{integral} = \frac{\left(23\sqrt{2}\sqrt{\frac{2-3x}{7+5x}}\sqrt{-\frac{-5+2x}{7+5x}}(7+5x)\right)\text{Subst}\left(\int\frac{1}{(4-5x^2)\sqrt{1-\frac{31x^2}{11}}\sqrt{1-\frac{39x^2}{22}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{7+5x}}\right)}{11\sqrt{2-3x}\sqrt{-5+2x}}$$

$$= \frac{23\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}}$$

Mathematica [A] (verified)

Time = 3.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int\frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= -\frac{62\sqrt{1+4x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticPi}\left(-\frac{55}{69},\arcsin\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{7+5x}}\right),-\frac{39}{23}\right)}{3\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{1+4x}{7+5x}}}$$

```
[In] Integrate[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-62*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticPi[-55/69, ArcSin[(Sqr
t[23/11]*Sqrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(3*Sqrt[253]*Sqrt[-5 + 2*x
]*Sqrt[(1 + 4*x)/(7 + 5*x)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.62

method	result
default	$-\frac{62\left(F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-\Pi\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},-\frac{69}{55},\frac{i\sqrt{897}}{39}\right)\right)\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{3}\,\sqrt{13}\,(-2+3x)\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{1+4\cdots}}{29601(40x^3-34x^2-151x-35)}$
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{14\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}F\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}+\frac{10\sqrt{-\cdots}}{\cdots}\cdots\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+\cdots}}$

[In] int((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -62/29601*(EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2)))*((1+4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*13^(1/2)*(-2+3*x)*(-253*(7+5*x)/(-2+3*x))^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(1/2)*(7+5*x)^(1/2)/(40*x^3-34*x^2-151*x-35)

Fricas [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx=\int \frac{\sqrt{5\,x+7}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)

Sympy [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

```
[In] integrate((7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}}\,dx$$

```
[In] int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.104 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 71

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{2\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

```
[Out] 2/253*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF
((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897
^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {176, 429}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{2\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (2*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -
39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
```

```
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{\frac{2}{253}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{2\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [A] (verified)

Time = 3.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = -\frac{2\sqrt{1+4x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{7+5x}}\right), -\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{1+4x}{7+5x}}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
[Out] (-2*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticF[ArcSin[(Sqrt[23/11]*S
qrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(1 +
4*x)/(7 + 5*x)])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

method	result	siz
default	$-\frac{2F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{3}\,\sqrt{13}\,(-2+3x)\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\sqrt{2-3x}\,\sqrt{7+5x}}{9867(40x^3-34x^2-151x-35)}$	13
elliptic	$\frac{2\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\,\left(-\frac{2}{3}+x\right)^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,F\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}$	13

```
[In] int(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/9867*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*((1+
4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*13^(1/2)*(-
2+3*x)*(-253*(7+5*x)/(-2+3*x))^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(
1/2)*(7+5*x)^(1/2)/(40*x^3-34*x^2-151*x-35)
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{1}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4
 - 182*x^3 - 385*x^2 + 197*x + 70), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}\,dx$$

```
[In] integrate(1/(7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{1}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{1}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

3.105 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 195

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{10\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{22}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{62}{39}\right)}{713\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} + \frac{2\sqrt{\frac{3}{143}}(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{31\sqrt{-5+2x}\sqrt{1+4x}}$$

```
[Out] 2/4433*(2-3*x)*EllipticF(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),1/39*I*
897^(1/2))*429^(1/2)*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)/(-5+2
*x)^(1/2)/(1+4*x)^(1/2)+10/27807*EllipticE(1/22*858^(1/2)*(1+4*x)^(1/2)/(7+
5*x)^(1/2),1/39*2418^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)
/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {177, 176, 429, 182, 433, 506, 422}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{6\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} - \frac{5\sqrt{\frac{22}{31}}\sqrt{4x+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{1209\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}} + \frac{10\sqrt{\frac{22}{31}}\sqrt{4x+1}E\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}} - \frac{10\sqrt{2x-5}\sqrt{4x+1}}{897\sqrt{2-3x}\sqrt{5x+7}}$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-10*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (10*
Sqrt[22/31]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqr
t[7 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) + (6*
Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/
23])/(31*Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (5*Sqrt[22/3
1]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x
]], 39/62])/(1209*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 177

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3}{31}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx\\ &\quad+\frac{5}{31}\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx\end{aligned}$$

$$= \frac{\left(5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx,x,\frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{1209\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}} + \frac{\left(3\sqrt{\frac{2}{253}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{31\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{6\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{\left(5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{23x^2}{22}}\sqrt{1+\frac{31x^2}{11}}}\,dx,x,\frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{1209\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}} + \frac{\left(5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{23x^2}{22}}\sqrt{1+\frac{31x^2}{11}}}\,dx,x,\frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{429\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= -\frac{10\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{6\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} - \frac{5\sqrt{\frac{22}{31}}\sqrt{1+4x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{1209\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} - \frac{\left(10\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{23x^2}{22}}}{\left(1+\frac{31x^2}{11}\right)^{3/2}}\,dx,x,\frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{897\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= -\frac{10\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{10\sqrt{\frac{22}{31}}\sqrt{1+4x}E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} + \frac{6\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} - \frac{5\sqrt{\frac{22}{31}}\sqrt{1+4x}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{1209\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}}$$

Mathematica [A] (verified)

Time = 18.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(1705\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)-55\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\text{arcsi}\right.\right.}{305877\sqrt{2-3x}\sqrt{7+5x}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*
x + 8*x^2) - 55*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x
 + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62
] - 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2
)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(3058
77*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^
2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(155) = 310.

Time = 1.62 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.23

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{7252\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}F\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{8505521739\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}+\frac{140\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$
default	$-\frac{2\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(1116\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,x^{2}F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-495\sqrt{-\frac{253(\cdots}{-2\cdots}}\right.}{\cdots}$

[In] int(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2)*(7252/8505521739*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+140/654270903*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-200/9269*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*El

```
lipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*Ellipt
icE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(
1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)
*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)-10/27807*(-120*x^3+350*x^2-105*x-50)/((x+7
/5)*(-120*x^3+350*x^2-105*x-50))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5
 - 70*x^4 - 3199*x^3 - 1710*x^2 + 1729*x + 490), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}\,(5x+7)^{3/2}}\,dx$$

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

3.106 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 288

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} + \frac{358120\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2319687747\sqrt{-5+2x}} - \frac{179060\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{59479173\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{103964\sqrt{7+5x}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1918683\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

```
[Out] -50/83421*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-895300/2
319687747*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+358120/2
319687747*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+103964/4
85426799*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*Ellipt
icF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*
897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-1
79060/2319687747*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39
*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2
*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {178, 1613, 1616, 12, 176, 429, 182, 435}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{179060\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{59479173\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{103964\sqrt{5x+7}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1918683\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}} + \frac{358120\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2319687747\sqrt{2x-5}} - \frac{895300\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2319687747\sqrt{5x+7}} - \frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-50*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) -
(895300*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5*
x]) + (358120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2319687747*Sqrt[-
5 + 2*x]) - (179060*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*Ell
ipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(594791
73*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (103964*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1918683*Sqrt[25
3]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 178

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b^2*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(
b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}+\frac{\int \frac{11928-4270x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{83421}\\
&= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}\\
&\quad+\frac{\int \frac{41179978+16294460x-21487200x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{2319687747}\\
&= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}\\
&\quad+\frac{358120\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2319687747\sqrt{-5+2x}}-\frac{\int -\frac{15083097120}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{556725059280}\\
&\quad+\frac{1969660\int \frac{\sqrt{2-3x}}{(-5+2x)^{3/2}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{59479173}\\
&= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}}\\
&\quad+\frac{358120\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2319687747\sqrt{-5+2x}}+\frac{51982\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx}{1918683}\\
&\quad-\frac{\left(179060\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{-\frac{7+5x}{-5+2x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-\frac{39x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{-5+2x}}\right)}{59479173\sqrt{-\frac{2-3x}{-5+2x}}\sqrt{7+5x}}
\end{aligned}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} + \frac{358120\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2319687747\sqrt{-5+2x}} - \frac{179060\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{59479173\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{\left(51982\sqrt{\frac{2}{253}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{2}}\sqrt{1+\frac{31x^2}{23}}}\,dx, x, \frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{1918683\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2319687747\sqrt{7+5x}} + \frac{358120\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2319687747\sqrt{-5+2x}} - \frac{179060\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{59479173\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} + \frac{103964\sqrt{7+5x}F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1918683\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [A] (verified)

Time = 31.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.85

$$\int\frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(1705\sqrt{\frac{7+5x}{-2+3x}}(-671560-2797991x-294854x^2+608600x^3)-984830\sqrt{682}(-2+3\right.}{255165652}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-671560
- 2797991*x - 294854*x^2 + 608600*x^3) - 984830*Sqrt[682]*(-2 + 3*x)*(7 + 5
*x)^2*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 28819*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2
*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62]))/(25516565217*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*
Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{83421\left(x+\frac{7}{5}\right)^2}-\frac{179060\left(-120x^3+350x^2-105x-50\right)}{2319687747\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}+\frac{82359956\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	$-\frac{2\left(72514890\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,F\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^3-44317350\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2\cdots}{-2+3\cdots}}\cdots\right.}{\cdots}$

```
[In] int(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] (-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+
4*x)^(1/2)/(7+5*x)^(1/2)*(-2/83421*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2
)/(x+7/5)^2-179060/2319687747*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^
3+350*x^2-105*x-50))^(1/2)+82359956/709539128989119*(-3795*(x+7/5)/(-2/3+x)
)^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(
-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*
(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+2506840/54579932999163*(-3
795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2
```

```
139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(
1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-3
1/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)
))-3581200/773229249*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+
x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)
/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/3
9*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*
897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39
*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^
6 + 3850*x^5 - 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{5/2}}\,dx$$

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

3.107 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 968

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}}$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

$$+\frac{b(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{df^2h\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+\frac{b\sqrt{bg-ah}(adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\right.\right.}{df^2\sqrt{be-af}h^2\sqrt{a+bx}\sqrt{c+dx}}$$

$$-\frac{2\sqrt{bc-ad}\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\right.}{dh\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] b*(a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x
+g)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d
*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-a*h+b*g)^(1/2)*((-e*h+f*g)*(
b*x+a)/(-a*h+b*g)/(f*x+e))^(1/2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1
/2)/d/f^2/h^2/(-a*f+b*e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)-2*(b*x+a)*Ellipt
icPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+
d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-a*
d+b*c)^(1/2)*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*
((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/d/h/(d*x+c)^(1/2)/(f*x+e)^(1/
2)+b*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(h*x+g)^(1/2)/d/h/(f*x+e)^(1/2)+b*(-c*f+d*
e)*(-2*a*f*h+b*e*h+b*f*g)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*
```

```
g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2)
)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/d/f^2/h/(-a*h
+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/
(b*x+a))^(1/2)-b*EllipticE((-e*h+f*g)^(1/2)*(d*x+c)^(1/2)/(-c*h+d*g)^(1/2)/
(f*x+e)^(1/2),((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-c*h+d*
g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*((-c*f+d*e)*(h*x+g)/(-c*h+d*g)/(f*x
+e))^(1/2)/d/f/h/(-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/
2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 968, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {172, 179, 182, 435, 171, 551, 176, 430}

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)b}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

$$+\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)b}{df^2h\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$+\frac{\sqrt{bg-ah}(adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\frac{\sqrt{b}\cdots}{\sqrt{b}\cdots}\right.\right.}{df^2\sqrt{be-af}h^2\sqrt{a+bx}\sqrt{c+dx}}$$

$$+\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}b}{dh\sqrt{e+fx}}$$

$$-\frac{2\sqrt{bc-ad}\sqrt{ch-dg}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-\cdots}{(bc-\cdots}\right.}{dh\sqrt{c+dx}\sqrt{e+fx}}$$

```
[In] Int[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(d*h*Sqrt[e + f*x]) - (b*Sqrt
[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*
g - c*h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt
[d*g - c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g -
e*h))])/(d*f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqr
t[g + h*x]) + (b*(d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
- a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g
 - e*h))/((d*e - c*f)*(b*g - a*h)))])/(d*f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e
*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

```
 + (b*Sqrt[b*g - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*
h)*(a + b*x))/((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g -
 c*h)*(e + f*x))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), Arc
Sin[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e
 - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))])/(d*f^2*Sqrt[b*e - a*f]*h^2
*Sqrt[a + b*x]*Sqrt[c + d*x]) - (2*Sqrt[b*c - a*d]*Sqrt[-(d*g) + c*h]*(a +
b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h
)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c -
a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a
 + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*h*Sqrt[
c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*x]*
(Sqrt[c + d*x]/(Sqrt[e + f*x]*Sqrt[g + h*x])), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
```

```
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\text{integral} = \frac{b \int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{d}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{(b(de-cf)(fg-eh))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx}{2dfh}$$
$$+ \frac{(b(de-cf)(bfg+beh-2afh))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{2df^2h}$$
$$+ \frac{(b(adfh-b(dfg+deh-cfh)))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}\,dx}{2df^2h}$$
$$- \frac{\left(2(bc-ad)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\sqrt{1+\frac{(be-af)x^2}{fg-eh}}}\,dx, x\right.}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}}$$
$$- \frac{2\sqrt{bc-ad}\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)\right.}{dh\sqrt{c+dx}\sqrt{e+fx}}$$
$$+ \frac{\left(b(adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\right)\text{Subst}\left(\int \frac{}{(h-fx^2)\sqrt{1+\frac{(-be}{bg}}}\right.}{df^2h\sqrt{a+bx}\sqrt{c+dx}}$$
$$+ \frac{\left(b(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x\right.}{df^2h(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$
$$+ \frac{\left(b(de-cf)(fg-eh)\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(-be+af)x^2}{bc-ad}}}{\sqrt{1-\frac{(fg-eh)x^2}{dg-ch}}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{df(-de+cf)h\sqrt{\frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}} + \frac{b(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{df^2h\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}} + \frac{b\sqrt{bg-ah}(adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}(e+fx)\Pi\left(\frac{f(bg-ah)}{(be-af)h};\sin^{-1}\left(\right.\right.}{df^2\sqrt{be-af}h^2\sqrt{a+bx}\sqrt{c+dx}} - \frac{2\sqrt{bc-ad}\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)|\right.}{dh\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7319 vs. 2(968) = 1936.

Time = 30.31 (sec) , antiderivative size = 7319, normalized size of antiderivative = 7.56

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 1541, normalized size of antiderivative = 1.59

method	result	size
elliptic	Expression too large to display	1541
default	Expression too large to display	17031

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*a^2*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d
))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b
)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*
(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+
```

```
c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+4*a*b*(g/h-a
/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+
e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2
)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c
/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-
a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)
/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b
+e/f)/(-c/d+g/h))^(1/2)))+b^2*((x+a/b)*(x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d
)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b
)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/
h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/
(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)
)+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-
c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*
d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)
/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+
g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral((a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.108 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 228

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\mathrm{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right),\frac{(be-af)(dg}{(bc-ad)(fg}\right.}{\sqrt{bc-ad}\,h\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] 2*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)
^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f
*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*(
(-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/h/(-a*d+b*c)^(1/2)/(d*x+c)^(1/
2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {171, 551}

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\mathrm{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch}{(bc-ad)(fg-eh}\right.}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

```
[In] Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi
[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])
/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d
)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

integral

$$= \frac{\left(2(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{(h-bx^2)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\sqrt{1+\frac{(be-af)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{g+hx}}{\sqrt{a+bx}}\right)}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$= \frac{2\sqrt{-dg+ch}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right.}{\sqrt{bc-ad}h\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 583 vs. 2(228) = 456.

Time = 31.71 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\sqrt{\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}}(c+dx)^{3/2}\left(\frac{ad\sqrt{\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}(g+hx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right),\frac{(bc-ad)(-fg+eh)}{(de-cf)(bg-ah)}\right)}{(dg-ch)(c+dx)\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}}+\frac{bc\sqrt{\frac{(dg-ch)(e}{(fg-eh)(c}}}{}\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]*(c + d*x)^(3/2)*(
(a*d*Sqrt[((d*g - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*Ellipt
icF[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c
 - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/((d*g - c*h)*(c + d*x)*
Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*c*Sqrt[((d*g
 - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*EllipticF[ArcSin[Sqrt
[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g)
 + e*h))/((d*e - c*f)*(b*g - a*h))])/((-(d*g) + c*h)*(c + d*x)*Sqrt[((-(d*e
) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*(f*g - e*h)*Sqrt[((-(d*e
) + c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(c + d*x)^2)]*Elli
pticPi[(d*(-(f*g) + e*h))/((d*e - c*f)*h), ArcSin[Sqrt[((-(d*e) + c*f)*(g +
 h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)
*(b*g - a*h))])/((d*e - c*f)*h)))/(d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(209) = 418$.

Time = 1.29 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.72

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\left(\frac{2a\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^{2}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}},\right.}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}\right.}{}$
default	Expression too large to display

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*a*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))
^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*
(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x
+c/d)*(x+e/f)*(x+g/h))^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/
d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*b*(g/h-a/b)*
((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+a/b)*(x+e/f)
/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/d))^(1/2)/(-
g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(1/2)*(-c/d*E
```

```
llipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)
/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)*(x+a/b)/(-g
/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f
)/(-c/d+g/h))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
[Out] integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
[Out] integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.109 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 161

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}\sqrt{e+fx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{be-af}\sqrt{fg-eh}\sqrt{c+dx}}$$

```
[Out] -2*(1/(1+(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a)))^(1/2)*(1+(-a*f+b*e)*(h*x+g
)/(-e*h+f*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(h*x+g)^(1/2)/(-e*h+
f*g)^(1/2)/(b*x+a)^(1/2)/(1+(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2),((
-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*((-e*h+f*g)*(d*x+c)/(-c*
h+d*g)/(f*x+e))^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {176, 430}

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Elli
pticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x]
)], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[b*g - a*
h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x)))])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{\left(2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

Mathematica [A] (verified)

Time = 23.05 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.41

$$\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\frac{(-bc+ad)(-fg+eh)}{(be-af)(dg-ch)}\right)}{(bg-ah)\sqrt{c+dx}\sqrt{e+fx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqr
t[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[
ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c)
+ a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.68

method	result
default	$-\frac{2\sqrt{\frac{(ch-dg)(bx+a)}{(ah-gb)(dx+c)}}\,\sqrt{\frac{(ad-bc)(fx+e)}{(af-be)(dx+c)}}\,\sqrt{\frac{(ad-bc)(hx+g)}{(ah-gb)(dx+c)}}\,F\left(\sqrt{\frac{(ch-dg)(bx+a)}{(ah-gb)(dx+c)}},\sqrt{\frac{(cf-de)(ah-gb)}{(af-be)(ch-dg)}}\right)\left(a\,d^2h\,x^2-b\,d^2g\,x^2+2acdhx-2bcdgx+\right.}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,(ch-dg)(ad-bc)}$
elliptic	$\frac{2\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\,\left(\frac{g}{h}-\frac{a}{b}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\,\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\,\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\,F\left(\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}},\right.}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}}$

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*((c*h-d*g)*(b*x+
a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a
*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-
b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*(a*d^2
*h*x^2-b*d^2*g*x^2+2*a*c*d*h*x-2*b*c*d*g*x+a*c^2*h-b*c^2*g)/(c*h-d*g)/(a*d-
b*c)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*d*f*h*x
^4 + a*c*e*g + (b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^3 + ((b*d*e + (b*c +
 a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^2 + (a*c*e*h + (a*c*f + (b*c + a*
d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.110 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 429

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2b\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{2d\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

```
[Out] -2*d*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2
),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)
/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+b*g)^(1/2)/(-e*h+
f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2*b
*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-
(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-e*h+f*g)^(1/2)*(d*x+c
)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)/(-a*d+b*c)/(-a*f+b*e
)/(-a*h+b*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1
/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {177, 176, 430, 182, 435}

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}} - \frac{2b\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\,E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*
g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a
*h)))])/((b*c - a*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]) - (2*d*Sqrt[((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*
h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*
h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 177

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad}\\
&= -\frac{\left(2d\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(bc-ad)(fg-eh)\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}\\
&\quad-\frac{\left(2b\sqrt{c+dx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}}{\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx,x,\frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(bc-ad)(be-af)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}\\
&= -\frac{2b\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}\\
&\quad-\frac{2d\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4121 vs. 2(429) = 858.

Time = 41.91 (sec) , antiderivative size = 4121, normalized size of antiderivative = 9.61

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*b^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)
*(b*g - a*h)*Sqrt[a + b*x]) - (2*(-((b*(c + d*x)^(3/2)*(f + (d*e)/(c + d*x)
 - (c*f)/(c + d*x))*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x))*Sqrt[a + ((c +
d*x)*(b - (b*c)/(c + d*x)))/d])/(Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))
/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])) + ((c + d*x)*Sqrt[f + (
d*e)/(c + d*x) - (c*f)/(c + d*x)]*Sqrt[h + (d*g)/(c + d*x) - (c*h)/(c + d*x
)]*Sqrt[(b - (b*c)/(c + d*x) + (a*d)/(c + d*x))*(f + (d*e)/(c + d*x) - (c*f
)/(c + d*x))*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x))]*Sqrt[a + ((c + d*x)*(
b - (b*c)/(c + d*x)))/d]*(((b*c - a*d)*f*(b*g - a*h)*(-(d*g) + c*h)*Sqrt[f
+ (d*e)/(c + d*x) - (c*f)/(c + d*x)])/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[b - (
b*c)/(c + d*x) + (a*d)/(c + d*x)]*Sqrt[h + (d*g)/(c + d*x) - (c*h)/(c + d*x
)]) - ((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*h*Sqrt[h + (d*g)/(c + d*x) -
(c*h)/(c + d*x)])/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[b - (b*c)/(c + d*x) + (a*
d)/(c + d*x)]*Sqrt[f + (d*e)/(c + d*x) - (c*f)/(c + d*x)]))*((b*d^2*e*g*Sqr
t[((b*c - a*d)*(-(d*g) + c*h)*(b/(b*c - a*d) - (c + d*x)^(-1)))/(-(b*d*g) +
 a*d*h)]*(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))*Sqrt[(-(h/(-(d*g) + c*h)) +
 (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*(((-(b*d*g) + a*d*h
)*EllipticE[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)
))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*
g) + a*h))])/((b*c - a*d)*(-(d*g) + c*h)) - (b*EllipticF[ArcSin[Sqrt[((d*e
- c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c
 - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(b*c - a*d)))/(Sq
rt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g)
+ c*h))]*Sqrt[-((-b + (b*c - a*d)/(c + d*x))*(-f + (-(d*e) + c*f)/(c + d*x)
)*(-h + (-(d*g) + c*h)/(c + d*x)))]) - (b*c*d*f*g*Sqrt[((b*c - a*d)*(-(d*g)
 + c*h)*(b/(b*c - a*d) - (c + d*x)^(-1)))/(-(b*d*g) + a*d*h)]*(-(f/(-(d*e)
+ c*f)) + (c + d*x)^(-1))*Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-
(d*e) + c*f) - h/(-(d*g) + c*h))]*(((-(b*d*g) + a*d*h)*EllipticE[ArcSin[Sqr
t[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]
], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/((b*c - a
*d)*(-(d*g) + c*h)) - (b*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c +
 d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h)
)/((-(d*e) + c*f)*(-(b*g) + a*h))])/(b*c - a*d)))/(Sqrt[(-(f/(-(d*e) + c*f)
) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*Sqrt[-((-b +
(b*c - a*d)/(c + d*x))*(-f + (-(d*e) + c*f)/(c + d*x))*(-h + (-(d*g) + c*h)
```

```
/(c + d*x)))]) - (b*c*d*e*h*Sqrt[((b*c - a*d)*(-(d*g) + c*h)*(b/(b*c - a*d)
 - (c + d*x)^(-1)))/(-(b*d*g) + a*d*h)]*(-(f/(-(d*e) + c*f)) + (c + d*x)^(-
1))*Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*
g) + c*h))]*(((-(b*d*g) + a*d*h)*EllipticE[ArcSin[Sqrt[((d*e - c*f)*(h + (d
*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g
) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/((b*c - a*d)*(-(d*g) + c*h)) -
(b*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x
)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b
*g) + a*h))])/(b*c - a*d)))/(Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-
(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*Sqrt[-((-b + (b*c - a*d)/(c + d*x))
*(-f + (-(d*e) + c*f)/(c + d*x))*(-h + (-(d*g) + c*h)/(c + d*x)))]) + (b*c^
2*f*h*Sqrt[((b*c - a*d)*(-(d*g) + c*h)*(b/(b*c - a*d) - (c + d*x)^(-1)))/(-
(b*d*g) + a*d*h)]*(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))*Sqrt[(-(h/(-(d*g)
+ c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*(((-(b*d*g
) + a*d*h)*EllipticE[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/
(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c
*f)*(-(b*g) + a*h))])/((b*c - a*d)*(-(d*g) + c*h)) - (b*EllipticF[ArcSin[Sq
rt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))
]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(b*c - a
*d)))/(Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h
/(-(d*g) + c*h))]*Sqrt[-((-b + (b*c - a*d)/(c + d*x))*(-f + (-(d*e) + c*f)/
(c + d*x))*(-h + (-(d*g) + c*h)/(c + d*x)))]) + (b*d*f*g*Sqrt[(-(b/(b*c - a
*d)) + (c + d*x)^(-1))/(-(b/(b*c - a*d)) + h/(-(d*g) + c*h))]*Sqrt[(-(f/(-(
d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*(-
(h/(-(d*g) + c*h)) + (c + d*x)^(-1))*EllipticF[ArcSin[Sqrt[((-(d*e) + c*f)*
(-h - (d*g)/(c + d*x) + (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d
)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(Sqrt[(-(h/(-(d*g) + c*
h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*Sqrt[-((-b + (
b*c - a*d)/(c + d*x))*(-f + (-(d*e) + c*f)/(c + d*x))*(-h + (-(d*g) + c*h)/
(c + d*x)))]) + (b*d*e*h*Sqrt[(-(b/(b*c - a*d)) + (c + d*x)^(-1))/(-(b/(b*c
 - a*d)) + h/(-(d*g) + c*h))]*Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(
-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*(-(h/(-(d*g) + c*h)) + (c + d*x)^(
-1))*EllipticF[ArcSin[Sqrt[((-(d*e) + c*f)*(-h - (d*g)/(c + d*x) + (c*h)/(c
 + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f
)*(-(b*g) + a*h))])/(Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e)
 + c*f) - h/(-(d*g) + c*h))]*Sqrt[-((-b + (b*c - a*d)/(c + d*x))*(-f + (-(d
*e) + c*f)/(c + d*x))*(-h + (-(d*g) + c*h)/(c + d*x)))]) - (b*c*f*h*Sqrt[(-
(b/(b*c - a*d)) + (c + d*x)^(-1))/(-(b/(b*c - a*d)) + h/(-(d*g) + c*h))]*Sq
rt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g)
+ c*h))]*(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))*EllipticF[ArcSin[Sqrt[((-(d
*e) + c*f)*(-h - (d*g)/(c + d*x) + (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]],
((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(Sqrt[(-(h/(
-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*Sqr
t[-((-b + (b*c - a*d)/(c + d*x))*(-f + (-(d*e) + c*f)/(c + d*x))*(-h + (-(d
*g) + c*h)/(c + d*x)))]) - (a*d*f*h*Sqrt[(-(b/(b*c - a*d)) + (c + d*x)^(-1)
```

```
)/(-(b/(b*c - a*d)) + h/(-(d*g) + c*h))]*Sqrt[(-(f/(-(d*e) + c*f)) + (c + d
*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*(-(h/(-(d*g) + c*h)) +
(c + d*x)^(-1))*EllipticF[ArcSin[Sqrt[((-(d*e) + c*f)*(-h - (d*g)/(c + d*x)
 + (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-
(d*e) + c*f)*(-(b*g) + a*h))])/(Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))
/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*Sqrt[-((-b + (b*c - a*d)/(c + d*x))
*(-f + (-(d*e) + c*f)/(c + d*x))*(-h + (-(d*g) + c*h)/(c + d*x)))]))))/(Sqrt
[b - (b*c)/(c + d*x) + (a*d)/(c + d*x)]*(b*d*f*g + b*d*e*h - b*c*f*h - a*d*
f*h + (b*d^2*e*g)/(c + d*x) - (b*c*d*f*g)/(c + d*x) - (b*c*d*e*h)/(c + d*x)
 + (b*c^2*f*h)/(c + d*x))*Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))/d]*Sqr
t[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])))/(d*(b*c - a*d)*(b*e - a*f)*(b
*g - a*h))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2199 vs. $2(391) = 782$.

Time = 1.69 (sec) , antiderivative size = 2200, normalized size of antiderivative = 5.13

method	result	size
elliptic	Expression too large to display	2200
default	Expression too large to display	9326

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)
^(1/2)/(h*x+g)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*
c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a
^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/((x+a/b)*(b*d*f*h
*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*
e*g))^(1/2)+2*((a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g
+b^2*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^
2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b*c*f*g+b*d*e*g)*b/(a^3*d*f*h-a^2*b
*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*
g))*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/
d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+
c/d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))
^(1/2)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(
g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+2*(-b*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f
*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+
a*b^2*d*e*g-b^3*c*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)*b/(a^3*d*f*h-a^2*b*c
*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)
)*(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+
a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g/h+a/b)/(x+c/
```

```
d))^(1/2)/(-g/h+c/d)/(-c/d+a/b)/(b*d*f*h*(x+a/b)*(x+c/d)*(x+e/f)*(x+g/h))^(
1/2)*(-c/d*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/
d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(c/d-a/b)*EllipticPi(((-g/h+c/d)
*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(-g/h+a/b)/(-g/h+c/d),((e/f-c/d)*(g/h-a/
b)/(-a/b+e/f)/(-c/d+g/h))^(1/2)))-2*b^2*d*f*h/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*
d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*((x+a/b)*(
x+e/f)*(x+g/h)+(g/h-a/b)*((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2)*(x+c
/d)^2*((-c/d+a/b)*(x+e/f)/(-e/f+a/b)/(x+c/d))^(1/2)*((-c/d+a/b)*(x+g/h)/(-g
/h+a/b)/(x+c/d))^(1/2)*((a*c/b/d-g/h*a/b+g/h*c/d+c^2/d^2)/(-g/h+c/d)/(-c/d+
a/b)*EllipticF(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/
h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))+(-a/b+e/f)*EllipticE(((-g/h+c/d)*(x+a/
b)/(-g/h+a/b)/(x+c/d))^(1/2),((e/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1
/2))/(-c/d+a/b)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/b/d/f/h/(-g/h+c/d)*Ellipt
icPi(((-g/h+c/d)*(x+a/b)/(-g/h+a/b)/(x+c/d))^(1/2),(g/h-a/b)/(-c/d+g/h),((e
/f-c/d)*(g/h-a/b)/(-a/b+e/f)/(-c/d+g/h))^(1/2))))/(b*d*f*h*(x+a/b)*(x+c/d)*
(x+e/f)*(x+g/h))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d*f*h
*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a*b*d)*f)*h)*x^4 + ((
b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)
*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*f)*g + (a^2*c*f + (2
*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*a*b*c + a^2*d)*e)*g)
*x), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

3.111 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 786

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2d^3\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2b^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(be-af)(bg-ah)\sqrt{a+bx}}$$
$$+\frac{2b(a^2d^2fh-abd^2(fg+eh)+b^2(2d^2eg+c^2fh-cd(fg+eh)))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(be-af)(de-cf)(bg-ah)(dg-ch)\sqrt{a+bx}}$$
$$-\frac{2\sqrt{fg-eh}(a^2d^2fh-abd^2(fg+eh)+b^2(2d^2eg+c^2fh-cd(fg+eh)))\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\arcsin\right.}{(bc-ad)^2(be-af)(de-cf)\sqrt{bg-ah}(dg-ch)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$-\frac{4bd\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)^2\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

```
[Out] -2*d^3*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-
c*h+d*g)/(d*x+c)^(1/2)-2*b^3*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*
d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*b*(a^2*d^2*f*h-a*b*d^2*(e*h+
f*g)+b^2*(2*d^2*e*g+c^2*f*h-c*d*(e*h+f*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*
x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(-a*h+b*g)/(-c*h+d*g)/(b*x+a)
^(1/2)-4*b*d*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x
+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)
*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*h+b*g)^(1
/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))
^(1/2)-2*(a^2*d^2*f*h-a*b*d^2*(e*h+f*g)+b^2*(2*d^2*e*g+c^2*f*h-c*d*(e*h+f*g
)))*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2)
,(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-e*h+f*g)^(1/2)*(d*
```

```
x+c)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)/(-a*d+b*c)^2/(-a*
f+b*e)/(-c*f+d*e)/(-c*h+d*g)/(-a*h+b*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-c*f+d*e
)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Rubi [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
[Out] Defer[Int][1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Mathematica [A] (verified)

Time = 34.40 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\frac{2\sqrt{c+dx}\left(-b\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)(g+hx)\left(a^3d^3fh-ab\right.\right.}{}$$

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
[Out] (2*Sqrt[c + d*x]*(-(b*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]
*(e + f*x)*(g + h*x)*(a^3*d^3*f*h - a*b^2*d^3*(-(e*g) + f*g*x + e*h*x) - a^
2*b*d^3*(e*h + f*(g - h*x)) + b^3*(c^3*f*h + 2*d^3*e*g*x + c*d^2*(e*g - f*g
*x - e*h*x) - c^2*d*(f*g + e*h - f*h*x)))) + (c + d*x)*(b^2*(a^2*d^2*f*h -
a*b*d^2*(f*g + e*h) + b^2*(2*d^2*e*g + c^2*f*h - c*d*(f*g + e*h)))*Sqrt[((b
*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e + f*x)*(g + h*x) + b*(f*g
- e*h)*(a + b*x)*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g
 - e*h)^2*(a + b*x)^2))]*((a^2*d^2*f*h - a*b*d^2*(f*g + e*h) + b^2*(2*d^2*e
*g + c^2*f*h - c*d*(f*g + e*h)))*EllipticE[ArcSin[Sqrt[((-(b*e) + a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d
*g - c*h))] - 2*b*d*(d*e - c*f)*(b*g - a*h)*EllipticF[ArcSin[Sqrt[((-(b*e)
+ a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h))/((b*
e - a*f)*(d*g - c*h))]))))/(b*(b*c - a*d)^2*(b*e - a*f)*(d*e - c*f)*(d*g -
c*h)^2*(a + b*x)^(3/2)*(((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)))^(3
/2)*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7102 vs. $2(724) = 1448$.

Time = 3.00 (sec) , antiderivative size = 7103, normalized size of antiderivative = 9.04

method	result	size
elliptic	Expression too large to display	7103
default	Expression too large to display	22970

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d^2*f
*h*x^6 + a^2*c^2*e*g + (b^2*d^2*f*g + (b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)
*h)*x^5 + ((b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)*g + (2*(b^2*c*d + a*b*d^2)
*e + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*h)*x^4 + ((2*(b^2*c*d + a*b*d^2)*e
+ (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*g + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e
 + 2*(a*b*c^2 + a^2*c*d)*f)*h)*x^3 + (((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e +
2*(a*b*c^2 + a^2*c*d)*f)*g + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*h)*x^2 +
 (a^2*c^2*e*h + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)),
 x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Giac [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)), x)
```

3.112 $\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 319

$$\begin{aligned}\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx &= \frac{e^2(e+fx)^{1+n}}{bdf^3(1+n)}+\frac{(bc+ad)e(e+fx)^{1+n}}{b^2d^2f^2(1+n)}\\&\quad+\frac{(b^2c^2+abcd+a^2d^2)\,(e+fx)^{1+n}}{b^3d^3f(1+n)}-\frac{2e(e+fx)^{2+n}}{bdf^3(2+n)}\\&\quad-\frac{(bc+ad)(e+fx)^{2+n}}{b^2d^2f^2(2+n)}+\frac{(e+fx)^{3+n}}{bdf^3(3+n)}\\&\quad-\frac{a^4(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{b^3(bc-ad)(be-af)(1+n)}\\&\quad+\frac{c^4(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{d^3(bc-ad)(de-cf)(1+n)}\end{aligned}$$

```
[Out] e^2*(f*x+e)^(1+n)/b/d/f^3/(1+n)+(a*d+b*c)*e*(f*x+e)^(1+n)/b^2/d^2/f^2/(1+n)
+(a^2*d^2+a*b*c*d+b^2*c^2)*(f*x+e)^(1+n)/b^3/d^3/f/(1+n)-2*e*(f*x+e)^(2+n)/
b/d/f^3/(2+n)-(a*d+b*c)*(f*x+e)^(2+n)/b^2/d^2/f^2/(2+n)+(f*x+e)^(3+n)/b/d/f
^3/(3+n)-a^4*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/b
^3/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^4*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d
*(f*x+e)/(-c*f+d*e))/d^3/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {186, 45, 70}

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = -\frac{a^4(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{b^3(n+1)(bc-ad)(be-af)} + \frac{(a^2d^2+abcd+b^2c^2)\,(e+fx)^{n+1}}{b^3d^3f(n+1)} + \frac{e(ad+bc)(e+fx)^{n+1}}{b^2d^2f^2(n+1)} - \frac{(ad+bc)(e+fx)^{n+2}}{b^2d^2f^2(n+2)} + \frac{c^4(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{d^3(n+1)(bc-ad)(de-cf)} + \frac{e^2(e+fx)^{n+1}}{bdf^3(n+1)} - \frac{2e(e+fx)^{n+2}}{bdf^3(n+2)} + \frac{(e+fx)^{n+3}}{bdf^3(n+3)}$$

```
[In] Int[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*d*f^3*(1 + n)) + ((b*c + a*d)*e*(e + f*x)^(1 + n
))/(b^2*d^2*f^2*(1 + n)) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(e + f*x)^(1 + n)
)/(b^3*d^3*f*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*d*f^3*(2 + n)) - ((b*c +
 a*d)*(e + f*x)^(2 + n))/(b^2*d^2*f^2*(2 + n)) + (e + f*x)^(3 + n)/(b*d*f^3
*(3 + n)) - (a^4*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
 + f*x))/(b*e - a*f)])/(b^3*(b*c - a*d)*(b*e - a*f)*(1 + n)) + (c^4*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(
d^3*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
```

```
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(b^2c^2+abcd+a^2d^2)(e+fx)^n}{b^3d^3} - \frac{(bc+ad)x(e+fx)^n}{b^2d^2} + \frac{x^2(e+fx)^n}{bd} + \frac{a^4(e+fx)^n}{b^3(bc-ad)(a+bx)} + \frac{c^4(e+fx)^n}{d^3(-bc+ad)(c+dx)}\right) dx$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} + \frac{\int x^2(e+fx)^n\,dx}{bd} + \frac{a^4\int\frac{(e+fx)^n}{a+bx}\,dx}{b^3(bc-ad)} - \frac{c^4\int\frac{(e+fx)^n}{c+dx}\,dx}{d^3(bc-ad)} - \frac{(bc+ad)\int x(e+fx)^n\,dx}{b^2d^2}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{a^4(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^3(bc-ad)(be-af)(1+n)} + \frac{c^4(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{d^3(bc-ad)(de-cf)(1+n)} + \frac{\int\left(\frac{e^2(e+fx)^n}{f^2}-\frac{2e(e+fx)^{1+n}}{f^2}+\frac{(e+fx)^{2+n}}{f^2}\right)dx}{bd} - \frac{(bc+ad)\int\left(-\frac{e(e+fx)^n}{f}+\frac{(e+fx)^{1+n}}{f}\right)dx}{b^2d^2}$$

$$= \frac{e^2(e+fx)^{1+n}}{bdf^3(1+n)} + \frac{(bc+ad)e(e+fx)^{1+n}}{b^2d^2f^2(1+n)} + \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{2e(e+fx)^{2+n}}{bdf^3(2+n)} - \frac{(bc+ad)(e+fx)^{2+n}}{b^2d^2f^2(2+n)} + \frac{(e+fx)^{3+n}}{bdf^3(3+n)} - \frac{a^4(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^3(bc-ad)(be-af)(1+n)} + \frac{c^4(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{d^3(bc-ad)(de-cf)(1+n)}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{a^4d^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)}+\frac{-((bc-ad)(-de+cf)(a^2d^2f^2(6+5n+n^2)+abdf(3+n)(cf(2+n)+d(e-\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((a^4*d^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f
*x))/(b*e - a*f)])/((b*c - a*d)*(b*e - a*f))) + (-((b*c - a*d)*(-(d*e) + c*
f)*(a^2*d^2*f^2*(6 + 5*n + n^2) + a*b*d*f*(3 + n)*(c*f*(2 + n) + d*(e - f*(
1 + n)*x)) + b^2*(c^2*f^2*(6 + 5*n + n^2) + c*d*f*(3 + n)*(e - f*(1 + n)*x)
 + d^2*(2*e^2 - 2*e*f*(1 + n)*x + f^2*(2 + 3*n + n^2)*x^2)))) + b^3*c^4*f^3
*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*
f)])/((-(b*c) + a*d)*f^3*(-(d*e) + c*f)*(2 + n)*(3 + n))))/(b^3*d^3*(1 + n)
)
```

Maple [F]

$$\int \frac{x^4(fx+e)^n}{(bx+a)\,(dx+c)}dx$$

```
[In] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^4}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**4*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^4}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^4}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^4\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.113 $\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 216

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = -\frac{e(e+fx)^{1+n}}{bdf^2(1+n)} - \frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{(e+fx)^{2+n}}{bdf^2(2+n)} + \frac{a^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)} - \frac{c^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{d^2(bc-ad)(de-cf)(1+n)}$$

```
[Out] -e*(f*x+e)^(1+n)/b/d/f^2/(1+n)-(a*d+b*c)*(f*x+e)^(1+n)/b^2/d^2/f/(1+n)+(f*x
+e)^(2+n)/b/d/f^2/(2+n)+a^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e
)/(-a*f+b*e))/b^2/(-a*d+b*c)/(-a*f+b*e)/(1+n)-c^3*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {186, 45, 70}

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{a^3(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{b^2(n+1)(bc-ad)(be-af)} - \frac{(ad+bc)(e+fx)^{n+1}}{b^2d^2f(n+1)} - \frac{c^3(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{d^2(n+1)(bc-ad)(de-cf)} - \frac{e(e+fx)^{n+1}}{bdf^2(n+1)} + \frac{(e+fx)^{n+2}}{bdf^2(n+2)}$$

```
[In] Int[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*d*f^2*(1 + n))) - ((b*c + a*d)*(e + f*x)^(1 + n)
)/(b^2*d^2*f*(1 + n)) + (e + f*x)^(2 + n)/(b*d*f^2*(2 + n)) + (a^3*(e + f*x
)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b
^2*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (c^3*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*(b*c - a*d)*(d*e - c*
f)*(1 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(-bc-ad)(e+fx)^n}{b^2d^2} + \frac{x(e+fx)^n}{bd} - \frac{a^3(e+fx)^n}{b^2(bc-ad)(a+bx)} \right. \\
&\qquad \left. - \frac{c^3(e+fx)^n}{d^2(-bc+ad)(c+dx)} \right) dx \\
&= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{\int x(e+fx)^n\,dx}{bd} - \frac{a^3\int \frac{(e+fx)^n}{a+bx}\,dx}{b^2(bc-ad)} + \frac{c^3\int \frac{(e+fx)^n}{c+dx}\,dx}{d^2(bc-ad)} \\
&= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)} \\
&\quad - \frac{c^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{d^2(bc-ad)(de-cf)(1+n)} + \frac{\int \left(-\frac{e(e+fx)^n}{f} + \frac{(e+fx)^{1+n}}{f}\right)dx}{bd} \\
&= -\frac{e(e+fx)^{1+n}}{bdf^2(1+n)} - \frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{(e+fx)^{2+n}}{bdf^2(2+n)} \\
&\quad + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)} \\
&\quad - \frac{c^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{d^2(bc-ad)(de-cf)(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$$

$$= \frac{(e+fx)^{1+n}\left(\frac{a^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{be-af} + \frac{(bc-ad)(-de+cf)(bcf(2+n)+adf(2+n)+bd(e-f(1+n)x))-b^2c^3f^2(2+n)}{d^2f^2(de-cf)(2+n)}\right.}{b^2(bc-ad)(1+n)}$$

```
[In] Integrate[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((a^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(b*e - a*f) + ((b*c - a*d)*(-(d*e) + c*f)*(b*c*f*(2 + n) + a*d
*f*(2 + n) + b*d*(e - f*(1 + n)*x)) - b^2*c^3*f^2*(2 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*f^2*(d*e - c*f)*(2 + n))
))/(b^2*(b*c - a*d)*(1 + n))
```

Maple [F]

$$\int \frac{x^3 (fx+e)^n}{(bx+a)\,(dx+c)}dx$$

```
[In] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
[Out] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^3/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^3(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

```
[In] integrate(x**3*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral(x**3*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^3\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.114 $\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{(e+fx)^{1+n}}{bdf(1+n)} - \frac{a^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{b(bc-ad)(be-af)(1+n)} + \frac{c^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{d(bc-ad)(de-cf)(1+n)}$$

```
[Out] (f*x+e)^(1+n)/b/d/f/(1+n)-a^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x
+e)/(-a*f+b*e))/b/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^2*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {186, 70}

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = -\frac{a^2(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{b(n+1)(bc-ad)(be-af)} + \frac{c^2(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{d(n+1)(bc-ad)(de-cf)} + \frac{(e+fx)^{n+1}}{bdf(n+1)}$$

```
[In] Int[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*d*f*(1 + n)) - (a^2*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b*(b*c - a*d)*(b*e - a*f)*(
1 + n)) + (c^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e +
 f*x))/(d*e - c*f)])/(d*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(e+fx)^n}{bd} + \frac{a^2(e+fx)^n}{b(bc-ad)(a+bx)} + \frac{c^2(e+fx)^n}{d(-bc+ad)(c+dx)}\right) dx \\
&= \frac{(e+fx)^{1+n}}{bdf(1+n)} + \frac{a^2 \int \frac{(e+fx)^n}{a+bx} \, dx}{b(bc-ad)} - \frac{c^2 \int \frac{(e+fx)^n}{c+dx} \, dx}{d(bc-ad)} \\
&= \frac{(e+fx)^{1+n}}{bdf(1+n)} - \frac{a^2(e+fx)^{1+n} \, {}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b(bc-ad)(be-af)(1+n)} \\
&\quad + \frac{c^2(e+fx)^{1+n} \, {}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{d(bc-ad)(de-cf)(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)} \, dx$$
$$= \frac{(e+fx)^{1+n} \left(a^2df(-de+cf) \operatorname{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right) + (be-af)\left(-((bc-ad)(-\right.\right.}{bd(bc-ad)f(be-af)(de-cf)(1+n)}$$

```
[In] Integrate[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(a^2*d*f*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 +
n, (b*(e + f*x))/(b*e - a*f)] + (b*e - a*f)*(-((b*c - a*d)*(-(d*e) + c*f))
+ b*c^2*f*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])))/
(b*d*(b*c - a*d)*f*(b*e - a*f)*(d*e - c*f)*(1 + n))
```

Maple [F]

$$\int \frac{x^2(fx+e)^n}{(bx+a)\,(dx+c)}dx$$

```
[In] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
[Out] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^2}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^2/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^2(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

```
[In] integrate(x**2*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral(x**2*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^2}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^2}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^2\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.115 $\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)} - \frac{c(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

```
[Out] a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)/(
-a*f+b*e)/(1+n)-c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*
e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {162, 70}

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{a(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)} - \frac{c(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}$$

```
[In] Int[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e
- a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n)) - (c*(e + f*x)^(1 + n)*Hypergeom
```

```
etric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e - c
*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rubi steps

$$\text{integral} = -\frac{a\int\frac{(e+fx)^n}{a+bx}\,dx}{bc-ad}+\frac{c\int\frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}$$
$$=\frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)}-\frac{c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int\frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx$$
$$=\frac{(e+fx)^{1+n}\left(a(-de+cf)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)+c(be-af)\,\text{Hypergeometric2F}\right.}{(bc-ad)(be-af)(-de+cf)(1+n)}$$

```
[In] Integrate[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(a*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*
(e + f*x))/(b*e - a*f)] + c*(b*e - a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

Maple [F]

$$\int \frac{x(fx+e)^n}{(bx+a)\,(dx+c)}dx$$

[In] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)

[Out] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)

Fricas [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral((f*x + e)^n*x/(b*d*x^2 + a*c + (b*c + a*d)*x), x)

Sympy [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

[In] integrate(x*(f*x+e)**n/(b*x+a)/(d*x+c),x)

[Out] Integral(x*(e + f*x)**n/((a + b*x)*(c + d*x)), x)

Maxima [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)

Giac [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.116 $\int \frac{(e+fx)^n}{(a+bx)(c+dx)} dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)} dx = -\frac{b(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)} + \frac{d(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

```
[Out] -b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)/
(-a*f+b*e)/(1+n)+d*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d
*e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {88, 70}

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)} dx = \frac{d(e+fx)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)} - \frac{b(e+fx)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}$$

```
[In] Int[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n))) + (d*(e + f*x)^(1 + n)*Hyperg
```

```
eometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e
- c*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{(e+fx)^n}{a+bx}\,dx}{bc-ad} - \frac{d\int \frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}$$

$$= -\frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)} + \frac{d(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx$$

$$= \frac{(e+fx)^{1+n}\left(b(de-cf)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)+d(-be+af)\,\text{Hypergeometric2F}\right.}{(bc-ad)(be-af)(-de+cf)(1+n)}$$

```
[In] Integrate[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(b*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
+ f*x))/(b*e - a*f)] + d*(-(b*e) + a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

Maple [F]

$$\int \frac{(fx+e)^n}{(bx+a)\,(dx+c)}dx$$

```
[In] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
[Out] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral((f*x + e)^n/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

```
[In] integrate((f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral((e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((e + f*x)^n/((a + b*x)*(c + d*x)),x)
[Out] int((e + f*x)^n/((a + b*x)*(c + d*x)), x)
```

3.117 $\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \frac{b^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{a(bc-ad)(be-af)(1+n)} - \frac{d^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{c(bc-ad)(de-cf)(1+n)} - \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{ace(1+n)}$$

```
[Out] b^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/a/(-a*d+b*
c)/(-a*f+b*e)/(1+n)-d^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-
c*f+d*e))/c/(-a*d+b*c)/(-c*f+d*e)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2
+n],1+f*x/e)/a/c/e/(1+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {186, 67, 70}

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \frac{b^2(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{a(n+1)(bc-ad)(be-af)} - \frac{d^2(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{c(n+1)(bc-ad)(de-cf)} - \frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{ace(n+1)}$$

```
[In] Int[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] (b^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/(a*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (d^2*(e + f*x)^(1 + n)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c*(b*c - a*d)*
(d*e - c*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n
, 1 + (f*x)/e])/(a*c*e*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(e+fx)^n}{acx}+\frac{b^2(e+fx)^n}{a(-bc+ad)(a+bx)}+\frac{d^2(e+fx)^n}{c(bc-ad)(c+dx)}\right)dx\\
&= \frac{\int \frac{(e+fx)^n}{x}\,dx}{ac}-\frac{b^2\int \frac{(e+fx)^n}{a+bx}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{(e+fx)^n}{c+dx}\,dx}{c(bc-ad)}\\
&= \frac{b^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a(bc-ad)(be-af)(1+n)}\\
&\quad -\frac{d^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{c(bc-ad)(de-cf)(1+n)}\\
&\quad -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ace(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = -\frac{(e+fx)^{1+n}\left(b^2ce(de-cf)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)+(-be+af)\left(ad^2e\,\text{Hypergeo}\right.\right.}{ac(-bc+ad)e(-be}$$

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] -(((e + f*x)^(1 + n)*(b^2*c*e*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n
, (b*(e + f*x))/(b*e - a*f)] + (-(b*e) + a*f)*(a*d^2*e*Hypergeometric2F1[1,
 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)] - (b*c - a*d)*(-(d*e) + c*f)*Hype
rgeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])))/(a*c*(-(b*c) + a*d)*e*(-(b*e
) + a*f)*(-(d*e) + c*f)*(1 + n)))
```

Maple [F]

$$\int \frac{(fx+e)^n}{x\,(bx+a)\,(dx+c)}dx$$

```
[In] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(e+fx)^n}{x\,(a+bx)\,(c+dx)}\,dx$$

```
[In] integrate((f*x+e)**n/x/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral((e + f*x)**n/(x*(a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(e+f\,x)^n}{x\,(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((e + f*x)^n/(x*(a + b*x)*(c + d*x)),x)
[Out] int((e + f*x)^n/(x*(a + b*x)*(c + d*x)), x)
```

3.118 $\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\begin{aligned}\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx &= -\frac{b^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)}\\ &\quad+\frac{d^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{c^2(bc-ad)(de-cf)(1+n)}\\ &\quad+\frac{(bc+ad)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{a^2c^2e(1+n)}\\ &\quad+\frac{f(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{fx}{e}\right)}{ace^2(1+n)}\end{aligned}$$

```
[Out] -b^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/a^2/(-a*d
+b*c)/(-a*f+b*e)/(1+n)+d^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)
/(-c*f+d*e))/c^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)+(a*d+b*c)*(f*x+e)^(1+n)*hyperg
eom([1, 1+n],[2+n],1+f*x/e)/a^2/c^2/e/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1
+n],[2+n],1+f*x/e)/a/c/e^2/(1+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {186, 67, 70}

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$$
$$= -\frac{b^3(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{a^2(n+1)(bc-ad)(be-af)} + \frac{(ad+bc)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{a^2c^2e(n+1)} + \frac{d^3(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{c^2(n+1)(bc-ad)(de-cf)} + \frac{f(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{fx}{e}+1\right)}{ace^2(n+1)}$$

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

```
[Out] -((b^3*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f)*(1 + n))) + (d^3*(e + f*x)^(1 + n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c^2*(b*c
- a*d)*(d*e - c*f)*(1 + n)) + ((b*c + a*d)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*c^2*e*(1 + n)) + (f*(e + f*x)^(1 +
n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*c*e^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(e+fx)^n}{acx^2} + \frac{(-bc-ad)(e+fx)^n}{a^2c^2x} - \frac{b^3(e+fx)^n}{a^2(-bc+ad)(a+bx)} - \frac{d^3(e+fx)^n}{c^2(bc-ad)(c+dx)}\right) dx$$

$$= \frac{\int \frac{(e+fx)^n}{x^2} dx}{ac} + \frac{b^3 \int \frac{(e+fx)^n}{a+bx} dx}{a^2(bc-ad)} - \frac{d^3 \int \frac{(e+fx)^n}{c+dx} dx}{c^2(bc-ad)} - \frac{(bc+ad) \int \frac{(e+fx)^n}{x} dx}{a^2c^2}$$

$$= -\frac{b^3(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)} + \frac{d^3(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{c^2(bc-ad)(de-cf)(1+n)} + \frac{(bc+ad)(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2c^2e(1+n)} + \frac{f(e+fx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ace^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.80

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)} dx = \frac{(e+fx)^{1+n}\left(-\frac{b^3 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)} + \frac{-\frac{d^3 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(-de+cf)} + \frac{(bc+ad)e \text{Hypergeometric2F1}(}{}}{c^2}\right.}{1+n}$$

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((b^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))
/(b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f))) + (-((d^3*Hypergeometric2F1[1
, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(-(d*e) + c*f))) +
 ((b*c + a*d)*e*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e] + a*c*f*Hyp
ergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*e^2))/c^2))/(1 + n)
```

Maple [F]

$$\int \frac{(fx+e)^n}{x^2\,(bx+a)\,(dx+c)}dx$$

```
[In] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^4 + a*c*x^2 + (b*c + a*d)*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**n/x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \frac{(e+f\,x)^n}{x^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

[In] int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x)

[Out] int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)), x)

3.119 $\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 167

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)} + \frac{(3a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh))\,(a+bx)^{2+m}}{b^4(2+m)} - \frac{(3adfh-b(dfg+deh+cfh))(a+bx)^{3+m}}{b^4(3+m)} + \frac{dfh(a+bx)^{4+m}}{b^4(4+m)}$$

```
[Out] (-a*d+b*c)*(-a*f+b*e)*(-a*h+b*g)*(b*x+a)^(1+m)/b^4/(1+m)+(3*a^2*d*f*h+b^2*(
c*e*h+c*f*g+d*e*g)-2*a*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(2+m)/b^4/(2+m)-(3*a*
d*f*h-b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(3+m)/b^4/(3+m)+d*f*h*(b*x+a)^(4+m)/b^
4/(4+m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {147}

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \frac{(a+bx)^{m+2}\,(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg))}{b^4(m+2)} + \frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)} - \frac{(a+bx)^{m+3}(3adfh-b(cfh+deh+dfg))}{b^4(m+3)} + \frac{dfh(a+bx)^{m+4}}{b^4(m+4)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(1 + m))/(b^4*(1 + m)) + ((3
*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(
a + b*x)^(2 + m))/(b^4*(2 + m)) - ((3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*
(a + b*x)^(3 + m))/(b^4*(3 + m)) + (d*f*h*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^m}{b^3} + \frac{(3a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh))\,(a+bx)^{1+m}}{b^3} + \frac{(-3adfh+b(dfg+deh+cfh))(a+bx)^{2+m}}{b^3} + \frac{dfh(a+bx)^{3+m}}{b^3}\right) dx \\
&= \frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)} + \frac{(3a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh))\,(a+bx)^{2+m}}{b^4(2+m)} - \frac{(3adfh-b(dfg+deh+cfh))(a+bx)^{3+m}}{b^4(3+m)} + \frac{dfh(a+bx)^{4+m}}{b^4(4+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}\left(\frac{(bc-ad)(be-af)(bg-ah)}{1+m} + \frac{(3a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh))(a+bx)}{2+m} + \frac{(-3adfh+b(dfg+deh+cfh))(a+bx}{3+m}\right.}{b^4}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(((b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(1 + m) + ((3*a^2
*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(a +
b*x))/(2 + m) + ((-3*a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(a + b*x)^2)/(3 +
 m) + (d*f*h*(a + b*x)^3)/(4 + m)))/b^4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(167) = 334$.

Time = 1.62 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.35

method	result
gosper	$-\frac{(bx+a)^{1+m}\left(-b^3dfh\,m^3x^3-b^3cfh\,m^3x^2-b^3deh\,m^3x^2-b^3dfg\,m^3x^2-6b^3dfh\,m^2x^3+3a\,b^2dfh\,m^2x^2-b^3ceh\,m^3x-b^3cfg\,m^3\right.}{}$
norman	$\frac{(adfhm+bcfhm+bdehm+bdfgm+4bcfh+4bdeh+4bdfg)x^3e^{m\ln(bx+a)}}{b(m^2+7m+12)}+\frac{(a\,b^2ceh\,m^3+a\,b^2cfg\,m^3+a\,b^2deg\,m^3+b^3ceg\,m^3-}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(b*x+a)^(1+m)/(m^4+10*m^3+35*m^2+50*m+24)*(-b^3*d*f*h*m^3*x^3-b^3*c*
f*h*m^3*x^2-b^3*d*e*h*m^3*x^2-b^3*d*f*g*m^3*x^2-6*b^3*d*f*h*m^2*x^3+3*a*b^2
*d*f*h*m^2*x^2-b^3*c*e*h*m^3*x-b^3*c*f*g*m^3*x-7*b^3*c*f*h*m^2*x^2-b^3*d*e*
g*m^3*x-7*b^3*d*e*h*m^2*x^2-7*b^3*d*f*g*m^2*x^2-11*b^3*d*f*h*m*x^3+2*a*b^2*
c*f*h*m^2*x+2*a*b^2*d*e*h*m^2*x+2*a*b^2*d*f*g*m^2*x+9*a*b^2*d*f*h*m*x^2-b^3
*c*e*g*m^3-8*b^3*c*e*h*m^2*x-8*b^3*c*f*g*m^2*x-14*b^3*c*f*h*m*x^2-8*b^3*d*e
*g*m^2*x-14*b^3*d*e*h*m*x^2-14*b^3*d*f*g*m*x^2-6*b^3*d*f*h*x^3-6*a^2*b*d*f*
h*m*x+a*b^2*c*e*h*m^2+a*b^2*c*f*g*m^2+10*a*b^2*c*f*h*m*x+a*b^2*d*e*g*m^2+10
*a*b^2*d*e*h*m*x+10*a*b^2*d*f*g*m*x+6*a*b^2*d*f*h*x^2-9*b^3*c*e*g*m^2-19*b^
3*c*e*h*m*x-19*b^3*c*f*g*m*x-8*b^3*c*f*h*x^2-19*b^3*d*e*g*m*x-8*b^3*d*e*h*x
^2-8*b^3*d*f*g*x^2-2*a^2*b*c*f*h*m-2*a^2*b*d*e*h*m-2*a^2*b*d*f*g*m-6*a^2*b*
d*f*h*x+7*a*b^2*c*e*h*m+7*a*b^2*c*f*g*m+8*a*b^2*c*f*h*x+7*a*b^2*d*e*g*m+8*a
*b^2*d*e*h*x+8*a*b^2*d*f*g*x-26*b^3*c*e*g*m-12*b^3*c*e*h*x-12*b^3*c*f*g*x-1
2*b^3*d*e*g*x+6*a^3*d*f*h-8*a^2*b*c*f*h-8*a^2*b*d*e*h-8*a^2*b*d*f*g+12*a*b^
2*c*e*h+12*a*b^2*c*f*g+12*a*b^2*d*e*g-24*b^3*c*e*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(167) = 334$.

Time = 0.26 (sec) , antiderivative size = 877, normalized size of antiderivative = 5.25

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \frac{(ab^3cegm^3+(b^4dfhm^3+6\,b^4dfhm^2+11\,b^4dfhm+6\,b^4dfh)x^4+(8\,b^4dfg+(b^4dfg+(b^4de+(b^4c+ab^3d}{}$$

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*e*g*m^3 + (b^4*d*f*h*m^3 + 6*b^4*d*f*h*m^2 + 11*b^4*d*f*h*m + 6*b^
4*d*f*h)*x^4 + (8*b^4*d*f*g + (b^4*d*f*g + (b^4*d*e + (b^4*c + a*b^3*d)*f)*
```

```
h)*m^3 + (7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + 3*a*b^3*d)*f)*h)*m^2 + 8*(b
^4*d*e + b^4*c*f)*h + 2*(7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + a*b^3*d)*f)*
h)*m)*x^3 - (a^2*b^2*c*e*h + (a^2*b^2*c*f - (9*a*b^3*c - a^2*b^2*d)*e)*g)*m
^2 + (12*b^4*c*e*h + ((b^4*d*e + (b^4*c + a*b^3*d)*f)*g + (a*b^3*c*f + (b^4
*c + a*b^3*d)*e)*h)*m^3 + ((8*b^4*d*e + (8*b^4*c + 5*a*b^3*d)*f)*g + ((8*b^
4*c + 5*a*b^3*d)*e + (5*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m^2 + 12*(b^4*d*e + b^
4*c*f)*g + ((19*b^4*d*e + (19*b^4*c + 4*a*b^3*d)*f)*g + ((19*b^4*c + 4*a*b^
3*d)*e + (4*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m)*x^2 + 4*(3*(2*a*b^3*c - a^2*b^2
*d)*e - (3*a^2*b^2*c - 2*a^3*b*d)*f)*g - 2*(2*(3*a^2*b^2*c - 2*a^3*b*d)*e -
 (4*a^3*b*c - 3*a^4*d)*f)*h + (((26*a*b^3*c - 7*a^2*b^2*d)*e - (7*a^2*b^2*c
 - 2*a^3*b*d)*f)*g + (2*a^3*b*c*f - (7*a^2*b^2*c - 2*a^3*b*d)*e)*h)*m + (24
*b^4*c*e*g + (a*b^3*c*e*h + (a*b^3*c*f + (b^4*c + a*b^3*d)*e)*g)*m^3 + (((9
*b^4*c + 7*a*b^3*d)*e + (7*a*b^3*c - 2*a^2*b^2*d)*f)*g - (2*a^2*b^2*c*f - (
7*a*b^3*c - 2*a^2*b^2*d)*e)*h)*m^2 + 2*(((13*b^4*c + 6*a*b^3*d)*e + 2*(3*a*
b^3*c - 2*a^2*b^2*d)*f)*g + (2*(3*a*b^3*c - 2*a^2*b^2*d)*e - (4*a^2*b^2*c -
 3*a^3*b*d)*f)*h)*m)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50
*b^4*m + 24*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8221 vs. $2(160) = 320$.

Time = 1.74 (sec) , antiderivative size = 8221, normalized size of antiderivative = 49.23

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] Piecewise((a**m*(c*e*g*x + c*e*h*x**2/2 + c*f*g*x**2/2 + c*f*h*x**3/3 + d*e
*g*x**2/2 + d*e*h*x**3/3 + d*f*g*x**3/3 + d*f*h*x**4/4), Eq(b, 0)), (6*a**3
*d*f*h*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7
*x**3) + 11*a**3*d*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3) - 2*a**2*b*c*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
 6*b**7*x**3) - 2*a**2*b*d*e*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) - 2*a**2*b*d*f*g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*f*h*x*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d*f*h*x/(6*a**3*b**4
+ 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*e*h/(6*a**3*b**
4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*f*g/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*c*f*h*x/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*d*e*g/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*d*e
*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b*
*2*d*f*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*f*h*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
```

*x**2 + 6*b**7*x**3) + 18*a*b**2*d*f*h*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c*e*g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*e*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*f*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*c*f*h*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*d*e*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*e*h*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*f*g*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*f*h*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(m, -4)), (-6*a**3*d*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*c*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*c*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*e*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*e*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*f*g*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*f*g/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*f*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*f*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*b**2*c*e*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*b**2*c*f*g/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*b**2*d*e*g/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*e*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*f*h*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c*e*g/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*f*g*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*c*f*h*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*d*e*g*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*e*h*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*g*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*h*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2), Eq(m, -3)), (6*a**3*d*f*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*e*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*f*g/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*f*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*e*h/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*f*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*d*e*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*d*e*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*d*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b

```
**5*x) - 4*a*b**2*d*f*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*f
*h*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c*e*g/(2*a*b**4 + 2*b**5*x) + 2*b**3
*c*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*g*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*h*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e
*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e*h*x**2/(2*a*b**4 + 2*b
**5*x) + 2*b**3*d*f*g*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d*f*h*x**3/(2*a*b**
4 + 2*b**5*x), Eq(m, -2)), (-a**3*d*f*h*log(a/b + x)/b**4 + a**2*c*f*h*log(
a/b + x)/b**3 + a**2*d*e*h*log(a/b + x)/b**3 + a**2*d*f*g*log(a/b + x)/b**3
 + a**2*d*f*h*x/b**3 - a*c*e*h*log(a/b + x)/b**2 - a*c*f*g*log(a/b + x)/b**
2 - a*c*f*h*x/b**2 - a*d*e*g*log(a/b + x)/b**2 - a*d*e*h*x/b**2 - a*d*f*g*x
/b**2 - a*d*f*h*x**2/(2*b**2) + c*e*g*log(a/b + x)/b + c*e*h*x/b + c*f*g*x/
b + c*f*h*x**2/(2*b) + d*e*g*x/b + d*e*h*x**2/(2*b) + d*f*g*x**2/(2*b) + d*
f*h*x**3/(3*b), Eq(m, -1)), (-6*a**4*d*f*h*(a + b*x)**m/(b**4*m**4 + 10*b**
4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*c*f*h*m*(a + b*x)**
m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*
b*c*f*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
 24*b**4) + 2*a**3*b*d*e*h*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*b*d*e*h*(a + b*x)**m/(b**4*m**4 + 1
0*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*d*f*g*m*(a + b
*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*
a**3*b*d*f*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**
4*m + 24*b**4) + 6*a**3*b*d*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
+ 35*b**4*m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*e*h*m**2*(a + b*x)**m/(
b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**
2*c*e*h*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) - 12*a**2*b**2*c*e*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
5*b**4*m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*f*g*m**2*(a + b*x)**m/(b**
4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*c
*f*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
24*b**4) - 12*a**2*b**2*c*f*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b**2*c*f*h*m**2*x*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*
c*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) - a**2*b**2*d*e*g*m**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 +
 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*d*e*g*m*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 12*a**2*b**2
*d*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
24*b**4) - 2*a**2*b**2*d*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
+ 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*d*e*h*m*x*(a + b*x)**m/
(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b*
*2*d*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) - 8*a**2*b**2*d*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4
*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d*f*h*m**2*x**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 - 3*a**2*b**2*d*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
```

```
4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*g*m**3*(a + b*x)**m/(b**4*m**4 +
 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c*e*g*m**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 + 26*a*b**3*c*e*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 24*a*b**3*c*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*
m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*h*m**3*x*(a + b*x)*
*m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b*
*3*c*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 12*a*b**3*c*e*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m
**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*g*m**3*x*(a + b*x)**
m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b**
3*c*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b
**4*m + 24*b**4) + 12*a*b**3*c*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m*
*3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*h*m**3*x**2*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 5*a*b
**3*c*f*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
 50*b**4*m + 24*b**4) + 4*a*b**3*c*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*g*m**3*x*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7
*a*b**3*d*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 12*a*b**3*d*e*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b
**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*h*m**3*x**2*(a
+ b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
 5*a*b**3*d*e*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*e*h*m*x**2*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*g*m**3*x
**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + 5*a*b**3*d*f*g*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
5*b**4*m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*f*g*m*x**2*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*h
*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*
m + 24*b**4) + 3*a*b**3*d*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m
**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a*b**3*d*f*h*m*x**3*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*
c*e*g*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**
4*m + 24*b**4) + 9*b**4*c*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*b**4*c*e*g*m*x*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b**4*c*e*
g*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + b**4*c*e*h*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*e*h*m**2*x**2*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*c*e*h*m
*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 12*b**4*c*e*h*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*f*g*m**3*x**2*(a + b*x)**m/(b**4*m*
```

```
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*g*m**2
*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 19*b**4*c*f*g*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*
b**4*m**2 + 50*b**4*m + 24*b**4) + 12*b**4*c*f*g*x**2*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*f*h*m**3*x
**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + 7*b**4*c*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*
b**4*m**2 + 50*b**4*m + 24*b**4) + 14*b**4*c*f*h*m*x**3*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*h*x*
*3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + b**4*d*e*g*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*e*g*m**2*x**2*(a + b*x)**m/(b**4*m
**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*d*e*g*m*
x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 12*b**4*d*e*g*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + b**4*d*e*h*m**3*x**3*(a + b*x)**m/(b**4*m**
4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*b**4*d*e*h*m**2*
x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 14*b**4*d*e*h*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*e*h*x**3*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*d*f*g*m**3*x**
3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b*
*4) + 7*b**4*d*f*g*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 14*b**4*d*f*g*m*x**3*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*f*g*x**3
*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**
4) + b**4*d*f*h*m**3*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d*f*h*m**2*x**4*(a + b*x)**m/(b**4*m**
4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 11*b**4*d*f*h*m*x*
*4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 6*b**4*d*f*h*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(167) = 334$.

Time = 0.23 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.84

$$\begin{aligned}
&\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx \\
&= \frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m deg}{(m^2+3m+2)b^2} + \frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m cfg}{(m^2+3m+2)b^2} \\
&+ \frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m ceh}{(m^2+3m+2)b^2} + \frac{(bx+a)^{m+1}ceg}{b(m+1)} \\
&+ \frac{((m^2+3m+2)b^3x^3+(m^2+m)ab^2x^2-2a^2bmx+2a^3)(bx+a)^m dfg}{(m^3+6m^2+11m+6)b^3} \\
&+ \frac{((m^2+3m+2)b^3x^3+(m^2+m)ab^2x^2-2a^2bmx+2a^3)(bx+a)^m deh}{(m^3+6m^2+11m+6)b^3} \\
&+ \frac{((m^2+3m+2)b^3x^3+(m^2+m)ab^2x^2-2a^2bmx+2a^3)(bx+a)^m cfh}{(m^3+6m^2+11m+6)b^3} \\
&+ \frac{((m^3+6m^2+11m+6)b^4x^4+(m^3+3m^2+2m)ab^3x^3-3(m^2+m)a^2b^2x^2+6a^3bmx-6a^4)(bx+}{(m^4+10m^3+35m^2+50m+24)b^4}
\end{aligned}$$

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d*e*g/((m^2 + 3*m + 2)*b^2) +
 (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*f*g/((m^2 + 3*m + 2)*b^2)
+ (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*e*h/((m^2 + 3*m + 2)*b^2)
 + (b*x + a)^(m + 1)*c*e*g/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 +
m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d*f*g/((m^3 + 6*m^2 + 11*m
+ 6)*b^3) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x +
2*a^3)*(b*x + a)^m*d*e*h/((m^3 + 6*m^2 + 11*m + 6)*b^3) + ((m^2 + 3*m + 2)*
b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*f*h/((m^
3 + 6*m^2 + 11*m + 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m
^2 + 2*m)*a*b^3*x^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x +
 a)^m*d*f*h/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1626 vs. 2(167) = 334.

Time = 0.28 (sec) , antiderivative size = 1626, normalized size of antiderivative = 9.74

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d*f*h*m^3*x^4 + (b*x + a)^m*b^4*d*f*g*m^3*x^3 + (b*x + a)^
m*b^4*d*e*h*m^3*x^3 + (b*x + a)^m*b^4*c*f*h*m^3*x^3 + (b*x + a)^m*a*b^3*d*f
```

```
*h*m^3*x^3 + 6*(b*x + a)^m*b^4*d*f*h*m^2*x^4 + (b*x + a)^m*b^4*d*e*g*m^3*x^
2 + (b*x + a)^m*b^4*c*f*g*m^3*x^2 + (b*x + a)^m*a*b^3*d*f*g*m^3*x^2 + (b*x
+ a)^m*b^4*c*e*h*m^3*x^2 + (b*x + a)^m*a*b^3*d*e*h*m^3*x^2 + (b*x + a)^m*a*
b^3*c*f*h*m^3*x^2 + 7*(b*x + a)^m*b^4*d*f*g*m^2*x^3 + 7*(b*x + a)^m*b^4*d*e
*h*m^2*x^3 + 7*(b*x + a)^m*b^4*c*f*h*m^2*x^3 + 3*(b*x + a)^m*a*b^3*d*f*h*m^
2*x^3 + 11*(b*x + a)^m*b^4*d*f*h*m*x^4 + (b*x + a)^m*b^4*c*e*g*m^3*x + (b*x
 + a)^m*a*b^3*d*e*g*m^3*x + (b*x + a)^m*a*b^3*c*f*g*m^3*x + (b*x + a)^m*a*b
^3*c*e*h*m^3*x + 8*(b*x + a)^m*b^4*d*e*g*m^2*x^2 + 8*(b*x + a)^m*b^4*c*f*g*
m^2*x^2 + 5*(b*x + a)^m*a*b^3*d*f*g*m^2*x^2 + 8*(b*x + a)^m*b^4*c*e*h*m^2*x
^2 + 5*(b*x + a)^m*a*b^3*d*e*h*m^2*x^2 + 5*(b*x + a)^m*a*b^3*c*f*h*m^2*x^2
- 3*(b*x + a)^m*a^2*b^2*d*f*h*m^2*x^2 + 14*(b*x + a)^m*b^4*d*f*g*m*x^3 + 14
*(b*x + a)^m*b^4*d*e*h*m*x^3 + 14*(b*x + a)^m*b^4*c*f*h*m*x^3 + 2*(b*x + a)
^m*a*b^3*d*f*h*m*x^3 + 6*(b*x + a)^m*b^4*d*f*h*x^4 + (b*x + a)^m*a*b^3*c*e*
g*m^3 + 9*(b*x + a)^m*b^4*c*e*g*m^2*x + 7*(b*x + a)^m*a*b^3*d*e*g*m^2*x + 7
*(b*x + a)^m*a*b^3*c*f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*f*g*m^2*x + 7*(b*x
 + a)^m*a*b^3*c*e*h*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*e*h*m^2*x - 2*(b*x + a)
^m*a^2*b^2*c*f*h*m^2*x + 19*(b*x + a)^m*b^4*d*e*g*m*x^2 + 19*(b*x + a)^m*b^
4*c*f*g*m*x^2 + 4*(b*x + a)^m*a*b^3*d*f*g*m*x^2 + 19*(b*x + a)^m*b^4*c*e*h*
m*x^2 + 4*(b*x + a)^m*a*b^3*d*e*h*m*x^2 + 4*(b*x + a)^m*a*b^3*c*f*h*m*x^2 -
 3*(b*x + a)^m*a^2*b^2*d*f*h*m*x^2 + 8*(b*x + a)^m*b^4*d*f*g*x^3 + 8*(b*x +
 a)^m*b^4*d*e*h*x^3 + 8*(b*x + a)^m*b^4*c*f*h*x^3 + 9*(b*x + a)^m*a*b^3*c*e
*g*m^2 - (b*x + a)^m*a^2*b^2*d*e*g*m^2 - (b*x + a)^m*a^2*b^2*c*f*g*m^2 - (b
*x + a)^m*a^2*b^2*c*e*h*m^2 + 26*(b*x + a)^m*b^4*c*e*g*m*x + 12*(b*x + a)^m
*a*b^3*d*e*g*m*x + 12*(b*x + a)^m*a*b^3*c*f*g*m*x - 8*(b*x + a)^m*a^2*b^2*d
*f*g*m*x + 12*(b*x + a)^m*a*b^3*c*e*h*m*x - 8*(b*x + a)^m*a^2*b^2*d*e*h*m*x
 - 8*(b*x + a)^m*a^2*b^2*c*f*h*m*x + 6*(b*x + a)^m*a^3*b*d*f*h*m*x + 12*(b*
x + a)^m*b^4*d*e*g*x^2 + 12*(b*x + a)^m*b^4*c*f*g*x^2 + 12*(b*x + a)^m*b^4*
c*e*h*x^2 + 26*(b*x + a)^m*a*b^3*c*e*g*m - 7*(b*x + a)^m*a^2*b^2*d*e*g*m -
7*(b*x + a)^m*a^2*b^2*c*f*g*m + 2*(b*x + a)^m*a^3*b*d*f*g*m - 7*(b*x + a)^m
*a^2*b^2*c*e*h*m + 2*(b*x + a)^m*a^3*b*d*e*h*m + 2*(b*x + a)^m*a^3*b*c*f*h*
m + 24*(b*x + a)^m*b^4*c*e*g*x + 24*(b*x + a)^m*a*b^3*c*e*g - 12*(b*x + a)^
m*a^2*b^2*d*e*g - 12*(b*x + a)^m*a^2*b^2*c*f*g + 8*(b*x + a)^m*a^3*b*d*f*g
- 12*(b*x + a)^m*a^2*b^2*c*e*h + 8*(b*x + a)^m*a^3*b*d*e*h + 8*(b*x + a)^m*
a^3*b*c*f*h - 6*(b*x + a)^m*a^4*d*f*h)/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 +
 50*b^4*m + 24*b^4)
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 819, normalized size of antiderivative = 4.90

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$$
$$= \frac{x\,(a+b\,x)^m\,(24\,b^4\,c\,e\,g+9\,b^4\,c\,e\,g\,m^2+b^4\,c\,e\,g\,m^3+26\,b^4\,c\,e\,g\,m+12\,a\,b^3\,c\,e\,h\,m+12\,a\,b^3\,c\,f\,g\,m+12\,a}{}$$
$$-\frac{(a+b\,x)^m\,(6\,a^4\,d\,f\,h+12\,a^2\,b^2\,c\,e\,h+12\,a^2\,b^2\,c\,f\,g+12\,a^2\,b^2\,d\,e\,g-24\,a\,b^3\,c\,e\,g-8\,a^3\,b\,c\,f\,h-8\,a^3\,b\,d}{}$$
$$+\frac{x^3\,(a+b\,x)^m\,(m^2+3\,m+2)\,\left(4\,b\,c\,f\,h+4\,b\,d\,e\,h+4\,b\,d\,f\,g+a\,d\,f\,h\,m+b\,c\,f\,h\,m+b\,d\,e\,h\,m+b\,d\,f\,g}{b\,(m^4+10\,m^3+35\,m^2+50\,m+24)}$$
$$+\frac{x^2\,(m+1)\,\left(a+b\,x\right)^m\,(12\,b^2\,c\,e\,h+12\,b^2\,c\,f\,g+12\,b^2\,d\,e\,g+b^2\,c\,e\,h\,m^2+b^2\,c\,f\,g\,m^2+b^2\,d\,e\,g\,m^2+7\,b^2}{b^2}$$
$$+\frac{d\,f\,h\,x^4\,(a+b\,x)^m\,(m^3+6\,m^2+11\,m+6)}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x),x)
```

```
[Out] (x*(a + b*x)^m*(24*b^4*c*e*g + 9*b^4*c*e*g*m^2 + b^4*c*e*g*m^3 + 26*b^4*c*e
*g*m + 12*a*b^3*c*e*h*m + 12*a*b^3*c*f*g*m + 12*a*b^3*d*e*g*m + 6*a^3*b*d*f
*h*m + 7*a*b^3*c*e*h*m^2 + 7*a*b^3*c*f*g*m^2 + 7*a*b^3*d*e*g*m^2 + a*b^3*c*
e*h*m^3 + a*b^3*c*f*g*m^3 + a*b^3*d*e*g*m^3 - 8*a^2*b^2*c*f*h*m - 8*a^2*b^2
*d*e*h*m - 8*a^2*b^2*d*f*g*m - 2*a^2*b^2*c*f*h*m^2 - 2*a^2*b^2*d*e*h*m^2 -
2*a^2*b^2*d*f*g*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x
)^m*(6*a^4*d*f*h + 12*a^2*b^2*c*e*h + 12*a^2*b^2*c*f*g + 12*a^2*b^2*d*e*g -
 24*a*b^3*c*e*g - 8*a^3*b*c*f*h - 8*a^3*b*d*e*h - 8*a^3*b*d*f*g - 26*a*b^3*
c*e*g*m - 2*a^3*b*c*f*h*m - 2*a^3*b*d*e*h*m - 2*a^3*b*d*f*g*m - 9*a*b^3*c*e
*g*m^2 - a*b^3*c*e*g*m^3 + 7*a^2*b^2*c*e*h*m + 7*a^2*b^2*c*f*g*m + 7*a^2*b^
2*d*e*g*m + a^2*b^2*c*e*h*m^2 + a^2*b^2*c*f*g*m^2 + a^2*b^2*d*e*g*m^2))/(b^
4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(3*m + m^2 + 2)*(
4*b*c*f*h + 4*b*d*e*h + 4*b*d*f*g + a*d*f*h*m + b*c*f*h*m + b*d*e*h*m + b*d
*f*g*m))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^2*(m + 1)*(a + b*x)^m
*(12*b^2*c*e*h + 12*b^2*c*f*g + 12*b^2*d*e*g + b^2*c*e*h*m^2 + b^2*c*f*g*m^
2 + b^2*d*e*g*m^2 + 7*b^2*c*e*h*m + 7*b^2*c*f*g*m + 7*b^2*d*e*g*m - 3*a^2*d
*f*h*m + a*b*c*f*h*m^2 + a*b*d*e*h*m^2 + a*b*d*f*g*m^2 + 4*a*b*c*f*h*m + 4*
a*b*d*e*h*m + 4*a*b*d*f*g*m))/(b^2*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (
d*f*h*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m
^4 + 24)
```

3.120 $\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx} \, dx = -\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)} + \frac{(dg-ch)(fg-eh)(a+bx)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{h^2(bg-ah)(1+m)}$$

```
[Out] -(b*x+a)^(1+m)*(a*d*f*h+b*(-c*f*h-d*e*h+d*f*g)*(2+m)-b*d*f*h*(1+m)*x)/b^2/h
^2/(1+m)/(2+m)+(-c*h+d*g)*(-e*h+f*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m]
,-h*(b*x+a)/(-a*h+b*g))/h^2/(-a*h+b*g)/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {152, 70}

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx} \, dx = \frac{(a+bx)^{m+1}(dg-ch)(fg-eh) \text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)} - \frac{(a+bx)^{m+1}(adfh-bh(m+2)(cf+de)+bdfg(m+2)-bdfh(m+1)x)}{b^2h^2(m+1)(m+2)}$$

```
[In] Int[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*d*f*h + b*d*f*g*(2 + m) - b*(d*e + c*f)*h*(2 + m) -
 b*d*f*h*(1 + m)*x))/(b^2*h^2*(1 + m)*(2 + m))) + ((d*g - c*h)*(f*g - e*h)*
(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g -
 a*h))])/(h^2*(b*g - a*h)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+bx)^{1+m}(adfh+bdfg(2+m)-b(de+cf)h(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)} \\ &\quad+\frac{((dg-ch)(fg-eh))\int\frac{(a+bx)^m}{g+hx}\,dx}{h^2} \\ &= -\frac{(a+bx)^{1+m}(adfh+bdfg(2+m)-b(de+cf)h(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)} \\ &\quad+\frac{(dg-ch)(fg-eh)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{h(a+bx)}{bg-ah}\right)}{h^2(bg-ah)(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int\frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx$$

$$=\frac{(a+bx)^{1+m}\left(\frac{-adfh+b(-dfg+deh+cfh)}{b^2(1+m)}+\frac{dfh(a+bx)}{b^2(2+m)}+\frac{(dg-ch)(fg-eh)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{h(a+bx)}{-bg+ah}\right)}{(bg-ah)(1+m)}\right)}{h^2}$$

```
[In] Integrate[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*((-(a*d*f*h) + b*(-(d*f*g) + d*e*h + c*f*h))/(b^2*(1 + m
)) + (d*f*h*(a + b*x))/(b^2*(2 + m)) + ((d*g - c*h)*(f*g - e*h)*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/((b*g - a*h)*(1 + m)
)))/h^2
```

Maple [F]

$$\int \frac{(bx+a)^m (dx+c)(fx+e)}{hx+g}dx$$

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(dx+c)(fx+e)(bx+a)^m}{hx+g}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((d*f*x^2 + c*e + (d*e + c*f)*x)*(b*x + a)^m/(h*x + g), x)
```

Sympy [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(a+bx)^m (c+dx)(e+fx)}{g+hx}\,dx$$

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] Integral((a + b*x)**m*(c + d*x)*(e + f*x)/(g + h*x), x)
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(dx+c)(fx+e)(bx+a)^m}{hx+g}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(dx+c)(fx+e)(bx+a)^m}{hx+g}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(e+f\,x)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)}{g+h\,x}\,dx$$

```
[In] int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x),x)
[Out] int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x), x)
```

3.121 $\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = -\frac{(de-cf)(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)} + \frac{(dg-ch)(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(fg-eh)(1+m)}$$

```
[Out] -(-c*f+d*e)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f*(b*x+a)/(-a*f+b*e))/(
-a*f+b*e)/(-e*h+f*g)/(1+m)+(-c*h+d*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m
],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-e*h+f*g)/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {162, 70}

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{(a+bx)^{m+1}(dg-ch)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)} - \frac{(a+bx)^{m+1}(de-cf)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

```
[In] Int[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] -(((d*e - c*f)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a
 + b*x))/(b*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m))) + ((d*g - c*h)*(
a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g -
a*h))])/((b*g - a*h)*(f*g - e*h)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(de-cf)\int \frac{(a+bx)^m}{e+fx}\,dx}{fg-eh}+\frac{(dg-ch)\int \frac{(a+bx)^m}{g+hx}\,dx}{fg-eh}\\
&= -\frac{(de-cf)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)}\\
&\quad+\frac{(dg-ch)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(fg-eh)(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$
\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{(a+bx)^{1+m}\left(-\frac{(de-cf)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)}{be-af}+\frac{(dg-ch)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{h(a+bx)}{-bg+ah}\right)}{bg-ah}\right)}{(fg-eh)(1+m)}
$$

```
[In] Integrate[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(((d*e - c*f)*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(a
 + b*x))/(-(b*e) + a*f)])/(b*e - a*f)) + ((d*g - c*h)*Hypergeometric2F1[1,
1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/(b*g - a*h)))/((f*g - e*h)*(1
+ m))
```

Maple [F]

$$\int \frac{(bx+a)^m (dx+c)}{(fx+e)(hx+g)}dx$$

[In] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)

[Out] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")

[Out] integral((d*x + c)*(b*x + a)^m/(f*h*x^2 + e*g + (f*g + e*h)*x), x)

Sympy [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(a+bx)^m (c+dx)}{(e+fx)(g+hx)}\,dx$$

[In] integrate((b*x+a)**m*(d*x+c)/(f*x+e)/(h*x+g),x)

[Out] Integral((a + b*x)**m*(c + d*x)/((e + f*x)*(g + h*x)), x)

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")

[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m\,(c+d\,x)}{(e+f\,x)\,(g+h\,x)}\,dx$$

```
[In] int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x)
[Out] int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)), x)
```

3.122 $\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)} dx$

Optimal result

Integrand size = 29, antiderivative size = 224

$$\begin{aligned}\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx &= \frac{d^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)} \\ &\quad - \frac{f^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(de-cf)(fg-eh)(1+m)} \\ &\quad + \frac{h^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(dg-ch)(fg-eh)(1+m)}\end{aligned}$$

```
[Out] d^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c
)/(-c*f+d*e)/(-c*h+d*g)/(1+m)-f^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f
*(b*x+a)/(-a*f+b*e))/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)/(1+m)+h^2*(b*x+a)^(1+
m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-c*h+d*g)/(-
e*h+f*g)/(1+m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used

$= \{186, 70\}$

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \frac{d^2(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)} - \frac{f^2(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)} + \frac{h^2(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(dg-ch)(fg-eh)}$$

```
[In] Int[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*(1 + m)) - (f^2*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f))])
/((b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(1 + m)) + (h^2*(a + b*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g - a*h)
*(d*g - c*h)*(f*g - e*h)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(a+bx)^m}{(de-cf)(dg-ch)(c+dx)} + \frac{f^2(a+bx)^m}{(de-cf)(-fg+eh)(e+fx)} + \frac{h^2(a+bx)^m}{(dg-ch)(fg-eh)(g+hx)}\right) dx$$

$$= \frac{d^2\int \frac{(a+bx)^m}{c+dx}\,dx}{(de-cf)(dg-ch)} - \frac{f^2\int \frac{(a+bx)^m}{e+fx}\,dx}{(de-cf)(fg-eh)} + \frac{h^2\int \frac{(a+bx)^m}{g+hx}\,dx}{(dg-ch)(fg-eh)}$$

$$= \frac{d^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)} - \frac{f^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a+bx)}{be-af}\right)}{(be-af)(de-cf)(fg-eh)(1+m)} + \frac{h^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(dg-ch)(fg-eh)(1+m)}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \frac{(a+bx)^{1+m}\left(\frac{d^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)(-de+cf)(-dg+ch)} + \frac{f^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)}{(be-af)(de-cf)(-fg+eh)} + \frac{h^2\,\text{Hypergeometric2F1}}{(bg-ah)(dg}\right.}{1+m}$$

```
[In] Integrate[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^(1 + m)*((d^2*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(
-(b*c) + a*d)])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) + (f^2*Hypergeo
metric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b*e) + a*f)])/((b*e - a*f)*(d*e
 - c*f)*(-(f*g) + e*h)) + (h^2*Hypergeometric2F1[1, 1 + m, 2 + m, (h*(a + b
*x))/(-(b*g) + a*h)])/((b*g - a*h)*(d*g - c*h)*(f*g - e*h))))/(1 + m)
```

Maple [F]

$$\int \frac{(bx+a)^m}{(dx+c)\,(fx+e)\,(hx+g)}dx$$

```
[In] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*f*h*x^3 + c*e*g + (d*f*g + (d*e + c*f)*h)*x^2 + (c*
e*h + (d*e + c*f)*g)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m/(d*x+c)/(f*x+e)/(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m}{(e+f\,x)\,(g+h\,x)\,(c+d\,x)}\,dx$$

```
[In] int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)),x)
[Out] int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)), x)
```

3.123 $\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{bx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{bx}{a}\right)}{a(bc-ad)(1+m)} - \frac{dx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{dx}{c}\right)}{c(bc-ad)(1+m)}$$

```
[Out] b*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b*x/a,-f*x/e)/a/(-a*d+b*c)/(1+m)
/((1+f*x/e)^n)-d*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-d*x/c,-f*x/e)/c/(
-a*d+b*c)/(1+m)/((1+f*x/e)^n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {186, 140, 138}

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{bx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{bx}{a}\right)}{a(m+1)(bc-ad)} - \frac{dx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{dx}{c}\right)}{c(m+1)(bc-ad)}$$

```
[In] Int[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (b*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b*x)/
a)])/(a*(b*c - a*d)*(1 + m)*(1 + (f*x)/e)^n) - (d*x^(1 + m)*(e + f*x)^n*App
ellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)])/(c*(b*c - a*d)*(1 + m)*
(1 + (f*x)/e)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{bx^m(e+fx)^n}{(bc-ad)(a+bx)} - \frac{dx^m(e+fx)^n}{(bc-ad)(c+dx)}\right) dx \\
&= \frac{b\int \frac{x^m(e+fx)^n}{a+bx}\,dx}{bc-ad} - \frac{d\int \frac{x^m(e+fx)^n}{c+dx}\,dx}{bc-ad} \\
&= \frac{\left(b(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{a+bx}\,dx}{bc-ad} - \frac{\left(d(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{c+dx}\,dx}{bc-ad} \\
&= \frac{bx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(bc-ad)(1+m)} \\
&\quad - \frac{dx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{dx}{c}\right)}{c(bc-ad)(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx$$
$$= \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\left(-bc\,\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{bx}{a}\right)+ad\,\text{AppellF1}\left(1+m,-n,1,2\right.\right.}{ac(-bc+ad)(1+m)}$$

```
[In] Integrate[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*(-(b*c*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((
b*x)/a)]) + a*d*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)]))/(a*
c*(-(b*c) + a*d)*(1 + m)*(1 + (f*x)/e)^n)
```

Maple [F]

$$\int \frac{x^m(fx+e)^n}{(bx+a)\,(dx+c)}dx$$

```
[In] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^m/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^m\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.124 $\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 266

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n)-bdfh(2+m+n)x)}{b^2d^2(2+m+n)(3+m+n)} + \frac{(a^2d^2fh(1+n)(2+n)+abd(1+n)(2cfh(1+m)-d(fg+eh)(3+m+n))+b^2(c^2fh(1+m)(2+m)}{}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(1+n)*(b*c*f*h*(2+m)+a*d*f*h*(2+n)-b*d*(e*h+f*g)*(3+
m+n)-b*d*f*h*(2+m+n)*x)/b^2/d^2/(2+m+n)/(3+m+n)+(a^2*d^2*f*h*(1+n)*(2+n)+a*
b*d*(1+n)*(2*c*f*h*(1+m)-d*(e*h+f*g)*(3+m+n))+b^2*(c^2*f*h*(1+m)*(2+m)-c*d*
(e*h+f*g)*(1+m)*(3+m+n)+d^2*e*g*(2+m+n)*(3+m+n)))*(b*x+a)^(1+m)*(d*x+c)^n*h
ypergeom([-n, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^3/d^2/(1+m)/(2+m+n)/(3+m+
n)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {152, 72, 71}

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)(a^2d^2fh(n+1)(n+2}{} - \frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*h*(2 + m) + a*d*f*h*(2 + n) -
 b*d*(f*g + e*h)*(3 + m + n) - b*d*f*h*(2 + m + n)*x))/(b^2*d^2*(2 + m + n)
*(3 + m + n))) + ((a^2*d^2*f*h*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*h*(1
+ m) - d*(f*g + e*h)*(3 + m + n)) + b^2*(c^2*f*h*(1 + m)*(2 + m) - c*d*(f*g
 + e*h)*(1 + m)*(3 + m + n) + d^2*e*g*(2 + m + n)*(3 + m + n)))*(a + b*x)^(
1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(b*c - a
*d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

integral =

$$-\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n)-bdfh(2+m+n)x)}{b^2d^2(2+m+n)(3+m+n)}$$

$$+\frac{(a^2d^2fh(1+n)(2+n)+abd(1+n)(2cfh(1+m)-d(fg+eh)(3+m+n))+b^2(c^2fh(1+m)(2+m}{b^2d^2(2+m+n)(3}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n)-bdfh(2+m+}{b^2d^2(2+m+n)(3+m+n)}$$

$$+\frac{\Big((a^2d^2fh(1+n)(2+n)+abd(1+n)(2cfh(1+m)-d(fg+eh)(3+m+n))+b^2(c^2fh(1+m}{}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n)-bdfh(2+m+}{b^2d^2(2+m+n)(3+m+n)}$$

$$+\frac{(a^2d^2fh(1+n)(2+n)+abd(1+n)(2cfh(1+m)-d(fg+eh)(3+m+n))+b^2(c^2fh(1+m)}{}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.73

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx$$

$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left((bc-ad)^2fh\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{d(a+bx)}{-bc+ad}\right)+b(}{}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*((b*c - a*d)^2*f*h*Hypergeometric2F1[1 + m,
-2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h -
d*(f*g + e*h))*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (d*(a + b*x))/(-(b*c
) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[1 + m, -n, 2 + m,
(d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*((b*(c + d*x))/(b*c - a*d
))^n)
```

Maple [F]

$$\int (bx+a)^m\,(dx+c)^n\,(fx+e)\,(hx+g)\,dx$$

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx=\int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="fricas")
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx=\int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx=\int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \int (e+f\,x)\,\left(g+h\,x\right)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n\,dx$$

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

3.125 $\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 245

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3bdfhx)}{12b^2d^2} + \frac{(bc-ad)\left(a^2d^2fh(6-5m+m^2)-2abd(2-m)(2d(fg+eh)-cfh(1+m))+b^2(12d^2eg-4cd(fg+\right.}{}$$

```
[Out] 1/12*(b*x+a)^(1+m)*(d*x+c)^(2-m)*(4*b*d*(e*h+f*g)-a*d*f*h*(3-m)-b*c*f*h*(2+
m)+3*b*d*f*h*x)/b^2/d^2+1/12*(-a*d+b*c)*(a^2*d^2*f*h*(m^2-5*m+6)-2*a*b*d*(2
-m)*(2*d*(e*h+f*g)-c*f*h*(1+m))+b^2*(12*d^2*e*g-4*c*d*(e*h+f*g)*(1+m)+c^2*f
*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([1+m, -1+
m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^4/d^2/(1+m)/((d*x+c)^m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {152, 72, 71}

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \frac{(bc-ad)(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m-1,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)(a^2d^2f}{} + \frac{(a+bx)^{m+1}(c+dx)^{2-m}(-adfh(3-m)-bcfh(m+2)+4bd(eh+fg)+3bdfhx)}{12b^2d^2}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(2 - m)*(4*b*d*(f*g + e*h) - a*d*f*h*(3 - m) -
 b*c*f*h*(2 + m) + 3*b*d*f*h*x))/(12*b^2*d^2) + ((b*c - a*d)*(a^2*d^2*f*h*(
6 - 5*m + m^2) - 2*a*b*d*(2 - m)*(2*d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(1
2*d^2*e*g - 4*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)
^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[-1 + m, 1 + m, 2 +
 m, -((d*(a + b*x))/(b*c - a*d))])/(12*b^4*d^2*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule steps

$$\text{integral} = \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3bdfhx)}{12b^2d^2}$$

$$+\frac{(a^2d^2fh(6-5m+m^2)-2abd(2-m)(2d(fg+eh)-cfh(1+m))+b^2(12d^2eg-4cd(fg+eh)(1+m)}{12b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3bdfhx)}{12b^2d^2}$$

$$+\frac{\Big((bc-ad)\left(a^2d^2fh(6-5m+m^2)-2abd(2-m)(2d(fg+eh)-cfh(1+m))+b^2(12d^2eg-4c\right.}{}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3bdfhx)}{12b^2d^2}$$
$$+ \frac{(bc-ad)\left(a^2d^2fh(6-5m+m^2)-2abd(2-m)(2d(fg+eh)-cfh(1+m))+b^2(12d^2eg-4c\right.}{}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.80

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{-1+m}\left((bc-ad)^2fh\,\text{Hypergeometric2F1}\left(-3+m,1+m,2+m,\frac{d(a+bx)}{-bc+a}\right.\right.}{}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*((b*(c + d*x))/(b*c - a*d))^(-1 + m)*(
(b*c - a*d)^2*f*h*Hypergeometric2F1[-3 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(
b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric2F1
[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(d*g
- c*h)*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)
])))/(b^3*d^2*(1 + m))
```

Maple [F]

$$\int (bx+a)^m (dx+c)^{1-m} (fx+e)(hx+g)\,dx$$

```
[In] int((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m+1}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**(1-m)*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m+1}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m+1}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \int (e+f\,x)\,(g+h\,x)\,(a+b\,x)^m\,(c+d\,x)^{1-m}\,dx$$

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m),x)
[Out] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m), x)
```

3.126 $\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 235

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfhx)}{6b^2d^2} + \frac{(a^2d^2fh(2-3m+m^2)-abd(1-m)(3d(fg+eh)-2cfh(1+m))+b^2(6d^2eg-3cd(fg+eh)(1+m)}{6b^3}$$

```
[Out] 1/6*(b*x+a)^(1+m)*(d*x+c)^(1-m)*(3*b*d*(e*h+f*g)-a*d*f*h*(2-m)-b*c*f*h*(2+m
)+2*b*d*f*h*x)/b^2/d^2+1/6*(a^2*d^2*f*h*(m^2-3*m+2)-a*b*d*(1-m)*(3*d*(e*h+f
*g)-2*c*f*h*(1+m))+b^2*(6*d^2*e*g-3*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)
))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([m, 1+m],[2+m],-d*(b*x+
a)/(-a*d+b*c))/b^3/d^2/(1+m)/((d*x+c)^m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {152, 72, 71}

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)(a^2d^2fh(m^2-3m+}{6b^3d^2} + \frac{(a+bx)^{m+1}(c+dx)^{1-m}(-adfh(2-m)-bcfh(m+2)+3bd(eh+fg)+2bdfhx)}{6b^2d^2}$$

```
[In] Int[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*(3*b*d*(f*g + e*h) - a*d*f*h*(2 - m) -
 b*c*f*h*(2 + m) + 2*b*d*f*h*x))/(6*b^2*d^2) + ((a^2*d^2*f*h*(2 - 3*m + m^2
) - a*b*d*(1 - m)*(3*d*(f*g + e*h) - 2*c*f*h*(1 + m)) + b^2*(6*d^2*e*g - 3*
c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*((b*(
c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))
/(b*c - a*d))])/(6*b^3*d^2*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfhx)}{6b^2d^2} + \frac{(a^2d^2fh(2-3m+m^2)-abd(1-m)(3d(fg+eh)-2cfh(1+m))+b^2(6d^2eg-3cd(fg+eh)(1+m)\cdots}{6b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfhx)}{6b^2d^2} + \frac{\Big((a^2d^2fh(2-3m+m^2)-abd(1-m)(3d(fg+eh)-2cfh(1+m))+b^2(6d^2eg-3cd(fg+eh)\cdots}{6b^2d^2\cdots}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfhx)}{6b^2d^2} + \frac{(a^2d^2fh(2-3m+m^2)-abd(1-m)(3d(fg+eh)-2cfh(1+m))+b^2(6d^2eg-3cd(fg+eh)}{6b^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.80

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \left((bc-ad)^2fh\,\text{Hypergeometric2F1}\left(-2+m,1+m,2+m,\frac{d(a+bx)}{-bc+ad}\right)\right.}{}$$

```
[In] Integrate[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*((b*c - a*d)^2*f*h*Hyperge
ometric2F1[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c
- a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (d
*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[
m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*(c + d*x
)^m)
```

Maple [F]

$$\int (bx+a)^m (fx+e)(hx+g)(dx+c)^{-m}\,dx$$

```
[In] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

```
[Out] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m/(d*x + c)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(f*x+e)*(h*x+g)/((d*x+c)**m),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \int \frac{(e+f\,x)\,\,(g+h\,x)\,\,{(a+b\,x)}^m}{{(c+d\,x)}^m}\,dx$$

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m,x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m, x)
```

3.127 $\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 261

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx$$
$$=\frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afhm)+d(bc-ad)fhmx\right)}{2bd^2(bc-ad)m}$$
$$-\frac{(b^2c^2fh(1+m)(2+m)-2bcd(1+m)(bfg+beh+afhm)+d^2(2b^2eg+2ab(fg+eh)m-a^2fh(1-}{2b^2d^2(bc-ad)m(}$$

```
[Out] 1/2*(b*x+a)^(1+m)*(2*b*d^2*e*g+b*c^2*f*h*(2+m)-c*d*(2*b*(e*h+f*g)+a*f*h*m)+
d*(-a*d+b*c)*f*h*m*x)/b/d^2/(-a*d+b*c)/m/((d*x+c)^m)-1/2*(b^2*c^2*f*h*(1+m)
*(2+m)-2*b*c*d*(1+m)*(a*f*h*m+b*e*h+b*f*g)+d^2*(2*b^2*e*g+2*a*b*(e*h+f*g)*m
-a^2*f*h*(1-m)*m))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([m, 1+m
],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/m/(1+m)/((d*x+c)^m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {151, 72, 71}

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx$$
$$=\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}$$
$$-\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)\left(d^2(a^2(-f)h(1-\right.}{2b^2d^2m(m+1)(}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(2*b*d^2*e*g + b*c^2*f*h*(2 + m) - c*d*(2*b*(f*g + e*h)
+ a*f*h*m) + d*(b*c - a*d)*f*h*m*x))/(2*b*d^2*(b*c - a*d)*m*(c + d*x)^m) -
((b^2*c^2*f*h*(1 + m)*(2 + m) - 2*b*c*d*(1 + m)*(b*f*g + b*e*h + a*f*h*m) +
 d^2*(2*b^2*e*g + 2*a*b*(f*g + e*h)*m - a^2*f*h*(1 - m)*m))*(a + b*x)^(1 +
m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a
 + b*x))/(b*c - a*d))])/(2*b^2*d^2*(b*c - a*d)*m*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

integral

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afhm)+d(bc-ad)fhmx\right)}{2bd^2(bc-ad)m} - \frac{(b^2c^2fh(1+m)(2+m)-2bcd(1+m)(bfg+beh+afhm)+d^2(2b^2eg+2ab(fg+eh)m-a^2fh(1-m}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afhm)+d(bc-ad)fhmx\right)}{2bd^2(bc-ad)m}$$
$$- \frac{\left((b^2c^2fh(1+m)(2+m)-2bcd(1+m)(bfg+beh+afhm)+d^2(2b^2eg+2ab(fg+eh)m-a\right.}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afhm)+d(bc-ad)fhmx\right)}{2bd^2(bc-ad)m}$$
$$- \frac{(b^2c^2fh(1+m)(2+m)-2bcd(1+m)(bfg+beh+afhm)+d^2(2b^2eg+2ab(fg+eh)m-a^2}{2b^2d^2(bc-ad)m(1}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.85

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(b(adfhm(c+dx)-b(2d^2eg+c^2fh(2+m)+cd(-2fg-2eh+fhmx)))+\frac{(a^2}{}\right.}{2b}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(b*(a*d*f*h*m*(c + d*x) - b*(2*d^2*e*g + c^2*f*h*(2 + m)
 + c*d*(-2*f*g - 2*e*h + f*h*m*x))) + ((a^2*d^2*f*h*(-1 + m)*m + 2*a*b*d*m*
(d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(2*d^2*e*g - 2*c*d*(f*g + e*h)*(1 + m
) + c^2*f*h*(2 + 3*m + m^2)))*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2
F1[m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(1 + m)))/(2*b^2*d^2*(-(
b*c) + a*d)*m*(c + d*x)^m)
```

Maple [F]

$$\int (bx+a)^m\,(dx+c)^{-1-m}\,(fx+e)\,(hx+g)\,dx$$

```
[In] int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m (c+dx)^{-1-m} (e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-1}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m (c+dx)^{-1-m} (e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**(-1-m)*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m (c+dx)^{-1-m} (e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-1}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Giac [F]

$$\int (a+bx)^m (c+dx)^{-1-m} (e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-1}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx=\int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^{m+1}}\,dx$$

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1), x)
```

3.128 $\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 203

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(bd^2eg+bc^2fh(2+m)-cd(b(fg+eh)+afh(1+m))+d(bc-ad)fh(1+m)x\right.}{bd^2(bc-ad)(1+m)}$$
$$-\frac{(adfhm+b(d(fg+eh)-cfh(2+m)))(a+bx)^m\left(-\frac{d(a+bx)}{bc-ad}\right)^{-m}(c+dx)^{-m}\,\text{Hypergeometric2F1}\left(-m,\right.}{bd^3m}$$

```
[Out] (b*x+a)^(1+m)*(d*x+c)^(-1-m)*(b*d^2*e*g+b*c^2*f*h*(2+m)-c*d*(b*(e*h+f*g)+a*
f*h*(1+m))+d*(-a*d+b*c)*f*h*(1+m)*x)/b/d^2/(-a*d+b*c)/(1+m)-(a*d*f*h*m+b*(d
*(e*h+f*g)-c*f*h*(2+m)))*(b*x+a)^m*hypergeom([-m, -m],[1-m],b*(d*x+c)/(-a*d
+b*c))/b/d^3/m/((-d*(b*x+a)/(-a*d+b*c))^m)/((d*x+c)^m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {148, 72, 71}

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx =$$
$$-\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-dfh(m+1)x(bc-ad)+acdfh(m+1)-b(c^2fh(m+2)-cd(eh+fg)+\right.}{bd^2(m+1)(bc-ad)}$$
$$-\frac{(a+bx)^m(c+dx)^{-m}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,\frac{b(c+dx)}{bc-ad}\right)(adfhm-bcfh(m+}{bd^3m}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*(a*c*d*f*h*(1 + m) - b*(d^2*e*g - c
*d*(f*g + e*h) + c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(1 + m)*x))/(b*d^2*(b
*c - a*d)*(1 + m))) - ((b*d*(f*g + e*h) + a*d*f*h*m - b*c*f*h*(2 + m))*(a +
 b*x)^m*Hypergeometric2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(b*d^3
*m*(-((d*(a + b*x))/(b*c - a*d)))^m*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\text{integral} = -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b(d^2eg-cd(fg+eh)+c^2fh(2+m))-d(bc-ad)fh(1+\right.}{bd^2(bc-ad)(1+m)} + \frac{(bd(fg+eh)+adfhm-bcfh(2+m))\int(a+bx)^m(c+dx)^{-1-m}\,dx}{bd^2}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b(d^2eg-cd(fg+eh)+c^2fh(2+m))-d(bc-ad\right.}{bd^2(bc-ad)(1+m)} + \frac{\left((bd(fg+eh)+adfhm-bcfh(2+m))(a+bx)^m\left(\frac{d(a+bx)}{-bc+ad}\right)^{-m}\right)\int(c+dx)^{-1-m}\left(-\frac{ad}{bc-ad}-\frac{b}{bc-}\right.}{bd^2}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b(d^2eg-cd(fg+eh)+c^2fh(2+m))-d(bc-ad)f\right.}{bd^2(bc-ad)(1+m)}$$
$$-\frac{(bd(fg+eh)+adfhm-bcfh(2+m))(a+bx)^m\left(-\frac{d(a+bx)}{bc-ad}\right)^{-m}(c+dx)^{-m}\,{}_2F_1\left(-m,-m;1-m\right.}{bd^3m}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^m(c+dx)^{-m}\left(-\frac{d(a+bx)(adfh(1+m)(c+dx)-b(d^2eg+c^2fh(2+m)+cd(-fg-eh+fh(1+m)x)))}{c+dx}+\frac{(bc-ad)(1+m)(-bd(fg+e\right.}{bd^3(bc-ad)(1+m)}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^m*(-((d*(a + b*x)*(a*d*f*h*(1 + m)*(c + d*x) - b*(d^2*e*g + c^2*
f*h*(2 + m) + c*d*(-(f*g) - e*h + f*h*(1 + m)*x))))/(c + d*x)) + ((b*c - a*
d)*(1 + m)*(-(b*d*(f*g + e*h)) - a*d*f*h*m + b*c*f*h*(2 + m))*Hypergeometri
c2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(m*((d*(a + b*x))/(-(b*c) +
 a*d))^m)))/(b*d^3*(b*c - a*d)*(1 + m)*(c + d*x)^m)
```

Maple [F]

$$\int (bx+a)^m (dx+c)^{-2-m} (fx+e)(hx+g)\,dx$$

```
[In] int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-2}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**(-2-m)*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-2}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-2}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = \int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^{m+2}}\,dx$$

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2), x)
```

3.129 $\int (a+bx)^m (c+dx)^{-3-m} (e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 246

$$\int (a+bx)^m (c+dx)^{-3-m}(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2+m) + ab^2(c(fg+eh) + deg(1+m)) -}{b^2(bc-ad)^2(1+m}$$
$$+\frac{fh(a+bx)^{3+m}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(3+m,3+m,4+m,-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(3+m)}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(-2-m)*(a^2*b*c*f*h*m-a^3*d*f*h*(1+m)-b^3*c*e*g*(2+m
)+a*b^2*(c*(e*h+f*g)+d*e*g*(1+m))-b*(a^2*d*f*h*(3+2*m)+b^2*(d*e*g+c*(e*h+f*
g)*(1+m))-a*b*(2*c*f*h*(1+m)+d*(e*h+f*g)*(2+m)))*x)/b^2/(-a*d+b*c)^2/(1+m)/
(2+m)+f*h*(b*x+a)^(3+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([3+m, 3+m],[4+m]
,-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^3/(3+m)/((d*x+c)^m)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {150, 72, 71}

$$\int (a+bx)^m (c+dx)^{-3-m}(e+fx)(g+hx)\,dx$$
$$= \frac{fh(a+bx)^{m+3}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m+3,m+3,m+4,-\frac{d(a+bx)}{bc-ad}\right)}{(m+3)(bc-ad)^3}$$
$$-\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^3(-d)fh(m+1) - bx(a^2dfh(2m+3) - ab(2cfh(m+1) + d(m+2)(eh+}{b^2(m+1)(m+2}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-2 - m)*(a^2*b*c*f*h*m - a^3*d*f*h*(1 + m)
- b^3*c*e*g*(2 + m) + a*b^2*(c*(f*g + e*h) + d*e*g*(1 + m)) - b*(a^2*d*f*h*
(3 + 2*m) + b^2*(d*e*g + c*(f*g + e*h)*(1 + m)) - a*b*(2*c*f*h*(1 + m) + d*
(f*g + e*h)*(2 + m)))*x))/(b^2*(b*c - a*d)^2*(1 + m)*(2 + m))) + (f*h*(a +
b*x)^(3 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[3 + m, 3 + m,
4 + m, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(3 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rubi steps

integral =

$$-\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm-a^3dfh(1+m)-b^3ceg(2+m)+ab^2(c(fg+eh)+deg(1+m))\right.}{b^2(bc-ad)^2(1+}$$

$$+\frac{(fh)\int(a+bx)^{2+m}(c+dx)^{-3-m}\,dx}{b^2}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2+m) + ab^2(c(fg+eh) + deg(1+\right.}{b^2(bc-ad}$$

$$+ \frac{\left(bfh(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\right)\int(a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-3-m}dx}{(bc-ad)^3}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2+m) + ab^2(c(fg+eh) + deg(1+\right.}{b^2(bc-ad}$$

$$+ \frac{fh(a+bx)^{3+m}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(3+m,3+m;4+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(3+m)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

$$\int(a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx =$$

$$-\frac{(a+bx)^m(c+dx)^{-2-m}\left(d^3(a+bx)\left(-a^3dfh(1+m)+a^2bfh(cm-d(3+2m)x)+ab^2(ceh+deg(1+\right.\right.}{}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^m*(c + d*x)^(-2 - m)*(d^3*(a + b*x)*(-(a^3*d*f*h*(1 + m)) + a^
2*b*f*h*(c*m - d*(3 + 2*m)*x) + a*b^2*(c*e*h + d*e*g*(1 + m) + d*f*g*(2 + m
)*x + d*e*h*(2 + m)*x + c*f*(g + 2*h*(1 + m)*x)) - b^3*(d*e*g*x + c*(e*g*(2
 + m) + f*g*(1 + m)*x + e*h*(1 + m)*x))) + ((b*c - a*d)^4*f*h*(1 + m)*Hyper
geometric2F1[-2 - m, -2 - m, -1 - m, (b*(c + d*x))/(b*c - a*d)])/((d*(a + b
*x))/(-(b*c) + a*d))^m))/(b^2*d^3*(b*c - a*d)^2*(1 + m)*(2 + m)))
```

Maple [F]

$$\int(bx+a)^m(dx+c)^{-3-m}(fx+e)(hx+g)\,dx$$

```
[In] int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**(-3-m)*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx=\int \frac{(e+f\,x)\,\left(g+h\,x\right)\,\left(a+b\,x\right)^m}{\left(c+d\,x\right)^{m+3}}\,dx$$

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3), x)
```

3.130 $\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 362

$$\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx$$
$$= \frac{(a^2d^2fh(6+5m+m^2) - abd(3+m)(d(fg+eh)+2cfh(1+m)) + b^2(2d^2eg + cd(fg+eh)(1+m) + c}{bd^2(bc-ad)^2(2+m)(3+m)}$$
$$+ \frac{(a^2d^2fh(6+5m+m^2) - abd(3+m)(d(fg+eh)+2cfh(1+m)) + b^2(2d^2eg + cd(fg+eh)(1+m) +}{d^2(bc-ad)^3(1+m)(2+m)(3+m)}$$
$$+ \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m) + b(d^2eg - cd(fg+eh) - c^2fh(2+m)) - d(bc-ad)fh(3+\right.}{bd^2(bc-ad)(3+m)}$$

```
[Out] (a^2*d^2*f*h*(m^2+5*m+6)-a*b*d*(3+m)*(d*(e*h+f*g)+2*c*f*h*(1+m))+b^2*(2*d^2
*e*g+c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(-2-m)
/b/d^2/(-a*d+b*c)^2/(2+m)/(3+m)+(a^2*d^2*f*h*(m^2+5*m+6)-a*b*d*(3+m)*(d*(e*
h+f*g)+2*c*f*h*(1+m))+b^2*(2*d^2*e*g+c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2
)))*(b*x+a)^(1+m)*(d*x+c)^(-1-m)/d^2/(-a*d+b*c)^3/(1+m)/(2+m)/(3+m)+(b*x+a)
^(1+m)*(d*x+c)^(-3-m)*(a*c*d*f*h*(3+m)+b*(d^2*e*g-c*d*(e*h+f*g)-c^2*f*h*(2+
m))-d*(-a*d+b*c)*f*h*(3+m)*x)/b/d^2/(-a*d+b*c)/(3+m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{151, 47, 37\}$

$$\int (a+bx)^m (c+dx)^{-4-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh(m^2+5m+6) - abd(m+3)(2cfh(m+1)+d(eh+fg)) + b^2(c^2fh(m^2\right.}{bd^2(m+2)(m+3)(bc-ad)^2}$$
$$+ \frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(a^2d^2fh(m^2+5m+6) - abd(m+3)(2cfh(m+1)+d(eh+fg)) + b^2(c^2fh(\right.}{d^2(m+1)(m+2)(m+3)(bc-ad)^3}$$
$$+ \frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(-dfh(m+3)x(bc-ad) + acdfh(m+3) + b(c^2(-f)h(m+2) - cd(eh+fg)\right.}{bd^2(m+3)(bc-ad)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 +
 m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))
*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(b*d^2*(b*c - a*d)^2*(2 + m)*(3 + m)
) + ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*
(1 + m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^
2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c - a*d)^3*(1 + m)*(2 +
m)*(3 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*c*d*f*h*(3 + m) + b*
(d^2*e*g - c*d*(f*g + e*h) - c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(3 + m)*x
))/(b*d^2*(b*c - a*d)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
```

```
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

integral

$$= \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m)+b(d^2eg-cd(fg+eh)-c^2fh(2+m))-d(bc-ad)fh(3+m\right.}{bd^2(bc-ad)(3+m)} + \frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+cd(fg+eh)(1+m)+}{bd^2(bc-ad)(3+m)}$$

$$= \frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+cd(fg+eh)(1+}{bd^2(bc-ad)^2(2+m)(3+m)} + \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m)+b(d^2eg-cd(fg+eh)-c^2fh(2+m))-d(bc-ad)\right.}{bd^2(bc-ad)(3+m)} + \frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+cd(fg+eh)(1}{d^2(bc-ad)^2(2+m)(3+m)}$$

$$= \frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+cd(fg+eh)(1+}{bd^2(bc-ad)^2(2+m)(3+m)} + \frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+cd(fg+eh)(1}{d^2(bc-ad)^3(1+m)(2+m)(3+m)} + \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m)+b(d^2eg-cd(fg+eh)-c^2fh(2+m))-d(bc-ad)\right.}{bd^2(bc-ad)(3+m)}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.61

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(adfh(3+m)(c+dx)+\frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+c}{(bc-ad)^2(}\right.}{bd^2(bc}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*d*f*h*(3 + m)*(c + d*x) + ((a^2*d^
2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 + m)) + b
^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
```

```
x)*(-(a*d*(1 + m)) + b*c*(2 + m) + b*d*x))/((b*c - a*d)^2*(1 + m)*(2 + m))
+ b*(d^2*e*g - c^2*f*h*(2 + m) - c*d*(e*h + f*(g + h*(3 + m)*x)))))/(b*d^2*
(b*c - a*d)*(3 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(362) = 724.

Time = 2.25 (sec) , antiderivative size = 894, normalized size of antiderivative = 2.47

method	result
gosper	$-\frac{(bx+a)^{1+m}(dx+c)^{-3-m}(a^2d^2fh\,m^2x^2-2abcdfh\,m^2x^2+b^2c^2fh\,m^2x^2+a^2d^2eh\,m^2x+a^2d^2fg\,m^2x+5a^2d^2fhm\,x^2-2abcdeh}{}$
parallelrisch	Expression too large to display

```
[In] int((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(-3-m)/(a^3*d^3*m^3-3*a^2*b*c*d^2*m^3+3*a*b^2*c^2*d*
m^3-b^3*c^3*m^3+6*a^3*d^3*m^2-18*a^2*b*c*d^2*m^2+18*a*b^2*c^2*d*m^2-6*b^3*c
^3*m^2+11*a^3*d^3*m-33*a^2*b*c*d^2*m+33*a*b^2*c^2*d*m-11*b^3*c^3*m+6*a^3*d^
3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)*(a^2*d^2*f*h*m^2*x^2-2*a*b*c*d*f
*h*m^2*x^2+b^2*c^2*f*h*m^2*x^2+a^2*d^2*e*h*m^2*x+a^2*d^2*f*g*m^2*x+5*a^2*d^
2*f*h*m*x^2-2*a*b*c*d*e*h*m^2*x-2*a*b*c*d*f*g*m^2*x-8*a*b*c*d*f*h*m*x^2-a*b
*d^2*e*h*m*x^2-a*b*d^2*f*g*m*x^2+b^2*c^2*e*h*m^2*x+b^2*c^2*f*g*m^2*x+3*b^2*
c^2*f*h*m*x^2+b^2*c*d*e*h*m*x^2+b^2*c*d*f*g*m*x^2+2*a^2*c*d*f*h*m*x+a^2*d^2
*e*g*m^2+4*a^2*d^2*e*h*m*x+4*a^2*d^2*f*g*m*x+6*a^2*d^2*f*h*x^2-2*a*b*c^2*f*
h*m*x-2*a*b*c*d*e*g*m^2-8*a*b*c*d*e*h*m*x-8*a*b*c*d*f*g*m*x-6*a*b*c*d*f*h*x
^2-2*a*b*d^2*e*g*m*x-3*a*b*d^2*e*h*x^2-3*a*b*d^2*f*g*x^2+b^2*c^2*e*g*m^2+4*
b^2*c^2*e*h*m*x+4*b^2*c^2*f*g*m*x+2*b^2*c^2*f*h*x^2+2*b^2*c*d*e*g*m*x+b^2*c
*d*e*h*x^2+b^2*c*d*f*g*x^2+2*b^2*d^2*e*g*x^2+a^2*c*d*e*h*m+a^2*c*d*f*g*m+6*
a^2*c*d*f*h*x+3*a^2*d^2*e*g*m+3*a^2*d^2*e*h*x+3*a^2*d^2*f*g*x-a*b*c^2*e*h*m
-a*b*c^2*f*g*m-2*a*b*c^2*f*h*x-8*a*b*c*d*e*g*m-10*a*b*c*d*e*h*x-10*a*b*c*d*
f*g*x-2*a*b*d^2*e*g*x+5*b^2*c^2*e*g*m+3*b^2*c^2*e*h*x+3*b^2*c^2*f*g*x+6*b^2
*c*d*e*g*x+2*a^2*c^2*f*h+a^2*c*d*e*h+a^2*c*d*f*g+2*a^2*d^2*e*g-3*a*b*c^2*e*
h-3*a*b*c^2*f*g-6*a*b*c*d*e*g+6*b^2*c^2*e*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. 2(362) = 724.

Time = 0.34 (sec) , antiderivative size = 1659, normalized size of antiderivative = 4.58

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*g*m^2 + ((b^3*c^2*d - 2*a*b^2*c*
d^2 + a^2*b*d^3)*f*h*m^2 + (2*b^3*d^3*e + (b^3*c*d^2 - 3*a*b^2*d^3)*f)*g +
((b^3*c*d^2 - 3*a*b^2*d^3)*e + 2*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*
f)*h + ((b^3*c*d^2 - a*b^2*d^3)*f*g + ((b^3*c*d^2 - a*b^2*d^3)*e + (3*b^3*c
^2*d - 8*a*b^2*c*d^2 + 5*a^2*b*d^3)*f)*h)*m)*x^4 + (((b^3*c^2*d - 2*a*b^2*c
*d^2 + a^2*b*d^3)*f*g + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c
^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*h)*m^2 + 4*(2*b^3*c*d^2*e + (b
^3*c^2*d - 3*a*b^2*c*d^2)*f)*g + 2*(2*(b^3*c^2*d - 3*a*b^2*c*d^2)*e + (b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 3*a^3*d^3)*f)*h + ((2*(b^3*c*d^2 - a*
b^2*d^3)*e + (5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*f)*g + ((5*b^3*c^2
*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*e + (3*b^3*c^3 - 7*a*b^2*c^2*d - a^2*b*c*
d^2 + 5*a^3*d^3)*f)*h)*m)*x^3 + ((((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
e + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + ((b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*f)*h)*m^2 + 3*(4*b^3*c^2*d*e + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 a^3*d^3)*f)*g + 3*(4*a^3*c*d^2*f + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2 + a^3*d^3)*e)*h + (((7*b^3*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*e + 4*(b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + (4*(b^3*c^3 - a*b^2*c^2*d
 - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 8*a^2*b*c^2*d + 7*a^3*c*d^2)*f)*
h)*m)*x^2 + (2*(3*a*b^2*c^3 - 3*a^2*b*c^2*d + a^3*c*d^2)*e - (3*a^2*b*c^3 -
 a^3*c^2*d)*f)*g + (2*a^3*c^3*f - (3*a^2*b*c^3 - a^3*c^2*d)*e)*h - ((a^2*b*
c^3 - a^3*c^2*d)*e*h - ((5*a*b^2*c^3 - 8*a^2*b*c^2*d + 3*a^3*c*d^2)*e - (a^
2*b*c^3 - a^3*c^2*d)*f)*g)*m + (((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*
h + ((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2
*b*c^2*d + a^3*c*d^2)*f)*g)*m^2 + 2*((3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + a^3*d^3)*e - 2*(3*a^2*b*c^2*d - a^3*c*d^2)*f)*g + 4*(2*a^3*c^2*d*f -
 (3*a^2*b*c^2*d - a^3*c*d^2)*e)*h + (((5*b^3*c^3 - a*b^2*c^2*d - 7*a^2*b*c*
d^2 + 3*a^3*d^3)*e + (3*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*f)*g + ((3
*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*e - 2*(a^2*b*c^3 - a^3*c^2*d)*f)*
h)*m)*x)*(b*x + a)^m*(d*x + c)^(-m - 4)/(6*b^3*c^3 - 18*a*b^2*c^2*d + 18*a^
2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*m^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m^2 + 11*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**(-4-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-4}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-4}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 1895, normalized size of antiderivative = 5.23

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 4),x)
[Out] - ((a + b*x)^m*(2*a^3*c^3*f*h + 6*a*b^2*c^3*e*g - 3*a^2*b*c^3*e*h - 3*a^2*b
*c^3*f*g + 2*a^3*c*d^2*e*g + a^3*c^2*d*e*h + a^3*c^2*d*f*g - 6*a^2*b*c^2*d*
e*g + 5*a*b^2*c^3*e*g*m - a^2*b*c^3*e*h*m - a^2*b*c^3*f*g*m + 3*a^3*c*d^2*e
*g*m + a^3*c^2*d*e*h*m + a^3*c^2*d*f*g*m + a*b^2*c^3*e*g*m^2 + a^3*c*d^2*e*
g*m^2 - 2*a^2*b*c^2*d*e*g*m^2 - 8*a^2*b*c^2*d*e*g*m))/((a*d - b*c)^3*(c + d
*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x^3*(a + b*x)^m*(6*a^3*d^3*f*h + 2
*b^3*c^3*f*h + 8*b^3*c*d^2*e*g + 4*b^3*c^2*d*e*h + 4*b^3*c^2*d*f*g + 5*a^3*
d^3*f*h*m + 3*b^3*c^3*f*h*m + a^3*d^3*f*h*m^2 + b^3*c^3*f*h*m^2 - 12*a*b^2*
c*d^2*e*h - 12*a*b^2*c*d^2*f*g - 6*a*b^2*c^2*d*f*h + 6*a^2*b*c*d^2*f*h - 2*
a*b^2*d^3*e*g*m + 3*a^2*b*d^3*e*h*m + 3*a^2*b*d^3*f*g*m + 2*b^3*c*d^2*e*g*m
 + 5*b^3*c^2*d*e*h*m + 5*b^3*c^2*d*f*g*m + a^2*b*d^3*e*h*m^2 + a^2*b*d^3*f*
g*m^2 + b^3*c^2*d*e*h*m^2 + b^3*c^2*d*f*g*m^2 - 2*a*b^2*c*d^2*e*h*m^2 - 2*a
*b^2*c*d^2*f*g*m^2 - a*b^2*c^2*d*f*h*m^2 - a^2*b*c*d^2*f*h*m^2 - 8*a*b^2*c*
d^2*e*h*m - 8*a*b^2*c*d^2*f*g*m - 7*a*b^2*c^2*d*f*h*m - a^2*b*c*d^2*f*h*m))
/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x*(a + b*x)^
m*(2*a^3*d^3*e*g + 6*b^3*c^3*e*g + 4*a^3*c*d^2*e*h + 4*a^3*c*d^2*f*g + 8*a^
```

```
3*c^2*d*f*h + 3*a^3*d^3*e*g*m + 5*b^3*c^3*e*g*m + a^3*d^3*e*g*m^2 + b^3*c^3
*e*g*m^2 + 6*a*b^2*c^2*d*e*g - 6*a^2*b*c*d^2*e*g - 12*a^2*b*c^2*d*e*h - 12*
a^2*b*c^2*d*f*g + 3*a*b^2*c^3*e*h*m + 3*a*b^2*c^3*f*g*m - 2*a^2*b*c^3*f*h*m
 + 5*a^3*c*d^2*e*h*m + 5*a^3*c*d^2*f*g*m + 2*a^3*c^2*d*f*h*m + a*b^2*c^3*e*
h*m^2 + a*b^2*c^3*f*g*m^2 + a^3*c*d^2*e*h*m^2 + a^3*c*d^2*f*g*m^2 - a*b^2*c
^2*d*e*g*m^2 - a^2*b*c*d^2*e*g*m^2 - 2*a^2*b*c^2*d*e*h*m^2 - 2*a^2*b*c^2*d*
f*g*m^2 - a*b^2*c^2*d*e*g*m - 7*a^2*b*c*d^2*e*g*m - 8*a^2*b*c^2*d*e*h*m - 8
*a^2*b*c^2*d*f*g*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 +
 6)) - (x^2*(a + b*x)^m*(3*a^3*d^3*e*h + 3*a^3*d^3*f*g + 3*b^3*c^3*e*h + 3*
b^3*c^3*f*g + 12*b^3*c^2*d*e*g + 12*a^3*c*d^2*f*h + 4*a^3*d^3*e*h*m + 4*a^3
*d^3*f*g*m + 4*b^3*c^3*e*h*m + 4*b^3*c^3*f*g*m + a^3*d^3*e*h*m^2 + a^3*d^3*
f*g*m^2 + b^3*c^3*e*h*m^2 + b^3*c^3*f*g*m^2 - 9*a*b^2*c^2*d*e*h - 9*a*b^2*c
^2*d*f*g - 9*a^2*b*c*d^2*e*h - 9*a^2*b*c*d^2*f*g + a^2*b*d^3*e*g*m + a*b^2*
c^3*f*h*m + 7*b^3*c^2*d*e*g*m + 7*a^3*c*d^2*f*h*m + a^2*b*d^3*e*g*m^2 + a*b
^2*c^3*f*h*m^2 + b^3*c^2*d*e*g*m^2 + a^3*c*d^2*f*h*m^2 - 2*a*b^2*c*d^2*e*g*
m^2 - a*b^2*c^2*d*e*h*m^2 - a*b^2*c^2*d*f*g*m^2 - a^2*b*c*d^2*e*h*m^2 - a^2
*b*c*d^2*f*g*m^2 - 2*a^2*b*c^2*d*f*h*m^2 - 8*a*b^2*c*d^2*e*g*m - 4*a*b^2*c^
2*d*e*h*m - 4*a*b^2*c^2*d*f*g*m - 4*a^2*b*c*d^2*e*h*m - 4*a^2*b*c*d^2*f*g*m
 - 8*a^2*b*c^2*d*f*h*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m
^3 + 6)) - (x^4*(a + b*x)^m*(2*b^3*d^3*e*g - 3*a*b^2*d^3*e*h - 3*a*b^2*d^3*
f*g + 6*a^2*b*d^3*f*h + b^3*c*d^2*e*h + b^3*c*d^2*f*g + 2*b^3*c^2*d*f*h - 6
*a*b^2*c*d^2*f*h - a*b^2*d^3*e*h*m - a*b^2*d^3*f*g*m + 5*a^2*b*d^3*f*h*m +
b^3*c*d^2*e*h*m + b^3*c*d^2*f*g*m + 3*b^3*c^2*d*f*h*m + a^2*b*d^3*f*h*m^2 +
 b^3*c^2*d*f*h*m^2 - 2*a*b^2*c*d^2*f*h*m^2 - 8*a*b^2*c*d^2*f*h*m))/((a*d -
b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

3.131 $\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 507

$$\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx$$
$$= \frac{(a^2d^2fh(12+7m+m^2) - 2abd(4+m)(d(fg+eh)+cfh(1+m)) + b^2(6d^2eg+2cd(fg+eh)(1+m)+c}{2bd^2(bc-ad)^2(3+m)(4+m)}$$
$$+ \frac{(a^2d^2fh(12+7m+m^2) - 2abd(4+m)(d(fg+eh)+cfh(1+m)) + b^2(6d^2eg+2cd(fg+eh)(1+m)}{d^2(bc-ad)^3(2+m)(3+m)(4+m)}$$
$$+ \frac{b(a^2d^2fh(12+7m+m^2) - 2abd(4+m)(d(fg+eh)+cfh(1+m)) + b^2(6d^2eg+2cd(fg+eh)(1+m)}{d^2(bc-ad)^4(1+m)(2+m)(3+m)(4+m)}$$
$$+ \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\,(acdfh(4+m) + b(2d^2eg - 2cd(fg+eh) - c^2fh(2+m)) - d(bc-ad)fh(4+}{2bd^2(bc-ad)(4+m)}$$

```
[Out] 1/2*(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*(
6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)
^(-3-m)/b/d^2/(-a*d+b*c)^2/(3+m)/(4+m)+(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4
+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*(6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*
(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(-2-m)/d^2/(-a*d+b*c)^3/(2+m)/(3+m)/(4+
m)+b*(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*
(6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c
)^(-1-m)/d^2/(-a*d+b*c)^4/(1+m)/(2+m)/(3+m)/(4+m)+1/2*(b*x+a)^(1+m)*(d*x+c)
^(-4-m)*(a*c*d*f*h*(4+m)+b*(2*d^2*e*g-2*c*d*(e*h+f*g)-c^2*f*h*(2+m))-d*(-a*
d+b*c)*f*h*(4+m)*x)/b/d^2/(-a*d+b*c)/(4+m)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {151, 47, 37}

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(a^2d^2fh(m^2+7m+12)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2(c^2fh(\right.}{2bd^2(m+3)(m+4)(bc-ad)^2}$$
$$+\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh(m^2+7m+12)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2(c^2f\right.}{d^2(m+2)(m+3)(m+4)(bc-ad)^3}$$
$$+\frac{b(a+bx)^{m+1}(c+dx)^{-m-1}\left(a^2d^2fh(m^2+7m+12)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2(c^2\right.}{d^2(m+1)(m+2)(m+3)(m+4)(bc-ad)^4}$$
$$+\frac{(a+bx)^{m+1}(c+dx)^{-m-4}\left(-dfh(m+4)x(bc-ad)+acdfh(m+4)+b(c^2(-f)h(m+2)-2cd(eh+f\right.}{2bd^2(m+4)(bc-ad)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1
+ m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2
)))*(a + b*x)^(1 + m)*(c + d*x)^(-3 - m))/(2*b*d^2*(b*c - a*d)^2*(3 + m)*(4
 + m)) + ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) +
c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 +
3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(d^2*(b*c - a*d)^3*(2 +
m)*(3 + m)*(4 + m)) + (b*(a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d
*(f*g + e*h) + c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m)
+ c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c
 - a*d)^4*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(
-4 - m)*(a*c*d*f*h*(4 + m) + b*(2*d^2*e*g - 2*c*d*(f*g + e*h) - c^2*f*h*(2
+ m)) - d*(b*c - a*d)*f*h*(4 + m)*x))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

integral

$$= \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(acdfh(4+m)+b(2d^2eg-2cd(fg+eh)-c^2fh(2+m))-d(bc-ad)fh(4+m\right.}{2bd^2(bc-ad)(4+m)}$$
$$+\frac{\left(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2(6d^2eg+2cd(fg+eh)(1+m)\right.}{2bd^2(bc-ad)(4+m)}$$

$$= \frac{\left(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2(6d^2eg+2cd(fg+eh)(1\right.}{2bd^2(bc-ad)^2(3+m)(4+m)}$$
$$+\frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(acdfh(4+m)+b(2d^2eg-2cd(fg+eh)-c^2fh(2+m))-d(bc-ad\right.}{2bd^2(bc-ad)(4+m)}$$
$$+\frac{\left(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2(6d^2eg+2cd(fg+eh)(\right.}{d^2(bc-ad)^2(3+m)(4+m)}$$

$$= \frac{\left(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2(6d^2eg+2cd(fg+eh)(1\right.}{2bd^2(bc-ad)^2(3+m)(4+m)}$$
$$+\frac{\left(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2(6d^2eg+2cd(fg+eh)(\right.}{d^2(bc-ad)^3(2+m)(3+m)(4+m)}$$
$$+\frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(acdfh(4+m)+b(2d^2eg-2cd(fg+eh)-c^2fh(2+m))-d(bc-ad\right.}{2bd^2(bc-ad)(4+m)}$$
$$+\frac{\left(b(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2(6d^2eg+2cd(fg+eh\right.}{d^2(bc-ad)^3(2+m)(3+m)(4+m}$$

$$= \frac{(a^2d^2fh(12+7m+m^2) - 2abd(4+m)(d(fg+eh)+cfh(1+m)) + b^2(6d^2eg+2cd(fg+eh)(1}{2bd^2(bc-ad)^2(3+m)(4+m)}$$
$$+ \frac{(a^2d^2fh(12+7m+m^2) - 2abd(4+m)(d(fg+eh)+cfh(1+m)) + b^2(6d^2eg+2cd(fg+eh)}{d^2(bc-ad)^3(2+m)(3+m)(4+m)}$$
$$+ \frac{b(a^2d^2fh(12+7m+m^2) - 2abd(4+m)(d(fg+eh)+cfh(1+m)) + b^2(6d^2eg+2cd(fg+eh}{d^2(bc-ad)^4(1+m)(2+m)(3+m)(4-}$$
$$+ \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\,(acdfh(4+m)+b(2d^2eg-2cd(fg+eh)-c^2fh(2+m))-d(bc-a}{2bd^2(bc-ad)(4+m)}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.55

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(adfh(4+m)(c+dx)+b(2d^2eg-c^2fh(2+m)-cd(2fg+2eh+fh(4+m)\right.}{}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-4 - m)*(a*d*f*h*(4 + m)*(c + d*x) + b*(2*d^2
*e*g - c^2*f*h*(2 + m) - c*d*(2*f*g + 2*e*h + f*h*(4 + m)*x)) + ((a^2*d^2*f
*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1 + m)) + b^2
*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(a^2*d^2*(2 + 3*m + m^2) - 2*a*b*d*(1 + m)*(c*(3 + m) + d*x) + b^2*(c^2*
(6 + 5*m + m^2) + 2*c*d*(3 + m)*x + 2*d^2*x^2)))/((b*c - a*d)^3*(1 + m)*(2
+ m)*(3 + m))))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2342 vs. 2(503) = 1006.

Time = 2.24 (sec) , antiderivative size = 2343, normalized size of antiderivative = 4.62

method	result	size
gosper	Expression too large to display	2343
parallelrisch	Expression too large to display	9664

```
[In] int((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(-4-m)/(a^4*d^4*m^4-4*a^3*b*c*d^3*m^4+6*a^2*b^2*c^2*
d^2*m^4-4*a*b^3*c^3*d*m^4+b^4*c^4*m^4+10*a^4*d^4*m^3-40*a^3*b*c*d^3*m^3+60*
a^2*b^2*c^2*d^2*m^3-40*a*b^3*c^3*d*m^3+10*b^4*c^4*m^3+35*a^4*d^4*m^2-140*a^
```

3*b*c*d^3*m^2+210*a^2*b^2*c^2*d^2*m^2-140*a*b^3*c^3*d*m^2+35*b^4*c^4*m^2+50*a^4*d^4*m-200*a^3*b*c*d^3*m+300*a^2*b^2*c^2*d^2*m-200*a*b^3*c^3*d*m+50*b^4*c^4*m+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d^2-96*a*b^3*c^3*d+24*b^4*c^4)*(a^3*d^3*f*h*m^3*x^2-3*a^2*b*c*d^2*f*h*m^3*x^2-a^2*b*d^3*f*h*m^2*x^3+3*a*b^2*c^2*d*f*h*m^3*x^2+2*a*b^2*c*d^2*f*h*m^2*x^3-b^3*c^3*f*h*m^3*x^2-b^3*c^2*d*f*h*m^2*x^3+a^3*d^3*e*h*m^3*x+a^3*d^3*f*g*m^3*x+8*a^3*d^3*f*h*m^2*x^2-3*a^2*b*c*d^2*e*h*m^3*x-3*a^2*b*c*d^2*f*g*m^3*x-23*a^2*b*c*d^2*f*h*m^2*x^2-2*a^2*b*d^3*e*h*m^2*x^2-2*a^2*b*d^3*f*g*m^2*x^2-7*a^2*b*d^3*f*h*m*x^3+3*a*b^2*c^2*d*e*h*m^3*x+3*a*b^2*c^2*d*f*g*m^3*x+22*a*b^2*c^2*d*f*h*m^2*x^2+4*a*b^2*c*d^2*e*h*m^2*x^2+4*a*b^2*c*d^2*f*g*m^2*x^2+10*a*b^2*c*d^2*f*h*m*x^3+2*a*b^2*d^3*e*h*m*x^3+2*a*b^2*d^3*f*g*m*x^3-b^3*c^3*e*h*m^3*x-b^3*c^3*f*g*m^3*x-7*b^3*c^3*f*h*m^2*x^2-2*b^3*c^2*d*e*h*m^2*x^2-2*b^3*c^2*d*f*g*m^2*x^2-3*b^3*c^2*d*f*h*m*x^3-2*b^3*c*d^2*e*h*m*x^3-2*b^3*c*d^2*f*g*m*x^3+2*a^3*c*d^2*f*h*m^2*x+a^3*d^3*e*g*m^3+7*a^3*d^3*e*h*m^2*x+7*a^3*d^3*f*g*m^2*x+19*a^3*d^3*f*h*m*x^2-4*a^2*b*c^2*d*f*h*m^2*x-3*a^2*b*c*d^2*e*g*m^3-22*a^2*b*c*d^2*e*h*m^2*x-22*a^2*b*c*d^2*f*g*m^2*x-58*a^2*b*c*d^2*f*h*m*x^2-3*a^2*b*d^3*e*g*m^2*x-10*a^2*b*d^3*e*h*m*x^2-10*a^2*b*d^3*f*g*m*x^2-12*a^2*b*d^3*f*h*x^3+2*a*b^2*c^3*f*h*m^2*x+3*a*b^2*c^2*d*e*g*m^3+23*a*b^2*c^2*d*e*h*m^2*x+23*a*b^2*c^2*d*f*g*m^2*x+53*a*b^2*c^2*d*f*h*m*x^2+6*a*b^2*c*d^2*e*g*m^2*x+20*a*b^2*c*d^2*e*h*m*x^2+20*a*b^2*c*d^2*f*g*m*x^2+8*a*b^2*c*d^2*f*h*x^3+6*a*b^2*d^3*e*g*m*x^2+8*a*b^2*d^3*e*h*x^3+8*a*b^2*d^3*f*g*x^3-b^3*c^3*e*g*m^3-8*b^3*c^3*e*h*m^2*x-8*b^3*c^3*f*g*m^2*x-14*b^3*c^3*f*h*m*x^2-3*b^3*c^2*d*e*g*m^2*x-10*b^3*c^2*d*e*h*m*x^2-10*b^3*c^2*d*f*g*m*x^2-2*b^3*c^2*d*f*h*x^3-6*b^3*c*d^2*e*g*m*x^2-2*b^3*c*d^2*e*h*x^3-2*b^3*c*d^2*f*g*x^3-6*b^3*d^3*e*g*x^3+a^3*c*d^2*e*h*m^2+a^3*c*d^2*f*g*m^2+10*a^3*c*d^2*f*h*m*x+6*a^3*d^3*e*g*m^2+14*a^3*d^3*e*h*m*x+14*a^3*d^3*f*g*m*x+12*a^3*d^3*f*h*x^2-2*a^2*b*c^2*d*e*h*m^2-2*a^2*b*c^2*d*f*g*m^2-20*a^2*b*c^2*d*f*h*m*x-21*a^2*b*c*d^2*e*g*m^2-53*a^2*b*c*d^2*e*h*m*x-53*a^2*b*c*d^2*f*g*m*x-56*a^2*b*c*d^2*f*h*x^2-9*a^2*b*d^3*e*g*m*x-8*a^2*b*d^3*e*h*x^2-8*a^2*b*d^3*f*g*x^2+a*b^2*c^3*e*h*m^2+a*b^2*c^3*f*g*m^2+10*a*b^2*c^3*f*h*m*x+24*a*b^2*c^2*d*e*g*m^2+58*a*b^2*c^2*d*e*h*m*x+58*a*b^2*c^2*d*f*g*m*x+34*a*b^2*c^2*d*f*h*x^2+30*a*b^2*c*d^2*e*g*m*x+34*a*b^2*c*d^2*e*h*x^2+34*a*b^2*c*d^2*f*g*x^2+6*a*b^2*d^3*e*g*x^2-9*b^3*c^3*e*g*m^2-19*b^3*c^3*e*h*m*x-19*b^3*c^3*f*g*m*x-8*b^3*c^3*f*h*x^2-21*b^3*c^2*d*e*g*m*x-8*b^3*c^2*d*e*h*x^2-8*b^3*c^2*d*f*g*x^2-24*b^3*c*d^2*e*g*x^2+2*a^3*c^2*d*f*h*m+3*a^3*c*d^2*e*h*m+3*a^3*c*d^2*f*g*m+8*a^3*c*d^2*f*h*x+11*a^3*d^3*e*g*m+8*a^3*d^3*e*h*x+8*a^3*d^3*f*g*x-2*a^2*b*c^3*f*h*m-10*a^2*b*c^2*d*e*h*m-10*a^2*b*c^2*d*f*g*m-34*a^2*b*c^2*d*f*h*x-42*a^2*b*c*d^2*e*g*m-34*a^2*b*c*d^2*e*h*x-34*a^2*b*c*d^2*f*g*x-6*a^2*b*d^3*e*g*x+7*a*b^2*c^3*e*h*m+7*a*b^2*c^3*f*g*m+8*a*b^2*c^3*f*h*x+57*a*b^2*c^2*d*e*g*m+56*a*b^2*c^2*d*e*h*x+56*a*b^2*c^2*d*f*g*x+24*a*b^2*c*d^2*e*g*x-26*b^3*c^3*e*g*m-12*b^3*c^3*e*h*x-12*b^3*c^3*f*g*x-36*b^3*c^2*d*e*g*x+2*a^3*c^2*d*f*h+2*a^3*c*d^2*e*h+2*a^3*c*d^2*f*g+6*a^3*d^3*e*g-8*a^2*b*c^3*f*h-8*a^2*b*c^2*d*e*h-8*a^2*b*c^2*d*f*g-24*a^2*b*c*d^2*e*g+12*a*b^2*c^3*e*h+12*a*b^2*c^3*f*g+36*a*b^2*c^2*d*e*g-24*b^3*c^3*e*g)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3441 vs. 2(503) = 1006.

Time = 0.57 (sec) , antiderivative size = 3441, normalized size of antiderivative = 6.79

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*e*g*m^3 + ((b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*h*m^2 + 2*(3*b^4*d^4*e + (b^4*c*
d^3 - 4*a*b^3*d^4)*f)*g + 2*((b^4*c*d^3 - 4*a*b^3*d^4)*e + (b^4*c^2*d^2 - 4
*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*f)*h + (2*(b^4*c*d^3 - a*b^3*d^4)*f*g + (2*(b
^4*c*d^3 - a*b^3*d^4)*e + (3*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + 7*a^2*b^2*d^4)*
f)*h)*m)*x^5 + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)
*f*h*m^3 + (2*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*g + (2*(b^4*c^2
*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + (8*b^4*c^3*d - 23*a*b^3*c^2*d^2 + 2
2*a^2*b^2*c*d^3 - 7*a^3*b*d^4)*f)*h)*m^2 + 10*(3*b^4*c*d^3*e + (b^4*c^2*d^2
 - 4*a*b^3*c*d^3)*f)*g + 10*((b^4*c^2*d^2 - 4*a*b^3*c*d^3)*e + (b^4*c^3*d -
 4*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3)*f)*h + (2*(3*(b^4*c*d^3 - a*b^3*d^4)*e
+ 2*(3*b^4*c^2*d^2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*f)*g + (4*(3*b^4*c^2*d^
2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*e + (17*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 5
5*a^2*b^2*c*d^3 - 12*a^3*b*d^4)*f)*h)*m)*x^4 + (((b^4*c^3*d - 3*a*b^3*c^2*d
^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*f*g + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a
^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^
4*d^4)*f)*h)*m^3 + ((3*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + 5*(2
*b^4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*f)*g + (5*(2*b^
4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (7*b^4*c^4 - 1
6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 8*a^4*d^4)*f)*h)*m^2 +
 20*(3*b^4*c^2*d^2*e + (b^4*c^3*d - 4*a*b^3*c^2*d^2)*f)*g + 4*(5*(b^4*c^3*d
 - 4*a*b^3*c^2*d^2)*e + (2*b^4*c^4 - 8*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 + 1
2*a^3*b*c*d^3 - 3*a^4*d^4)*f)*h + ((3*(9*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + a^2
*b^2*d^4)*e + (29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*f)*g + ((29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*e + (14*b^4*c^4 - 46*a*b^3*c^3*d + 15*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^
3 - 19*a^4*d^4)*f)*h)*m)*x^3 - ((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)
*e*h - (3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^2 - 2*a^4*c*d^3)*e
 - (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g)*m^2 + ((((b^4*c^3*d -
3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d
 + 2*a^3*b*c*d^3 - a^4*d^4)*f)*g + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^
3 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3
)*f)*h)*m^3 + ((3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - a^3*b*
d^4)*e + (8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 -
 7*a^4*d^4)*f)*g + ((8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^
```

```
3*b*c*d^3 - 7*a^4*d^4)*e + 5*(a*b^3*c^4 - 4*a^2*b^2*c^3*d + 5*a^3*b*c^2*d^2
 - 2*a^4*c*d^3)*f)*h)*m^2 + 4*(15*b^4*c^3*d*e + (3*b^4*c^4 - 12*a*b^3*c^3*d
 - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*f)*g + 4*((3*b^4*c^4 - 1
2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*e + 5*(4*a^
3*b*c^2*d^2 - a^4*c*d^3)*f)*h + (((47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2
*b^2*c*d^3 - 2*a^3*b*d^4)*e + (19*b^4*c^4 - 36*a*b^3*c^3*d - 15*a^2*b^2*c^2
*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*f)*g + ((19*b^4*c^4 - 36*a*b^3*c^3*d -
15*a^2*b^2*c^2*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*e + (4*a*b^3*c^4 - 41*a^2
*b^2*c^3*d + 66*a^3*b*c^2*d^2 - 29*a^4*c*d^3)*f)*h)*m)*x^2 + 2*(3*(4*a*b^3*
c^4 - 6*a^2*b^2*c^3*d + 4*a^3*b*c^2*d^2 - a^4*c*d^3)*e - (6*a^2*b^2*c^4 - 4
*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g - 2*((6*a^2*b^2*c^4 - 4*a^3*b*c^3*d + a^4*
c^2*d^2)*e - (4*a^3*b*c^4 - a^4*c^3*d)*f)*h + (((26*a*b^3*c^4 - 57*a^2*b^2*
c^3*d + 42*a^3*b*c^2*d^2 - 11*a^4*c*d^3)*e - (7*a^2*b^2*c^4 - 10*a^3*b*c^3*
d + 3*a^4*c^2*d^2)*f)*g - ((7*a^2*b^2*c^4 - 10*a^3*b*c^3*d + 3*a^4*c^2*d^2)
*e - 2*(a^3*b*c^4 - a^4*c^3*d)*f)*h)*m + (((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3
*a^3*b*c^2*d^2 - a^4*c*d^3)*e*h + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3
 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)
*f)*g)*m^3 + ((3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c
*d^3 - 2*a^4*d^4)*e + (7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 -
8*a^4*c*d^3)*f)*g + ((7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 - 8
*a^4*c*d^3)*e - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*h)*m^2 + 2
*(3*(4*b^4*c^4 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^
4)*e - 5*(6*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*f)*g - 10*((6*a^2*
b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*e - (4*a^3*b*c^3*d - a^4*c^2*d^2)*
f)*h + (((26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3
 - 11*a^4*d^4)*e + (12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 17
*a^4*c*d^3)*f)*g + ((12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 1
7*a^4*c*d^3)*e - 4*(2*a^2*b^2*c^4 - 5*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f)*h)*m)
*x)*(b*x + a)^m*(d*x + c)^(-m - 5)/(24*b^4*c^4 - 96*a*b^3*c^3*d + 144*a^2*b
^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^4 + 10*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^3 + 35*(b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^2 + 50*(b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**(-5-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-5}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-5}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 3720, normalized size of antiderivative = 7.34

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 5),x)
[Out] (x^5*(a + b*x)^m*(6*b^4*d^4*e*g - 8*a*b^3*d^4*e*h - 8*a*b^3*d^4*f*g + 2*b^4
*c*d^3*e*h + 2*b^4*c*d^3*f*g + 12*a^2*b^2*d^4*f*h + 2*b^4*c^2*d^2*f*h + a^2
*b^2*d^4*f*h*m^2 + b^4*c^2*d^2*f*h*m^2 - 8*a*b^3*c*d^3*f*h - 2*a*b^3*d^4*e*
h*m - 2*a*b^3*d^4*f*g*m + 2*b^4*c*d^3*e*h*m + 2*b^4*c*d^3*f*g*m + 7*a^2*b^2
*d^4*f*h*m + 3*b^4*c^2*d^2*f*h*m - 2*a*b^3*c*d^3*f*h*m^2 - 10*a*b^3*c*d^3*f
*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
) - (x*(a + b*x)^m*(6*a^4*d^4*e*g - 24*b^4*c^4*e*g + 10*a^4*c*d^3*e*h + 10*
a^4*c*d^3*f*g + 11*a^4*d^4*e*g*m - 26*b^4*c^4*e*g*m + 10*a^4*c^2*d^2*f*h +
6*a^4*d^4*e*g*m^2 - 9*b^4*c^4*e*g*m^2 + a^4*d^4*e*g*m^3 - b^4*c^4*e*g*m^3 +
 36*a^2*b^2*c^2*d^2*e*g + 2*a^2*b^2*c^4*f*h*m^2 + 2*a^4*c^2*d^2*f*h*m^2 - 2
4*a*b^3*c^3*d*e*g - 24*a^3*b*c*d^3*e*g - 40*a^3*b*c^3*d*f*h - 12*a*b^3*c^4*
e*h*m - 12*a*b^3*c^4*f*g*m + 17*a^4*c*d^3*e*h*m + 17*a^4*c*d^3*f*g*m + 60*a
^2*b^2*c^3*d*e*h + 60*a^2*b^2*c^3*d*f*g - 40*a^3*b*c^2*d^2*e*h - 40*a^3*b*c
^2*d^2*f*g - 7*a*b^3*c^4*e*h*m^2 - 7*a*b^3*c^4*f*g*m^2 - a*b^3*c^4*e*h*m^3
- a*b^3*c^4*f*g*m^3 + 8*a^2*b^2*c^4*f*h*m + 8*a^4*c*d^3*e*h*m^2 + 8*a^4*c*d
^3*f*g*m^2 + a^4*c*d^3*e*h*m^3 + a^4*c*d^3*f*g*m^3 + 12*a^4*c^2*d^2*f*h*m +
```

12*a*b^3*c^3*d*e*g*m^2 - 18*a^3*b*c*d^3*e*g*m^2 + 2*a*b^3*c^3*d*e*g*m^3 - 2*a^3*b*c*d^3*e*g*m^3 + 55*a^2*b^2*c^3*d*e*h*m + 55*a^2*b^2*c^3*d*f*g*m - 6 0*a^3*b*c^2*d^2*e*h*m - 60*a^3*b*c^2*d^2*f*g*m - 4*a^3*b*c^3*d*f*h*m^2 + 45 *a^2*b^2*c^2*d^2*e*g*m + 22*a^2*b^2*c^3*d*e*h*m^2 + 22*a^2*b^2*c^3*d*f*g*m^ 2 - 23*a^3*b*c^2*d^2*e*h*m^2 - 23*a^3*b*c^2*d^2*f*g*m^2 + 3*a^2*b^2*c^3*d*e *h*m^3 + 3*a^2*b^2*c^3*d*f*g*m^3 - 3*a^3*b*c^2*d^2*e*h*m^3 - 3*a^3*b*c^2*d^ 2*f*g*m^3 + 10*a*b^3*c^3*d*e*g*m - 40*a^3*b*c*d^3*e*g*m - 20*a^3*b*c^3*d*f* h*m + 9*a^2*b^2*c^2*d^2*e*g*m^2))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x)^m*(6*a^4*c*d^3*e*g - 8*a^3*b*c^4* f*h - 24*a*b^3*c^4*e*g + 2*a^4*c^3*d*f*h + 12*a^2*b^2*c^4*e*h + 12*a^2*b^2* c^4*f*g + 2*a^4*c^2*d^2*e*h + 2*a^4*c^2*d^2*f*g + a^2*b^2*c^4*e*h*m^2 + a^2 *b^2*c^4*f*g*m^2 + a^4*c^2*d^2*e*h*m^2 + a^4*c^2*d^2*f*g*m^2 - 8*a^3*b*c^3* d*e*h - 8*a^3*b*c^3*d*f*g - 26*a*b^3*c^4*e*g*m - 2*a^3*b*c^4*f*h*m + 11*a^4 *c*d^3*e*g*m + 2*a^4*c^3*d*f*h*m + 36*a^2*b^2*c^3*d*e*g - 24*a^3*b*c^2*d^2* e*g - 9*a*b^3*c^4*e*g*m^2 - a*b^3*c^4*e*g*m^3 + 7*a^2*b^2*c^4*e*h*m + 7*a^2 *b^2*c^4*f*g*m + 6*a^4*c*d^3*e*g*m^2 + a^4*c*d^3*e*g*m^3 + 3*a^4*c^2*d^2*e* h*m + 3*a^4*c^2*d^2*f*g*m + 57*a^2*b^2*c^3*d*e*g*m - 42*a^3*b*c^2*d^2*e*g*m - 2*a^3*b*c^3*d*e*h*m^2 - 2*a^3*b*c^3*d*f*g*m^2 + 24*a^2*b^2*c^3*d*e*g*m^2 - 21*a^3*b*c^2*d^2*e*g*m^2 + 3*a^2*b^2*c^3*d*e*g*m^3 - 3*a^3*b*c^2*d^2*e*g *m^3 - 10*a^3*b*c^3*d*e*h*m - 10*a^3*b*c^3*d*f*g*m))/((a*d - b*c)^4*(c + d* x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(8*b^4*c ^4*f*h - 12*a^4*d^4*f*h + 20*b^4*c^3*d*e*h + 20*b^4*c^3*d*f*g - 19*a^4*d^4* f*h*m + 14*b^4*c^4*f*h*m + 60*b^4*c^2*d^2*e*g - 8*a^4*d^4*f*h*m^2 + 7*b^4*c ^4*f*h*m^2 - a^4*d^4*f*h*m^3 + b^4*c^4*f*h*m^3 + 48*a^2*b^2*c^2*d^2*f*h + 3 *a^2*b^2*d^4*e*g*m^2 + 3*b^4*c^2*d^2*e*g*m^2 - 32*a*b^3*c^3*d*f*h + 48*a^3* b*c*d^3*f*h - 4*a^3*b*d^4*e*h*m - 4*a^3*b*d^4*f*g*m + 29*b^4*c^3*d*e*h*m + 29*b^4*c^3*d*f*g*m - 80*a*b^3*c^2*d^2*e*h - 80*a*b^3*c^2*d^2*f*g + 3*a^2*b^ 2*d^4*e*g*m - 5*a^3*b*d^4*e*h*m^2 - 5*a^3*b*d^4*f*g*m^2 - a^3*b*d^4*e*h*m^3 - a^3*b*d^4*f*g*m^3 + 27*b^4*c^2*d^2*e*g*m + 10*b^4*c^3*d*e*h*m^2 + 10*b^4 *c^3*d*f*g*m^2 + b^4*c^3*d*e*h*m^3 + b^4*c^3*d*f*g*m^3 + 3*a^2*b^2*c^2*d^2* f*h*m^2 - 6*a*b^3*c*d^3*e*g*m^2 - 66*a*b^3*c^2*d^2*e*h*m - 66*a*b^3*c^2*d^2 *f*g*m + 41*a^2*b^2*c*d^3*e*h*m + 41*a^2*b^2*c*d^3*f*g*m - 16*a*b^3*c^3*d*f *h*m^2 + 14*a^3*b*c*d^3*f*h*m^2 - 2*a*b^3*c^3*d*f*h*m^3 + 2*a^3*b*c*d^3*f*h *m^3 - 25*a*b^3*c^2*d^2*e*h*m^2 - 25*a*b^3*c^2*d^2*f*g*m^2 + 20*a^2*b^2*c*d ^3*e*h*m^2 + 20*a^2*b^2*c*d^3*f*g*m^2 - 3*a*b^3*c^2*d^2*e*h*m^3 - 3*a*b^3*c ^2*d^2*f*g*m^3 + 3*a^2*b^2*c*d^3*e*h*m^3 + 3*a^2*b^2*c*d^3*f*g*m^3 + 15*a^2 *b^2*c^2*d^2*f*h*m - 30*a*b^3*c*d^3*e*g*m - 46*a*b^3*c^3*d*f*h*m + 36*a^3*b *c*d^3*f*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m ^4 + 24)) - (x^2*(a + b*x)^m*(8*a^4*d^4*e*h + 8*a^4*d^4*f*g - 12*b^4*c^4*e* h - 12*b^4*c^4*f*g - 60*b^4*c^3*d*e*g + 20*a^4*c*d^3*f*h + 14*a^4*d^4*e*h*m + 14*a^4*d^4*f*g*m - 19*b^4*c^4*e*h*m - 19*b^4*c^4*f*g*m + 7*a^4*d^4*e*h*m ^2 + 7*a^4*d^4*f*g*m^2 - 8*b^4*c^4*e*h*m^2 - 8*b^4*c^4*f*g*m^2 + a^4*d^4*e* h*m^3 + a^4*d^4*f*g*m^3 - b^4*c^4*e*h*m^3 - b^4*c^4*f*g*m^3 + 48*a^2*b^2*c^ 2*d^2*e*h + 48*a^2*b^2*c^2*d^2*f*g + 48*a*b^3*c^3*d*e*h + 48*a*b^3*c^3*d*f* g - 32*a^3*b*c*d^3*e*h - 32*a^3*b*c*d^3*f*g + 2*a^3*b*d^4*e*g*m - 4*a*b^3*c

```
^4*f*h*m - 47*b^4*c^3*d*e*g*m + 29*a^4*c*d^3*f*h*m - 80*a^3*b*c^2*d^2*f*h +
 3*a^3*b*d^4*e*g*m^2 + a^3*b*d^4*e*g*m^3 - 5*a*b^3*c^4*f*h*m^2 - a*b^3*c^4*
f*h*m^3 - 12*b^4*c^3*d*e*g*m^2 - b^4*c^3*d*e*g*m^3 + 10*a^4*c*d^3*f*h*m^2 +
 a^4*c*d^3*f*h*m^3 + 3*a^2*b^2*c^2*d^2*e*h*m^2 + 3*a^2*b^2*c^2*d^2*f*g*m^2
+ 60*a*b^3*c^2*d^2*e*g*m - 15*a^2*b^2*c*d^3*e*g*m + 14*a*b^3*c^3*d*e*h*m^2
+ 14*a*b^3*c^3*d*f*g*m^2 - 16*a^3*b*c*d^3*e*h*m^2 - 16*a^3*b*c*d^3*f*g*m^2
+ 2*a*b^3*c^3*d*e*h*m^3 + 2*a*b^3*c^3*d*f*g*m^3 - 2*a^3*b*c*d^3*e*h*m^3 - 2
*a^3*b*c*d^3*f*g*m^3 + 41*a^2*b^2*c^3*d*f*h*m - 66*a^3*b*c^2*d^2*f*h*m + 27
*a*b^3*c^2*d^2*e*g*m^2 - 18*a^2*b^2*c*d^3*e*g*m^2 + 3*a*b^3*c^2*d^2*e*g*m^3
 - 3*a^2*b^2*c*d^3*e*g*m^3 + 15*a^2*b^2*c^2*d^2*e*h*m + 15*a^2*b^2*c^2*d^2*
f*g*m + 20*a^2*b^2*c^3*d*f*h*m^2 - 25*a^3*b*c^2*d^2*f*h*m^2 + 3*a^2*b^2*c^3
*d*f*h*m^3 - 3*a^3*b*c^2*d^2*f*h*m^3 + 36*a*b^3*c^3*d*e*h*m + 36*a*b^3*c^3*
d*f*g*m - 46*a^3*b*c*d^3*e*h*m - 46*a^3*b*c*d^3*f*g*m))/((a*d - b*c)^4*(c +
 d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (b*d*x^4*(a + b*x)^m*(
5*b*c - a*d*m + b*c*m)*(6*b^2*d^2*e*g + 12*a^2*d^2*f*h + 2*b^2*c^2*f*h + 7*
a^2*d^2*f*h*m + 3*b^2*c^2*f*h*m + a^2*d^2*f*h*m^2 + b^2*c^2*f*h*m^2 - 8*a*b
*d^2*e*h - 8*a*b*d^2*f*g + 2*b^2*c*d*e*h + 2*b^2*c*d*f*g - 2*a*b*d^2*e*h*m
- 2*a*b*d^2*f*g*m + 2*b^2*c*d*e*h*m + 2*b^2*c*d*f*g*m - 8*a*b*c*d*f*h - 10*
a*b*c*d*f*h*m - 2*a*b*c*d*f*h*m^2))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m
+ 35*m^2 + 10*m^3 + m^4 + 24))
```

3.132 $\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 815

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{(bc-ad)^2(adf+b(cf(2+m)-de(3+m)))(cfh(4+m)-d(fg+eh(3+m)))(c+dx)^{-3-m}(e+fx)^{1+m}}{d^4f^2(de-cf)(3+m)}$$
$$- \frac{b(bc-ad)(cfh(4+m)-d(fg+eh(3+m)))(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f^2}$$
$$+ \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df}$$
$$- \frac{(bc-ad)^2(3adfh-b(cfh(4+m)-d(fg+ehm)))(c+dx)^{-2-m}(e+fx)^{1+m}}{d^4f(de-cf)(2+m)}$$
$$+ \frac{(bc-ad)(cfh(4+m)-d(fg+eh(3+m)))\left(2a^2d^2f^2+2abdf(cf(1+m)-de(3+m))+b^2(c^2f^2(2+3}{d^4f^2(de-cf)^2(2+m)(3}$$
$$- \frac{(bc-ad)(adf-b(2de(2+m)-cf(3+2m)))(3adfh-b(cfh(4+m)-d(fg+ehm)))(c+dx)^{-1-m}(e}{d^4f(de-cf)^2(1+m)(2+m)}$$
$$- \frac{(bc-ad)(cfh(4+m)-d(fg+eh(3+m)))\left(2a^2d^2f^2+2abdf(cf(1+m)-de(3+m))+b^2(c^2f^2(2+3}{d^4f(de-cf)^3(1+m)(2+m}$$
$$- \frac{b^2(3adfh-b(cfh(4+m)-d(fg+ehm)))(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m\right.}{d^5fm}$$

```
[Out] (-a*d+b*c)^2*(a*d*f+b*(c*f*(2+m)-d*e*(3+m)))*(c*f*h*(4+m)-d*(f*g+e*h*(3+m))
)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^4/f^2/(-c*f+d*e)/(3+m)-b*(-a*d+b*c)*(c*f*h
*(4+m)-d*(f*g+e*h*(3+m)))*(b*x+a)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^3/f^2+h*(b
*x+a)^3*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d/f-(-a*d+b*c)^2*(3*a*d*f*h-b*(c*f*h*(
4+m)-d*(e*h*m+f*g)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)/(2+m)+(-
a*d+b*c)*(c*f*h*(4+m)-d*(f*g+e*h*(3+m)))*(2*a^2*d^2*f^2+2*a*b*d*f*(c*f*(1+m
```

```
)-d*e*(3+m))+b^2*(c^2*f^2*(m^2+3*m+2)-2*c*d*e*f*(m^2+4*m+3)+d^2*e^2*(m^2+5*
m+6)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^4/f^2/(-c*f+d*e)^2/(2+m)/(3+m)-(-a*d+
b*c)*(a*d*f-b*(2*d*e*(2+m)-c*f*(3+2*m)))*(3*a*d*f*h-b*(c*f*h*(4+m)-d*(e*h*m
+f*g)))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)^2/(1+m)/(2+m)-(-a*d+b
*c)*(c*f*h*(4+m)-d*(f*g+e*h*(3+m)))*(2*a^2*d^2*f^2+2*a*b*d*f*(c*f*(1+m)-d*e
*(3+m))+b^2*(c^2*f^2*(m^2+3*m+2)-2*c*d*e*f*(m^2+4*m+3)+d^2*e^2*(m^2+5*m+6))
)*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)-b^2*(3*
a*d*f*h-b*(c*f*h*(4+m)-d*(e*h*m+f*g)))*(f*x+e)^m*hypergeom([-m, -m],[1-m],-
f*(d*x+c)/(-c*f+d*e))/d^5/f/m/((d*x+c)^m)/((d*(f*x+e)/(-c*f+d*e))^m)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 803, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {158, 165, 91, 80, 72, 71, 92, 47, 37}

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{h(a+bx)^3(e+fx)^{m+1}(c+dx)^{-m-3}}{df}$$
$$+\frac{(bc-ad)^2(adf+bc(m+2)f-bde(m+3))(cfh(m+4)-d(fg+eh(m+3)))(e+fx)^{m+1}(c+dx)^{-m}}{d^4f^2(de-cf)(m+3)}$$
$$-\frac{b(bc-ad)(cfh(m+4)-d(fg+eh(m+3)))(a+bx)(e+fx)^{m+1}(c+dx)^{-m-3}}{d^3f^2}$$
$$-\frac{(bc-ad)^2(bdfg+3adfh+bdehm-bcfh(m+4))(e+fx)^{m+1}(c+dx)^{-m-2}}{d^4f(de-cf)(m+2)}$$
$$-\frac{(bc-ad)(dfg+deh(m+3)-cfh(m+4))\left(\left(d^2(m^2+5m+6)e^2-2cdf(m^2+4m+3)e+c^2f^2(m^2+\right.\right.}{d^4f^2(de-cf)^2(m+2)(}$$
$$-\frac{(bc-ad)(bdfg+3adfh+bdehm-bcfh(m+4))(adf+bc(2m+3)f-2bde(m+2))(e+fx)^{m+1}(c+}{d^4f(de-cf)^2(m+1)(m+2)}$$
$$+\frac{(bc-ad)(dfg+deh(m+3)-cfh(m+4))\left(\left(d^2(m^2+5m+6)e^2-2cdf(m^2+4m+3)e+c^2f^2(m^2+\right.\right.}{d^4f(de-cf)^3(m+1)(m+}$$
$$-\frac{b^2(bdfg+3adfh+bdehm-bcfh(m+4))(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-\right.}{d^5fm}$$

```
[In] Int[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)^2*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c*f*h*(4 + m) - d*(
f*g + e*h*(3 + m)))*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c
*f)*(3 + m)) - (b*(b*c - a*d)*(c*f*h*(4 + m) - d*(f*g + e*h*(3 + m)))*(a +
b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^3*f^2) + (h*(a + b*x)^3*(c +
d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*f) - ((b*c - a*d)^2*(b*d*f*g + 3*a*d*f*
h + b*d*e*h*m - b*c*f*h*(4 + m))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4
*f*(d*e - c*f)*(2 + m)) - ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) - c*f*h*(4 +
m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^2*(c^2*f^2*(
```

```
2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m + m^2)))*(c +
 d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c*f)^2*(2 + m)*(3 + m)) -
 ((b*c - a*d)*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))*(a*d*f -
2*b*d*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d
^4*f*(d*e - c*f)^2*(1 + m)*(2 + m)) + ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) -
 c*f*h*(4 + m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^
2*(c^2*f^2*(2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m +
 m^2)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^4*f*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))
*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f*(c + d*x))/(d*e - c*f))]
)/(d^5*f*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\text{integral} = \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} + \frac{\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(-3bceh+a(dfg-cfh(1+m)+deh(3+m))+(bdfg+3adfh+bd}{df}$$

$$= \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} - \frac{((bc-ad)(dfg+deh(3+m)-cfh(4+m)))\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m\,dx}{d^2f} + \frac{(bdfg+3adfh+bdehm-bcfh(4+m))\int (a+bx)^2(c+dx)^{-3-m}(e+fx)^m\,dx}{d^2f}$$

$$= \frac{b(bc-ad)(dfg+deh(3+m)-cfh(4+m))(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f^2} + \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} - \frac{(bc-ad)^2(bdfg+3adfh+bdehm-bcfh(4+m))(c+dx)^{-2-m}(e+fx)^{1+m}}{d^4f(de-cf)(2+m)} + \frac{((bc-ad)(dfg+deh(3+m)-cfh(4+m)))\int (c+dx)^{-4-m}(e+fx)^m\left(-a^2df-b(bce+acf(1\right.}{d^3f^2} + \frac{(bdfg+3adfh+bdehm-bcfh(4+m))\int (c+dx)^{-2-m}(e+fx)^m\left(-a^2d^2f+b^2c(cf(1+m)-d\right.}{d^4f(de-cf)(2+m)}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m)-cfh(4+m))(c+dx)^{-3-m}(e+}{d^4f^2(de-cf)(3+m)} + \frac{b(bc-ad)(dfg+deh(3+m)-cfh(4+m))(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f^2} + \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} - \frac{(bc-ad)^2(bdfg+3adfh+bdehm-bcfh(4+m))(c+dx)^{-2-m}(e+fx)^{1+m}}{d^4f(de-cf)(2+m)} - \frac{(bc-ad)(bdfg+3adfh+bdehm-bcfh(4+m))(adf-2bde(2+m)+bcf(3+2m))(c+dx)^{-1}}{d^4f(de-cf)^2(1+m)(2+m)} + \frac{(b^2(bdfg+3adfh+bdehm-bcfh(4+m)))\int (c+dx)^{-1-m}(e+fx)^m\,dx}{d^4f} - \frac{\left(((bc-ad)(dfg+deh(3+m)-cfh(4+m))\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d}-\frac{2f\left(b^2ce+a^2df+ab(cf(1+m)-d\right.}{de-cf}\right.\right.}{d^3f^2(3+m)}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m)-cfh(4+m))(c+dx)^{-3-m}(e}{d^4f^2(de-cf)(3+m)}$$
$$+\frac{b(bc-ad)(dfg+deh(3+m)-cfh(4+m))(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f^2}$$
$$+\frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df}$$
$$-\frac{(bc-ad)^2(bdfg+3adfh+bdehm-bcfh(4+m))(c+dx)^{-2-m}(e+fx)^{1+m}}{d^4f(de-cf)(2+m)}$$
$$+\frac{(bc-ad)(dfg+deh(3+m)-cfh(4+m))\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d}-\frac{2f(b^2ce+a^2df+ab(cf(1+m)-d}{de-cf}\right.}{d^3f^2(de-cf)(2+m)(3+m)}$$
$$-\frac{(bc-ad)(bdfg+3adfh+bdehm-bcfh(4+m))(adf-2bde(2+m)+bcf(3+2m))(c+dx)}{d^4f(de-cf)^2(1+m)(2+m)}$$
$$+\frac{\left((bc-ad)(dfg+deh(3+m)-cfh(4+m))\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d}-\frac{2f(b^2ce+a^2df+ab(cf(1+m)-}{de-cf}\right.\right.}{d^3f(de-cf)(2+m)(3+m)}$$
$$+\frac{\left(b^2(bdfg+3adfh+bdehm-bcfh(4+m))(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\right)\int(c+dx)^{-1-m}\left(\frac{de}{de-cf}+\right.}{d^4f}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m)-cfh(4+m))(c+dx)^{-3-m}(e}{d^4f^2(de-cf)(3+m)}$$
$$+\frac{b(bc-ad)(dfg+deh(3+m)-cfh(4+m))(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f^2}$$
$$+\frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df}$$
$$-\frac{(bc-ad)^2(bdfg+3adfh+bdehm-bcfh(4+m))(c+dx)^{-2-m}(e+fx)^{1+m}}{d^4f(de-cf)(2+m)}$$
$$+\frac{(bc-ad)(dfg+deh(3+m)-cfh(4+m))\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d}-\frac{2f(b^2ce+a^2df+ab(cf(1+m)-d}{de-cf}\right.}{d^3f^2(de-cf)(2+m)(3+m)}$$
$$-\frac{(bc-ad)(bdfg+3adfh+bdehm-bcfh(4+m))(adf-2bde(2+m)+bcf(3+2m))(c+dx)}{d^4f(de-cf)^2(1+m)(2+m)}$$
$$-\frac{(bc-ad)(dfg+deh(3+m)-cfh(4+m))\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d}-\frac{2f(b^2ce+a^2df+ab(cf(1+m)-d}{de-cf}\right.}{d^3f(de-cf)^2(1+m)(2+m)(3+m)}$$
$$-\frac{b^2(bdfg+3adfh+bdehm-bcfh(4+m))(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}{}_2F_1\left(-m,-m;1\right.}{d^5fm}$$

Mathematica [F]

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
[Out] Integrate[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x), x]
```

Maple [F]

$$\int (bx+a)^3\,(dx+c)^{-4-m}\,(fx+e)^m\,(hx+g)\,dx$$

```
[In] int((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
[Out] int((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)^3(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
[Out] integral((b^3*h*x^4 + a^3*g + (b^3*g + 3*a*b^2*h)*x^3 + 3*(a*b^2*g + a^2*b*
h)*x^2 + (3*a^2*b*g + a^3*h)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**3*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \int (bx+a)^3(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Giac [F]

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \int (bx+a)^3(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int \frac{(e+f\,x)^m\,(g+h\,x)\,(a+b\,x)^3}{(c+d\,x)^{m+4}}\,dx$$

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x)^3)/(c + d*x)^(m + 4),x)
[Out] int(((e + f*x)^m*(g + h*x)*(a + b*x)^3)/(c + d*x)^(m + 4), x)
```

3.133 $\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 572

$$
\begin{aligned}
&\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx \\
&= \frac{(bc-ad)(dg-ch)(adf+b(cf(2+m)-de(3+m)))(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f(de-cf)(3+m)} \\
&\quad - \frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f} - \frac{(bc-ad)^2h(c+dx)^{-2-m}(e+fx)^{1+m}}{d^3(de-cf)(2+m)} \\
&\quad - \frac{(dg-ch)\left(b^2(de-cf)(2+m)(cf(1+m)-de(3+m))-2df(b^2ce+a^2df+ab(cf(1+m)-de(3+m)\right.}{d^3f(de-cf)^2(2+m)(3+m)} \\
&\quad - \frac{(bc-ad)h(adf-b(2de(2+m)-cf(3+2m)))(c+dx)^{-1-m}(e+fx)^{1+m}}{d^3(de-cf)^2(1+m)(2+m)} \\
&\quad + \frac{(dg-ch)\left(b^2(de-cf)(2+m)(cf(1+m)-de(3+m))-2df(b^2ce+a^2df+ab(cf(1+m)-de(3+m)\right.}{d^3(de-cf)^3(1+m)(2+m)(3+m)} \\
&\quad - \frac{b^2h(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{f(c+dx)}{de-cf}\right)}{d^4m}
\end{aligned}
$$

```
[Out] (-a*d+b*c)*(-c*h+d*g)*(a*d*f+b*(c*f*(2+m)-d*e*(3+m)))*(d*x+c)^(-3-m)*(f*x+e
)^(1+m)/d^3/f/(-c*f+d*e)/(3+m)-b*(-c*h+d*g)*(b*x+a)*(d*x+c)^(-3-m)*(f*x+e)^
(1+m)/d^2/f-(-a*d+b*c)^2*h*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)/(2+m
)-(-c*h+d*g)*(b^2*(-c*f+d*e)*(2+m)*(c*f*(1+m)-d*e*(3+m))-2*d*f*(b^2*c*e+a^2
*d*f+a*b*(c*f*(1+m)-d*e*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^3/f/(-c*f+d
*e)^2/(2+m)/(3+m)-(-a*d+b*c)*h*(a*d*f-b*(2*d*e*(2+m)-c*f*(3+2*m)))*(d*x+c)^
(-1-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)^2/(1+m)/(2+m)+(-c*h+d*g)*(b^2*(-c*f+d*e
)*(2+m)*(c*f*(1+m)-d*e*(3+m))-2*d*f*(b^2*c*e+a^2*d*f+a*b*(c*f*(1+m)-d*e*(3+
m))))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)-b^2*h
*(f*x+e)^m*hypergeom([-m, -m],[1-m],-f*(d*x+c)/(-c*f+d*e))/d^4/m/((d*x+c)^m
)/((d*(f*x+e)/(-c*f+d*e))^m)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {165, 91, 80, 72, 71, 92, 47, 37}

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx =$$
$$-\frac{(dg-ch)(c+dx)^{-m-2}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df(a^2df+b(acf(\right.}{d^3f(m+2)(m+3)(de-cf)^2}$$
$$+\frac{(dg-ch)(c+dx)^{-m-1}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df(a^2df+b(acf(\right.}{d^3(m+1)(m+2)(m+3)(de-cf)^3}$$
$$+\frac{(bc-ad)(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}(adf+bcf(m+2)-bde(m+3))}{d^3f(m+3)(de-cf)}$$
$$-\frac{h(bc-ad)^2(c+dx)^{-m-2}(e+fx)^{m+1}}{d^3(m+2)(de-cf)}$$
$$-\frac{h(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{d^3(m+1)(m+2)(de-cf)^2}$$
$$-\frac{b(a+bx)(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d^2f}$$
$$-\frac{b^2h(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{f(c+dx)}{de-cf}\right)}{d^4m}$$

```
[In] Int[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(d*g - c*h)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c + d*x)^
(-3 - m)*(e + f*x)^(1 + m))/(d^3*f*(d*e - c*f)*(3 + m)) - (b*(d*g - c*h)*(a
 + b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^2*f) - ((b*c - a*d)^2*h*(c
 + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)*(2 + m)) - ((d*g - c*h
)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b
*(b*c*e + a*c*f*(1 + m) - a*d*e*(3 + m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1
+ m))/(d^3*f*(d*e - c*f)^2*(2 + m)*(3 + m)) - ((b*c - a*d)*h*(a*d*f - 2*b*d
*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d
*e - c*f)^2*(1 + m)*(2 + m)) + ((d*g - c*h)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(
1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b*(b*c*e + a*c*f*(1 + m) - a*d*e*(
3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*h*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f
*(c + d*x))/(d*e - c*f))])/(d^4*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m
)
```

<u>Rule 37</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\text{integral} = \frac{h\int(a+bx)^2(c+dx)^{-3-m}(e+fx)^m\,dx}{d} + \frac{(dg-ch)\int(a+bx)^2(c+dx)^{-4-m}(e+fx)^m\,dx}{d}$$

$$= -\frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f} - \frac{(bc-ad)^2h(c+dx)^{-2-m}(e+fx)^{1+m}}{d^3(de-cf)(2+m)} - \frac{(dg-ch)\int(c+dx)^{-4-m}(e+fx)^m\left(-a^2df-b(bce+acf(1+m)-ade(3+m))+b^2(de-cf)\right.}{d^2f} + \frac{h\int(c+dx)^{-2-m}(e+fx)^m\left(-a^2d^2f+b^2c(cf(1+m)-de(2+m))-2abd(cf(1+m)-de(2\right.}{d^3(de-cf)(2+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f(de-cf)(3+m)} - \frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f} - \frac{(bc-ad)^2h(c+dx)^{-2-m}(e+fx)^{1+m}}{d^3(de-cf)(2+m)} - \frac{(bc-ad)h(adf-2bde(2+m)+bcf(3+2m))(c+dx)^{-1-m}(e+fx)^{1+m}}{d^3(de-cf)^2(1+m)(2+m)} + \frac{(b^2h)\int(c+dx)^{-1-m}(e+fx)^m\,dx}{d^3} + \frac{\left((dg-ch)\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d}-\frac{2f\left(b^2ce+a^2df+ab(cf(1+m)-de(3+m))\right)}{de-cf}\right)\right)\int(c+dx)^{-3-m}(e+f}{d^2f(3+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f(de-cf)(3+m)}$$
$$- \frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f}$$
$$- \frac{(bc-ad)^2h(c+dx)^{-2-m}(e+fx)^{1+m}}{d^3(de-cf)(2+m)}$$
$$- \frac{(dg-ch)\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d} - \frac{2f(b^2ce+a^2df+ab(cf(1+m)-de(3+m)))}{de-cf}\right)(c+dx)^{-2-m}(e+fx)^{1+m}}{d^2f(de-cf)(2+m)(3+m)}$$
$$- \frac{(bc-ad)h(adf-2bde(2+m)+bcf(3+2m))(c+dx)^{-1-m}(e+fx)^{1+m}}{d^3(de-cf)^2(1+m)(2+m)}$$
$$- \frac{\left((dg-ch)\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d} - \frac{2f(b^2ce+a^2df+ab(cf(1+m)-de(3+m)))}{de-cf}\right)\right)\int(c+dx)^{-2-m}(e+fx)}{d^2(de-cf)(2+m)(3+m)}$$
$$+ \frac{\left(b^2h(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\right)\int(c+dx)^{-1-m}\left(\frac{de}{de-cf}+\frac{dfx}{de-cf}\right)^m dx}{d^3}$$
$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f(de-cf)(3+m)}$$
$$- \frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f}$$
$$- \frac{(bc-ad)^2h(c+dx)^{-2-m}(e+fx)^{1+m}}{d^3(de-cf)(2+m)} - \frac{(dg-ch)\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d} - \frac{2f(b^2ce+a^2df+ab(cf(1+}{de-cf}\right.}{d^2f(de-cf)(2+m)(3}$$
$$- \frac{(bc-ad)h(adf-2bde(2+m)+bcf(3+2m))(c+dx)^{-1-m}(e+fx)^{1+m}}{d^3(de-cf)^2(1+m)(2+m)}$$
$$+ \frac{(dg-ch)\left(\frac{b^2(2+m)(cf(1+m)-de(3+m))}{d} - \frac{2f(b^2ce+a^2df+ab(cf(1+m)-de(3+m)))}{de-cf}\right)(c+dx)^{-1-m}(e+fx)^{1+m}}{d^2(de-cf)^2(1+m)(2+m)(3+m)}$$
$$- \frac{b^2h(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m} {}_2F_1\left(-m,-m;1-m;-\frac{f(c+dx)}{de-cf}\right)}{d^4m}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.74

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{(c+dx)^{-3-m}(e+fx)^m\left(-d(dg-ch)(e+fx)\left(-((bc-ad)(de-cf)^2(1+m)(2+m)(adf+bcf(2+m)\right.\right.}{}$$

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^m*(-(d*(d*g - c*h)*(e + f*x)*(-((b*c - a*d)*(
d*e - c*f)^2*(1 + m)*(2 + m)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))) + b*d
*(d*e - c*f)^3*(1 + m)*(2 + m)*(3 + m)*(a + b*x) + (b^2*(d*e - c*f)*(2 + m)
*(c*f*(1 + m) - d*e*(3 + m)) + 2*d*f*(-(a^2*d*f) - b*(b*c*e + a*c*f*(1 + m)
 - a*d*e*(3 + m))))*(c + d*x)*(-(c*f*(2 + m)) + d*(e + e*m - f*x)))) - (d*e
 - c*f)*h*(3 + m)*(c + d*x)*(d*(b*c - a*d)^2*f*(d*e - c*f)*(1 + m)*(e + f*x
) - (c + d*x)*(d*(a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(2 + m)) + b^2
*(-(c^2*f^2*(1 + m)) + d^2*e^2*(2 + m)))*(e + f*x) - (b^2*(d*e - c*f)^3*(2
+ m)*Hypergeometric2F1[-1 - m, -1 - m, -m, (f*(c + d*x))/(-(d*e) + c*f)])/(
(d*(e + f*x))/(d*e - c*f))^m))))/(d^4*f*(d*e - c*f)^3*(1 + m)*(2 + m)*(3 +
m))
```

Maple [F]

$$\int (bx+a)^2 (dx+c)^{-4-m} (fx+e)^m (hx+g)\, dx$$

```
[In] int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
[Out] int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\, dx = \int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\, dx$$

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
[Out] integral((b^2*h*x^3 + a^2*g + (b^2*g + 2*a*b*h)*x^2 + (2*a*b*g + a^2*h)*x)*
(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**2*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Giac [F]

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int \frac{(e+f\,x)^m\,(g+h\,x)\,(a+b\,x)^2}{(c+d\,x)^{m+4}}\,dx$$

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4),x)
[Out] int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4), x)
```

3.134 $\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 363

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{(b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+m))+cdf(1+m)(fg-2eh(3+m)))+adf(cfh(1+}{d^2f(de-cf)^2(2+m)(3+m)}$$
$$-\frac{(b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+m))+cdf(1+m)(fg-2eh(3+m)))+adf(cfh(1}{d^2(de-cf)^3(1+m)(2+m)(3+m)}$$
$$-\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(cfh(2+m)+d(fg-eh(3+m)))+bd(de-cf)h(3+m}{d^2f(de-cf)(3+m)}$$

```
[Out] (b*(c^2*f^2*h*(m^2+3*m+2)-d^2*e*(3+m)*(f*g-e*h*(2+m))+c*d*f*(1+m)*(f*g-2*e*
h*(3+m)))+a*d*f*(c*f*h*(1+m)+d*(2*f*g-e*h*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(
1+m)/d^2/f/(-c*f+d*e)^2/(2+m)/(3+m)-(b*(c^2*f^2*h*(m^2+3*m+2)-d^2*e*(3+m)*(
f*g-e*h*(2+m))+c*d*f*(1+m)*(f*g-2*e*h*(3+m)))+a*d*f*(c*f*h*(1+m)+d*(2*f*g-e
*h*(3+m))))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^2/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)
-(d*x+c)^(-3-m)*(f*x+e)^(1+m)*(a*d*f*(-c*h+d*g)-b*c*(c*f*h*(2+m)+d*(f*g-e*h
*(3+m)))+b*d*(-c*f+d*e)*h*(3+m)*x)/d^2/f/(-c*f+d*e)/(3+m)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used

$= \{151, 47, 37\}$

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b(c^2f^2h(m^2+3m+2)+cdf(m+1)\right.}{d^2f(m+2)(m+3)(de-cf)^2}$$
$$- \frac{(c+dx)^{-m-1}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b(c^2f^2h(m^2+3m+2)+cdf(m+\right.}{d^2(m+1)(m+2)(m+3)(de-cf)^3}$$
$$- \frac{(c+dx)^{-m-3}(e+fx)^{m+1}(adf(dg-ch)-bc(cfh(m+2)-deh(m+3)+dfg)+bdh(m+3)x(de-cf}{d^2f(m+3)(de-cf)}$$

```
[In] Int[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m +
 m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 +
 m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^2*f*(d*e - c*f)^2*(2 + m)*(
3 + m)) - ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*
(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g -
2*e*h*(3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^2*(d*e - c*f)^3*(
1 + m)*(2 + m)*(3 + m)) - ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g
 - c*h) - b*c*(d*f*g + c*f*h*(2 + m) - d*e*h*(3 + m)) + b*d*(d*e - c*f)*h*(
3 + m)*x))/(d^2*f*(d*e - c*f)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
```

```
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

integral =

$$-\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(dfg+cfh(2+m)-deh(3+m))+bd(de-cf)h(3+m}{d^2f(de-cf)(3+m)}$$

$$-\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+m))+ca}{d^2f(de-cf)(3+m)}$$

$$=\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+m)}{d^2f(de-cf)^2(2+m)(3+m)}$$

$$-\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(dfg+cfh(2+m)-deh(3+m))+bd(de-cf)h}{d^2f(de-cf)(3+m)}$$

$$+\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+r}{d^2(de-cf)^2(2+m)(3+m)}$$

$$=\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+m)}{d^2f(de-cf)^2(2+m)(3+m)}$$

$$-\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+r}{d^2(de-cf)^3(1+m)(2+m)(3+}$$

$$-\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(dfg+cfh(2+m)-deh(3+m))+bd(de-cf)h}{d^2f(de-cf)(3+m)}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.63

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$

$$=\frac{(c+dx)^{-3-m}(e+fx)^{1+m}\left(adf(dg-ch)+\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)+d^2e(3+m)(-fg+eh(2+}{(de-cf)^2(1+m)(2-}\right.}{}$$

```
[In] Integrate[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g - c*h) + ((a*d*f*(2*d*f*g
 + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m + m^2) + d^2*e*(3
 + m)*(-(f*g) + e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 + m))))*(c + d
```

```
*x)*(c*f*(2 + m) - d*(e + e*m - f*x)))/((d*e - c*f)^2*(1 + m)*(2 + m)) - b*
(c^2*f*h*(2 + m) - d^2*e*h*(3 + m)*x + c*d*(-(e*h*(3 + m)) + f*(g + h*(3 +
m)*x)))))/(d^2*f*(-(d*e) + c*f)*(3 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(363) = 726.

Time = 2.26 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.50

method	result
gosper	$-\frac{(dx+c)^{-3-m}(fx+e)^{1+m}(-b\,c^2f^2h\,m^2x^2+2bcdefh\,m^2x^2-b\,d^2e^2h\,m^2x^2-a\,c^2f^2h\,m^2x+2acdefh\,m^2x-acd\,f^2hm\,x^2-a\,d^2e^2}{}$
parallelrisch	Expression too large to display

```
[In] int((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x+c)^(-3-m)*(f*x+e)^(1+m)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f*
m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3*e
^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3*f^
3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)*(-b*c^2*f^2*h*m^2*x^2+2*b*c*d*e*
f*h*m^2*x^2-b*d^2*e^2*h*m^2*x^2-a*c^2*f^2*h*m^2*x+2*a*c*d*e*f*h*m^2*x-a*c*d
*f^2*h*m*x^2-a*d^2*e^2*h*m^2*x+a*d^2*e*f*h*m*x^2-b*c^2*f^2*g*m^2*x-3*b*c^2*
f^2*h*m*x^2+2*b*c*d*e*f*g*m^2*x+8*b*c*d*e*f*h*m*x^2-b*c*d*f^2*g*m*x^2-b*d^2
*e^2*g*m^2*x-5*b*d^2*e^2*h*m*x^2+b*d^2*e*f*g*m*x^2-a*c^2*f^2*g*m^2-4*a*c^2*
f^2*h*m*x+2*a*c*d*e*f*g*m^2+8*a*c*d*e*f*h*m*x-2*a*c*d*f^2*g*m*x-a*c*d*f^2*h
*x^2-a*d^2*e^2*g*m^2-4*a*d^2*e^2*h*m*x+2*a*d^2*e*f*g*m*x+3*a*d^2*e*f*h*x^2-
2*a*d^2*f^2*g*x^2+2*b*c^2*e*f*h*m*x-4*b*c^2*f^2*g*m*x-2*b*c^2*f^2*h*x^2-2*b
*c*d*e^2*h*m*x+8*b*c*d*e*f*g*m*x+6*b*c*d*e*f*h*x^2-b*c*d*f^2*g*x^2-4*b*d^2*
e^2*g*m*x-6*b*d^2*e^2*h*x^2+3*b*d^2*e*f*g*x^2+a*c^2*e*f*h*m-5*a*c^2*f^2*g*m
-3*a*c^2*f^2*h*x-a*c*d*e^2*h*m+8*a*c*d*e*f*g*m+10*a*c*d*e*f*h*x-6*a*c*d*f^2
*g*x-3*a*d^2*e^2*g*m-3*a*d^2*e^2*h*x+2*a*d^2*e*f*g*x+b*c^2*e*f*g*m+2*b*c^2*
e*f*h*x-3*b*c^2*f^2*g*x-b*c*d*e^2*g*m-6*b*c*d*e^2*h*x+10*b*c*d*e*f*g*x-3*b*
d^2*e^2*g*x+3*a*c^2*e*f*h-6*a*c^2*f^2*g-a*c*d*e^2*h+6*a*c*d*e*f*g-2*a*d^2*e
^2*g-2*b*c^2*e^2*h+3*b*c^2*e*f*g-b*c*d*e^2*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(366) = 732.

Time = 0.31 (sec) , antiderivative size = 1608, normalized size of antiderivative = 4.43

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -((((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*h*m^2 - (3*b*d^3*e*f^2 - (
b*c*d^2 + 2*a*d^3)*f^3)*g + (6*b*d^3*e^2*f - 3*(2*b*c*d^2 + a*d^3)*e*f^2 +
(2*b*c^2*d + a*c*d^2)*f^3)*h - ((b*d^3*e*f^2 - b*c*d^2*f^3)*g - (5*b*d^3*e^
2*f - (8*b*c*d^2 + a*d^3)*e*f^2 + (3*b*c^2*d + a*c*d^2)*f^3)*h)*m)*x^4 + (a
*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3*e*f^2)*g*m^2 + (((b*d^3*e^2*f - 2*b*c*
d^2*e*f^2 + b*c^2*d*f^3)*g + (b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*
d + 2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*h)*m^2 - 4*(3*b*c*d^2*e*f^2 -
 (b*c^2*d + 2*a*c*d^2)*f^3)*g + 2*(3*b*d^3*e^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2
*d + 2*a*c*d^2)*e*f^2 + (b*c^3 + 2*a*c^2*d)*f^3)*h + ((3*b*d^3*e^2*f - 2*(4
*b*c*d^2 + a*d^3)*e*f^2 + (5*b*c^2*d + 2*a*c*d^2)*f^3)*g + (5*b*d^3*e^3 - (
b*c*d^2 - 3*a*d^3)*e^2*f - (7*b*c^2*d + 8*a*c*d^2)*e*f^2 + (3*b*c^3 + 5*a*c
^2*d)*f^3)*h)*m)*x^3 + (((b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*d +
2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*g + (a*c^3*f^3 + (b*c*d^2 + a*d^3
)*e^3 - (2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*h)*m^2 + 3*(
b*d^3*e^3 - 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + (b*c^3 + 4*a*c^2*d)*f^3)*g
- 3*(3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3 - (4*b*c*d^2 + a*d^3)*e^
3)*h + ((4*b*d^3*e^3 - (4*b*c*d^2 - a*d^3)*e^2*f - 4*(b*c^2*d + 2*a*c*d^2)*
e*f^2 + (4*b*c^3 + 7*a*c^2*d)*f^3)*g + (4*a*c^3*f^3 + (7*b*c*d^2 + 4*a*d^3)
*e^3 - 4*(2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - 4*a*c^2*d)*e*f^2)*h)*m)*x^2
 + (6*a*c^3*e*f^2 + (b*c^2*d + 2*a*c*d^2)*e^3 - 3*(b*c^3 + 2*a*c^2*d)*e^2*f
)*g - (3*a*c^3*e^2*f - (2*b*c^3 + a*c^2*d)*e^3)*h + ((5*a*c^3*e*f^2 + (b*c^
2*d + 3*a*c*d^2)*e^3 - (b*c^3 + 8*a*c^2*d)*e^2*f)*g + (a*c^2*d*e^3 - a*c^3*
e^2*f)*h)*m + (((a*c^3*f^3 + (b*c*d^2 + a*d^3)*e^3 - (2*b*c^2*d + a*c*d^2)*
e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*g + (a*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3
*e*f^2)*h)*m^2 + 2*(3*a*c^2*d*e*f^2 + 3*a*c^3*f^3 + (2*b*c*d^2 + a*d^3)*e^3
 - 3*(2*b*c^2*d + a*c*d^2)*e^2*f)*g - 4*(3*a*c^2*d*e^2*f - (2*b*c^2*d + a*c
*d^2)*e^3)*h + ((5*a*c^3*f^3 + (5*b*c*d^2 + 3*a*d^3)*e^3 - (8*b*c^2*d + 7*a
*c*d^2)*e^2*f + (3*b*c^3 - a*c^2*d)*e*f^2)*g + (3*a*c^3*e*f^2 + (2*b*c^2*d
+ 5*a*c*d^2)*e^3 - 2*(b*c^3 + 4*a*c^2*d)*e^2*f)*h)*m)*x)*(d*x + c)^(-m - 4)
*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^
3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2
*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2
*d*e*f^2 - c^3*f^3)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Giac [F]

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 1890, normalized size of antiderivative = 5.21

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x))/(c + d*x)^(m + 4),x)
```

```
[Out] ((e + f*x)^m*(2*b*c^3*e^3*h + 2*a*c*d^2*e^3*g + a*c^2*d*e^3*h + b*c^2*d*e^3
*g + 6*a*c^3*e*f^2*g - 3*a*c^3*e^2*f*h - 3*b*c^3*e^2*f*g - 6*a*c^2*d*e^2*f*
g + 3*a*c*d^2*e^3*g*m + a*c^2*d*e^3*h*m + b*c^2*d*e^3*g*m + 5*a*c^3*e*f^2*g
*m - a*c^3*e^2*f*h*m - b*c^3*e^2*f*g*m + a*c*d^2*e^3*g*m^2 + a*c^3*e*f^2*g*
m^2 - 2*a*c^2*d*e^2*f*g*m^2 - 8*a*c^2*d*e^2*f*g*m))/((c*f - d*e)^3*(c + d*x
)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*a*c^3*f^3*g + 2*a*d
^3*e^3*g + 4*a*c*d^2*e^3*h + 4*b*c*d^2*e^3*g + 8*b*c^2*d*e^3*h + 5*a*c^3*f^
3*g*m + 3*a*d^3*e^3*g*m + a*c^3*f^3*g*m^2 + a*d^3*e^3*g*m^2 - 6*a*c*d^2*e^2
*f*g + 6*a*c^2*d*e*f^2*g - 12*a*c^2*d*e^2*f*h - 12*b*c^2*d*e^2*f*g + 5*a*c*
d^2*e^3*h*m + 5*b*c*d^2*e^3*g*m + 2*b*c^2*d*e^3*h*m + 3*a*c^3*e*f^2*h*m + 3
*b*c^3*e*f^2*g*m - 2*b*c^3*e^2*f*h*m + a*c*d^2*e^3*h*m^2 + b*c*d^2*e^3*g*m^
2 + a*c^3*e*f^2*h*m^2 + b*c^3*e*f^2*g*m^2 - a*c*d^2*e^2*f*g*m^2 - a*c^2*d*e
*f^2*g*m^2 - 2*a*c^2*d*e^2*f*h*m^2 - 2*b*c^2*d*e^2*f*g*m^2 - 7*a*c*d^2*e^2*
f*g*m - a*c^2*d*e*f^2*g*m - 8*a*c^2*d*e^2*f*h*m - 8*b*c^2*d*e^2*f*g*m))/((c
*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x^4*(e + f*x)^m*
(2*a*d^3*f^3*g + a*c*d^2*f^3*h + b*c*d^2*f^3*g + 2*b*c^2*d*f^3*h - 3*a*d^3*
```

e*f^2*h - 3*b*d^3*e*f^2*g + 6*b*d^3*e^2*f*h - 6*b*c*d^2*e*f^2*h + a*c*d^2*f ^3*h*m + b*c*d^2*f^3*g*m + 3*b*c^2*d*f^3*h*m - a*d^3*e*f^2*h*m - b*d^3*e*f^ 2*g*m + 5*b*d^3*e^2*f*h*m + b*c^2*d*f^3*h*m^2 + b*d^3*e^2*f*h*m^2 - 2*b*c*d ^2*e*f^2*h*m^2 - 8*b*c*d^2*e*f^2*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11 *m + 6*m^2 + m^3 + 6)) + (x^2*(e + f*x)^m*(3*a*c^3*f^3*h + 3*a*d^3*e^3*h + 3*b*c^3*f^3*g + 3*b*d^3*e^3*g + 12*a*c^2*d*f^3*g + 12*b*c*d^2*e^3*h + 4*a*c ^3*f^3*h*m + 4*a*d^3*e^3*h*m + 4*b*c^3*f^3*g*m + 4*b*d^3*e^3*g*m + a*c^3*f^ 3*h*m^2 + a*d^3*e^3*h*m^2 + b*c^3*f^3*g*m^2 + b*d^3*e^3*g*m^2 - 9*a*c*d^2*e ^2*f*h - 9*a*c^2*d*e*f^2*h - 9*b*c*d^2*e^2*f*g - 9*b*c^2*d*e*f^2*g + 7*a*c^ 2*d*f^3*g*m + 7*b*c*d^2*e^3*h*m + a*d^3*e^2*f*g*m + b*c^3*e*f^2*h*m + a*c^2 *d*f^3*g*m^2 + b*c*d^2*e^3*h*m^2 + a*d^3*e^2*f*g*m^2 + b*c^3*e*f^2*h*m^2 - 2*a*c*d^2*e*f^2*g*m^2 - a*c*d^2*e^2*f*h*m^2 - a*c^2*d*e*f^2*h*m^2 - b*c*d^2 *e^2*f*g*m^2 - b*c^2*d*e*f^2*g*m^2 - 2*b*c^2*d*e^2*f*h*m^2 - 8*a*c*d^2*e*f^ 2*g*m - 4*a*c*d^2*e^2*f*h*m - 4*a*c^2*d*e*f^2*h*m - 4*b*c*d^2*e^2*f*g*m - 4 *b*c^2*d*e*f^2*g*m - 8*b*c^2*d*e^2*f*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4) *(11*m + 6*m^2 + m^3 + 6)) + (x^3*(e + f*x)^m*(2*b*c^3*f^3*h + 6*b*d^3*e^3* h + 8*a*c*d^2*f^3*g + 4*a*c^2*d*f^3*h + 4*b*c^2*d*f^3*g + 3*b*c^3*f^3*h*m + 5*b*d^3*e^3*h*m + b*c^3*f^3*h*m^2 + b*d^3*e^3*h*m^2 - 12*a*c*d^2*e*f^2*h - 12*b*c*d^2*e*f^2*g + 6*b*c*d^2*e^2*f*h - 6*b*c^2*d*e*f^2*h + 2*a*c*d^2*f^3 *g*m + 5*a*c^2*d*f^3*h*m + 5*b*c^2*d*f^3*g*m - 2*a*d^3*e*f^2*g*m + 3*a*d^3* e^2*f*h*m + 3*b*d^3*e^2*f*g*m + a*c^2*d*f^3*h*m^2 + b*c^2*d*f^3*g*m^2 + a*d ^3*e^2*f*h*m^2 + b*d^3*e^2*f*g*m^2 - 2*a*c*d^2*e*f^2*h*m^2 - 2*b*c*d^2*e*f^ 2*g*m^2 - b*c*d^2*e^2*f*h*m^2 - b*c^2*d*e*f^2*h*m^2 - 8*a*c*d^2*e*f^2*h*m - 8*b*c*d^2*e*f^2*g*m - b*c*d^2*e^2*f*h*m - 7*b*c^2*d*e*f^2*h*m))/((c*f - d* e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))

3.135 $\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\begin{aligned}\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx &= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)}\\ &\quad+\frac{(cfh(1+m)+d(2fg-eh(3+m)))(c+dx)^{-2-m}(e+fx)^{1+m}}{d(de-cf)^2(2+m)(3+m)}\\ &\quad-\frac{f(cfh(1+m)+d(2fg-eh(3+m)))(c+dx)^{-1-m}(e+fx)^{1+m}}{d(de-cf)^3(1+m)(2+m)(3+m)}\end{aligned}$$

```
[Out] -(-c*h+d*g)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)/(3+m)+(c*f*h*(1+m)+d*
(2*f*g-e*h*(3+m)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)^2/(2+m)/(3+m)-
f*(c*f*h*(1+m)+d*(2*f*g-e*h*(3+m)))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d/(-c*f+d*
e)^3/(1+m)/(2+m)/(3+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {80, 47, 37}

$$\begin{aligned}\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx &= -\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}\\ &\quad+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+2)(m+3)(de-cf)^2}\\ &\quad-\frac{f(c+dx)^{-m-1}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+1)(m+2)(m+3)(de-cf)^3}\end{aligned}$$

```
[In] Int[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] -(((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m)
)) + ((2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m))*(c + d*x)^(-2 - m)*(e + f*x
)^(1 + m))/(d*(d*e - c*f)^2*(2 + m)*(3 + m)) - (f*(2*d*f*g + c*f*h*(1 + m)
- d*e*h*(3 + m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)^3*(1
+ m)*(2 + m)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} \\
&\quad -\frac{(2dfg+cfh(1+m)-deh(3+m))\int (c+dx)^{-3-m}(e+fx)^m\,dx}{d(de-cf)(3+m)} \\
&= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} \\
&\quad +\frac{(2dfg+cfh(1+m)-deh(3+m))(c+dx)^{-2-m}(e+fx)^{1+m}}{d(de-cf)^2(2+m)(3+m)} \\
&\quad +\frac{(f(2dfg+cfh(1+m)-deh(3+m)))\int (c+dx)^{-2-m}(e+fx)^m\,dx}{d(de-cf)^2(2+m)(3+m)}
\end{aligned}
$$

$$= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} + \frac{(2dfg+cfh(1+m)-deh(3+m))(c+dx)^{-2-m}(e+fx)^{1+m}}{d(de-cf)^2(2+m)(3+m)} - \frac{f(2dfg+cfh(1+m)-deh(3+m))(c+dx)^{-1-m}(e+fx)^{1+m}}{d(de-cf)^3(1+m)(2+m)(3+m)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(-3-m)} - \frac{(-2dfg-h(de(-3-m)+cf(1+m)))\left(\frac{(c+dx)^{-2-m}(e+fx)^{1+m}}{(de-cf)(-2-m)}+\frac{f(c+dx)^{-1-m}(e+fx)^{1+m}}{(de-cf)^2(-2-m)(-1-m)}\right)}{d(de-cf)(-3-m)}$$

```
[In] Integrate[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(-3 - m))
 - ((-2*d*f*g - h*(d*e*(-3 - m) + c*f*(1 + m)))*(((c + d*x)^(-2 - m)*(e + f
*x)^(1 + m))/((d*e - c*f)*(-2 - m)) + (f*(c + d*x)^(-1 - m)*(e + f*x)^(1 +
m))/((d*e - c*f)^2*(-2 - m)*(-1 - m))))/(d*(d*e - c*f)*(-3 - m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(188) = 376.

Time = 1.92 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.71

method	result
gosper	$-\frac{(dx+c)^{-3-m}(fx+e)^{1+m}\left(-c^2f^2hm^2x+2cdefhm^2x-cdf^2hmx^2-d^2e^2hm^2x+d^2efhmx^2-c^2f^2gm^2-4c^2f^2hmx+2cdefg\right.}{c^3f^3m^3-3c^2def^2m^3+3cd^2e}$
parallelrisch	Expression too large to display

```
[In] int((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x+c)^(-3-m)*(f*x+e)^(1+m)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f*
m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3*e
^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3*f^
3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)*(-c^2*f^2*h*m^2*x+2*c*d*e*f*h*m^
2*x-c*d*f^2*h*m*x^2-d^2*e^2*h*m^2*x+d^2*e*f*h*m*x^2-c^2*f^2*g*m^2-4*c^2*f^2
*h*m*x+2*c*d*e*f*g*m^2+8*c*d*e*f*h*m*x-2*c*d*f^2*g*m*x-c*d*f^2*h*x^2-d^2*e^
```

```
2*g*m^2-4*d^2*e^2*h*m*x+2*d^2*e*f*g*m*x+3*d^2*e*f*h*x^2-2*d^2*f^2*g*x^2+c^2
*e*f*h*m-5*c^2*f^2*g*m-3*c^2*f^2*h*x-c*d*e^2*h*m+8*c*d*e*f*g*m+10*c*d*e*f*h
*x-6*c*d*f^2*g*x-3*d^2*e^2*g*m-3*d^2*e^2*h*x+2*d^2*e*f*g*x+3*c^2*e*f*h-6*c^
2*f^2*g-c*d*e^2*h+6*c*d*e*f*g-2*d^2*e^2*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(188) = 376$.

Time = 0.27 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.81

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{((2\,d^3f^3g-(d^3ef^2-cd^2f^3)hm-(3\,d^3ef^2-cd^2f^3)h)x^4+(cd^2e^3-2\,c^2de^2f+c^3ef^2)gm^2+(8\,cd^2f^3g}{}$$

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -((2*d^3*f^3*g - (d^3*e*f^2 - c*d^2*f^3)*h*m - (3*d^3*e*f^2 - c*d^2*f^3)*h)
*x^4 + (c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*g*m^2 + (8*c*d^2*f^3*g + (d^
3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*h*m^2 - 4*(3*c*d^2*e*f^2 - c^2*d*f^3)*
h - (2*(d^3*e*f^2 - c*d^2*f^3)*g - (3*d^3*e^2*f - 8*c*d^2*e*f^2 + 5*c^2*d*f
^3)*h)*m)*x^3 + (12*c^2*d*f^3*g + ((d^3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*
g + (d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m^2 + 3*(d^3*e^3 - 3
*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*h + ((d^3*e^2*f - 8*c*d^2*e*f^2 + 7
*c^2*d*f^3)*g + 4*(d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m)*x^2
 + 2*(c*d^2*e^3 - 3*c^2*d*e^2*f + 3*c^3*e*f^2)*g + (c^2*d*e^3 - 3*c^3*e^2*f
)*h + ((3*c*d^2*e^3 - 8*c^2*d*e^2*f + 5*c^3*e*f^2)*g + (c^2*d*e^3 - c^3*e^2
*f)*h)*m + (((d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*g + (c*d^2*e^3
 - 2*c^2*d*e^2*f + c^3*e*f^2)*h)*m^2 + 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d
*e*f^2 + 3*c^3*f^3)*g + 4*(c*d^2*e^3 - 3*c^2*d*e^2*f)*h + ((3*d^3*e^3 - 7*c
*d^2*e^2*f - c^2*d*e*f^2 + 5*c^3*f^3)*g + (5*c*d^2*e^3 - 8*c^2*d*e^2*f + 3*
c^3*e*f^2)*h)*m)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^
2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2
 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Giac [F]

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 869, normalized size of antiderivative = 4.62

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{x^2\,(e+f\,x)^m\,(h\,c^3\,f^3\,m^2+4\,h\,c^3\,f^3\,m+3\,h\,c^3\,f^3-h\,c^2\,d\,e\,f^2\,m^2-4\,h\,c^2\,d\,e\,f^2\,m-9\,h\,c^2\,d\,e\,f^2+g\,c^2\,d\,f^3}{}$$
$$+\frac{x\,(e+f\,x)^m\,(h\,c^3\,e\,f^2\,m^2+3\,h\,c^3\,e\,f^2\,m+g\,c^3\,f^3\,m^2+5\,g\,c^3\,f^3\,m+6\,g\,c^3\,f^3-2\,h\,c^2\,d\,e^2\,f\,m^2-8\,h\,c^2\,d}{}$$
$$+\frac{c\,e\,(e+f\,x)^m\,(-h\,c^2\,e\,f\,m-3\,h\,c^2\,e\,f+g\,c^2\,f^2\,m^2+5\,g\,c^2\,f^2\,m+6\,g\,c^2\,f^2+h\,c\,d\,e^2\,m+h\,c\,d\,e^2-2\,g\,c}{(c\,f-d\,e)^3\,(c+d\,x)^{m+4}\,(m^3+6\,m^2+1}$$
$$+\frac{d^2\,f^2\,x^4\,(e+f\,x)^m\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g+c\,f\,h\,m-d\,e\,h\,m)}{(c\,f-d\,e)^3\,(c+d\,x)^{m+4}\,(m^3+6\,m^2+11\,m+6)}$$
$$+\frac{d\,f\,x^3\,(e+f\,x)^m\,(4\,c\,f+c\,f\,m-d\,e\,m)\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g+c\,f\,h\,m-d\,e\,h\,m)}{(c\,f-d\,e)^3\,(c+d\,x)^{m+4}\,(m^3+6\,m^2+11\,m+6)}$$

```
[In] int(((e + f*x)^m*(g + h*x))/(c + d*x)^(m + 4),x)

[Out] (x^2*(e + f*x)^m*(3*c^3*f^3*h + 3*d^3*e^3*h + c^3*f^3*h*m^2 + d^3*e^3*h*m^2
 + 12*c^2*d*f^3*g + 4*c^3*f^3*h*m + 4*d^3*e^3*h*m - 9*c*d^2*e^2*f*h - 9*c^2
*d*e*f^2*h + 7*c^2*d*f^3*g*m + d^3*e^2*f*g*m + c^2*d*f^3*g*m^2 + d^3*e^2*f*
g*m^2 - 8*c*d^2*e*f^2*g*m - 4*c*d^2*e^2*f*h*m - 4*c^2*d*e*f^2*h*m - 2*c*d^2
*e*f^2*g*m^2 - c*d^2*e^2*f*h*m^2 - c^2*d*e*f^2*h*m^2))/((c*f - d*e)^3*(c +
d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*c^3*f^3*g + 2*d^
3*e^3*g + c^3*f^3*g*m^2 + d^3*e^3*g*m^2 + 4*c*d^2*e^3*h + 5*c^3*f^3*g*m + 3
*d^3*e^3*g*m - 6*c*d^2*e^2*f*g + 6*c^2*d*e*f^2*g - 12*c^2*d*e^2*f*h + 5*c*d
^2*e^3*h*m + 3*c^3*e*f^2*h*m + c*d^2*e^3*h*m^2 + c^3*e*f^2*h*m^2 - 7*c*d^2*
e^2*f*g*m - c^2*d*e*f^2*g*m - 8*c^2*d*e^2*f*h*m - c*d^2*e^2*f*g*m^2 - c^2*d
*e*f^2*g*m^2 - 2*c^2*d*e^2*f*h*m^2))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m
 + 6*m^2 + m^3 + 6)) + (c*e*(e + f*x)^m*(6*c^2*f^2*g + 2*d^2*e^2*g + c^2*f^
2*g*m^2 + d^2*e^2*g*m^2 + c*d*e^2*h - 3*c^2*e*f*h + 5*c^2*f^2*g*m + 3*d^2*e
^2*g*m - 6*c*d*e*f*g + c*d*e^2*h*m - c^2*e*f*h*m - 2*c*d*e*f*g*m^2 - 8*c*d*
e*f*g*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d^2
*f^2*x^4*(e + f*x)^m*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/((c*f
 - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d*f*x^3*(e + f*x)^
m*(4*c*f + c*f*m - d*e*m)*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/
((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

3.136 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx = -\frac{(Ab-aB)(c+dx)^{1+n}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\operatorname{AppellF1}\left(1+n,1,-p,2+n,\frac{b(c+dx)}{bc-ad},-\frac{f(c+dx)}{de-cf}\right)}{b(bc-ad)(1+n)} - \frac{B(c+dx)^{1+n}(e+fx)^{1+p}\operatorname{Hypergeometric2F1}\left(1,2+n+p,2+p,\frac{d(e+fx)}{de-cf}\right)}{b(de-cf)(1+p)}$$

```
[Out] -(A*b-B*a)*(d*x+c)^(1+n)*(f*x+e)^p*AppellF1(1+n,1,-p,2+n,b*(d*x+c)/(-a*d+b*
c),-f*(d*x+c)/(-c*f+d*e))/b/(-a*d+b*c)/(1+n)/((d*(f*x+e)/(-c*f+d*e))^p)-B*(
d*x+c)^(1+n)*(f*x+e)^(p+1)*hypergeom([1, 2+n+p],[2+p],d*(f*x+e)/(-c*f+d*e))
/b/(-c*f+d*e)/(p+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {163, 72, 71, 142, 141}

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx$$
$$= \frac{B(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\operatorname{Hypergeometric2F1}\left(n+1,-p,n+2,-\frac{f(c+dx)}{de-cf}\right)}{bd(n+1)} - \frac{(Ab-aB)(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\operatorname{AppellF1}\left(n+1,-p,1,n+2,-\frac{f(c+dx)}{de-cf},\frac{b(c+dx)}{bc-ad}\right)}{b(n+1)(bc-ad)}$$

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] -(((A*b - a*B)*(c + d*x)^(1 + n)*(e + f*x)^p*AppellF1[1 + n, -p, 1, 2 + n,
-((f*(c + d*x))/(d*e - c*f)), (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*(1
 + n)*((d*(e + f*x))/(d*e - c*f))^p)) + (B*(c + d*x)^(1 + n)*(e + f*x)^p*Hy
pergeometric2F1[1 + n, -p, 2 + n, -((f*(c + d*x))/(d*e - c*f))])/(b*d*(1 +
n)*((d*(e + f*x))/(d*e - c*f))^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{B\int (c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{a+bx}\,dx}{b}$$

$$= \frac{\left(B(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\right)\int (c+dx)^n\left(\frac{de}{de-cf}+\frac{dfx}{de-cf}\right)^p dx}{b} + \frac{\left((Ab-aB)(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\right)\int \frac{(c+dx)^n\left(\frac{de}{de-cf}+\frac{dfx}{de-cf}\right)^p}{a+bx}\,dx}{b}$$

$$= -\frac{(Ab-aB)(c+dx)^{1+n}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F_1\left(1+n;-p,1;2+n;-\frac{f(c+dx)}{de-cf},\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)(1+n)} + \frac{B(c+dx)^{1+n}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}{}_2F_1\left(1+n,-p;2+n;-\frac{f(c+dx)}{de-cf}\right)}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx$$

$$= \frac{(c+dx)^n(e+fx)^p\left(\frac{(Ab-aB)\left(\frac{b(c+dx)}{d(a+bx)}\right)^{-n}\left(\frac{b(e+fx)}{f(a+bx)}\right)^{-p}\text{AppellF1}\left(-n-p,-n,-p,1-n-p,\frac{-bc+ad}{d(a+bx)},\frac{-be+af}{f(a+bx)}\right)}{n+p} + \frac{bB\left(\frac{f(c+dx)}{-de+cf}\right)^{-n}(e+fx \cdots}{\cdots}\right.}{b^2}$$

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] ((c + d*x)^n*(e + f*x)^p*(((A*b - a*B)*AppellF1[-n - p, -n, -p, 1 - n - p,
(-(b*c) + a*d)/(d*(a + b*x)), (-(b*e) + a*f)/(f*(a + b*x))])/((n + p)*((b*(
c + d*x))/(d*(a + b*x)))^n*((b*(e + f*x))/(f*(a + b*x)))^p) + (b*B*(e + f*x
)*Hypergeometric2F1[-n, 1 + p, 2 + p, (d*(e + f*x))/(d*e - c*f)])/(f*(1 + p
)*((f*(c + d*x))/(-(d*e) + c*f))^n)))/b^2
```

Maple [F]

$$\int \frac{(Bx + A)(dx + c)^n (fx + e)^p}{bx + a} dx$$

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(A + Bx)(c + dx)^n(e + fx)^p}{a + bx} dx = \int \frac{(Bx + A)(dx + c)^n(fx + e)^p}{bx + a} dx$$

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="fricas")
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + Bx)(c + dx)^n(e + fx)^p}{a + bx} dx = \text{Timed out}$$

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + Bx)(c + dx)^n(e + fx)^p}{a + bx} dx = \int \frac{(Bx + A)(dx + c)^n(fx + e)^p}{bx + a} dx$$

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="maxima")
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Giac [F]

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx = \int \frac{(Bx+A)(dx+c)^n(fx+e)^p}{bx+a}\,dx$$

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="giac")
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx = \int \frac{(e+f\,x)^p\,(A+B\,x)\,(c+d\,x)^n}{a+b\,x}\,dx$$

```
[In] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x),x)
[Out] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x), x)
```

3.137 $\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 233

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = -\frac{d(Be-Af)(a+bx)^{1+m}(c+dx)^{-m}}{(bc-ad)f^2m} - \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,\text{Hypergeometric2F1}\left(1,-m,1-m,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m} - \frac{(aBdfm-b(Bde-Adf+Bcfm))(a+bx)^{1+m}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\text{Hypergeometric2F1}\left(m,1+m,$$

$$b(bc-ad)f^2m(1+m)$$

```
[Out] -d*(-A*f+B*e)*(b*x+a)^(1+m)/(-a*d+b*c)/f^2/m/((d*x+c)^m)-(-A*f+B*e)*(b*x+a)
^m*hypergeom([1, -m],[1-m],(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f^2/m/((d
*x+c)^m)-(a*B*d*f*m-b*(B*c*f*m-A*d*f+B*d*e))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d
+b*c))^m*hypergeom([m, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)/f^2/m
/(1+m)/((d*x+c)^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {161, 133, 80, 72, 71}

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx$$
$$= -\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\,\text{Hypergeometric2F1}\left(1,-m,1-m,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m}$$
$$-\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\text{Hypergeometric2F1}\left(m,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)(aBdfm-b(-Adf}{bf^2m(m+1)(bc-ad)}$$
$$-\frac{d(a+bx)^{m+1}(Be-Af)(c+dx)^{-m}}{f^2m(bc-ad)}$$

```
[In] Int[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] -((d*(B*e - A*f)*(a + b*x)^(1 + m))/((b*c - a*d)*f^2*m*(c + d*x)^m)) - ((B*
e - A*f)*(a + b*x)^m*Hypergeometric2F1[1, -m, 1 - m, ((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))])/(f^2*m*(c + d*x)^m) - ((a*B*d*f*m - b*(B*d*e -
A*d*f + B*c*f*m))*(a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeom
etric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*f^2
*m*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 161

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((g_.) + (h_.)*(x_
)))/((e_.) + (f_.)*(x_)), x_Symbol] :> Dist[(f*g - e*h)*((c*f - d*e)^(m + n
 + 1)/f^(m + n + 2)), Int[(a + b*x)^m/((c + d*x)^(m + 1)*(e + f*x)), x], x]
 + Dist[1/f^(m + n + 2), Int[((a + b*x)^m/(c + d*x)^(m + 1))*ExpandToSum[(f
^(m + n + 2)*(c + d*x)^(m + n + 1)*(g + h*x) - (f*g - e*h)*(c*f - d*e)^(m +
 n + 1))/(e + f*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGt
Q[m + n + 1, 0] && (LtQ[m, 0] || SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (a+bx)^m (c+dx)^{-1-m} (-Bde+Bcf+Adf+Bdfx)\,dx}{f^2} \\
&\quad + \frac{((Be-Af)(de-cf)) \int \frac{(a+bx)^m (c+dx)^{-1-m}}{e+fx}\,dx}{f^2} \\
&= -\frac{d(Be-Af)(a+bx)^{1+m}(c+dx)^{-m}}{(bc-ad)f^2m} \\
&\quad - \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,-m;1-m;\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m} \\
&\quad + \frac{(bd(-Bde+Bcf+Adf)-Bdf(-adm+bc(1+m)))\int (a+bx)^m(c+dx)^{-m}\,dx}{d(-bc+ad)f^2m} \\
&= -\frac{d(Be-Af)(a+bx)^{1+m}(c+dx)^{-m}}{(bc-ad)f^2m} \\
&\quad - \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,-m;1-m;\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m} \\
&\quad + \frac{\left((bd(-Bde+Bcf+Adf)-Bdf(-adm+bc(1+m)))(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\right)\int (a+bx)^m \left(\frac{}{bc\cdots}\right.}{d(-bc+ad)f^2m}
\end{aligned}$$

$$= -\frac{d(Be - Af)(a + bx)^{1+m}(c + dx)^{-m}}{(bc - ad)f^2m} - \frac{(Be - Af)(a + bx)^m(c + dx)^{-m}\,{}_2F_1\left(1, -m; 1 - m; \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m} - \frac{(aBdfm - b(Bde - Adf + Bcfm))(a + bx)^{1+m}(c + dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\,{}_2F_1\left(m, 1 + m; 2 + m; -\frac{d(a+bx)}{bc-ad}\right)}{b(bc - ad)f^2m(1 + m)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.75

$$\int \frac{(a + bx)^m(A + Bx)(c + dx)^{-m}}{e + fx}\,dx = \frac{(a + bx)^m(c + dx)^{-m}\left(b(Be - Af)(1 + m)\operatorname{Hypergeometric2F1}\left(1, m, 1 + m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right) + \left(\frac{b(c+dx)}{bc-ad}\right)^m\left(\ldots\right)\right)}{\ldots}$$

```
[In] Integrate[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] ((a + b*x)^m*(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[1, m, 1 + m, ((d*e -
c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))] + ((b*(c + d*x))/(b*c - a*d))^m*(-
(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[m, m, 1 + m, (d*(a + b*x))/(-(b*c)
 + a*d)]) + B*f*m*(a + b*x)*Hypergeometric2F1[m, 1 + m, 2 + m, (d*(a + b*x)
)/(-(b*c) + a*d)])))/(b*f^2*m*(1 + m)*(c + d*x)^m)
```

Maple [F]

$$\int \frac{(bx + a)^m (Bx + A)(dx + c)^{-m}}{fx + e}dx$$

```
[In] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

Fricas [F]

$$\int \frac{(a + bx)^m(A + Bx)(c + dx)^{-m}}{e + fx}\,dx = \int \frac{(Bx + A)(bx + a)^m}{(fx + e)(dx + c)^m}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*x+a)**m*(B*x+A)/((d*x+c)**m)/(f*x+e),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="maxima")
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m}{(e+f\,x)\,(c+d\,x)^m}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m),x)
[Out] int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m), x)
```

3.138 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 250

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \frac{2(Ab-aB)\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-n,-p,\frac{3}{2},-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2} + \frac{2B(a+bx)^{3/2}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-n,-p,\frac{5}{2},-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{3b^2}$$

```
[Out] 2/3*B*(b*x+a)^(3/2)*(d*x+c)^n*(f*x+e)^p*AppellF1(3/2,-n,-p,5/2,-d*(b*x+a)/(
-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)
/(-a*f+b*e))^p)+2*(A*b-B*a)*(d*x+c)^n*(f*x+e)^p*AppellF1(1/2,-n,-p,3/2,-d*(
b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))*(b*x+a)^(1/2)/b^2/((b*(d*x+c)/(-a*
d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {165, 145, 144, 143}

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(Ab-aB)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-n,-p,\frac{3}{2},-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2} + \frac{2B(a+bx)^{3/2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-n,-p,\frac{5}{2},-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{3b^2}$$

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*AppellF1[1/2, -n, -p,
3/2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(b^2*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (2*B*(a + b*x)^(
3/2)*(c + d*x)^n*(e + f*x)^p*AppellF1[3/2, -n, -p, 5/2, -((d*(a + b*x))/(b*
c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(3*b^2*((b*(c + d*x))/(b*c - a*d)
)^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\text{integral} = \frac{B\int \sqrt{a+bx}(c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b} + \frac{\left((Ab-aB)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^p\,dx}{b} + \frac{\left((Ab-aB)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^p}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{2(Ab-aB)\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right.}{b^2} + \frac{2B(a+bx)^{3/2}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{3}{2};-n,-p;\frac{5}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{3b^2}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\left(3(Ab-aB)\,\text{AppellF1}\left(\frac{1}{2},-n,-p,\frac{3}{2},\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right.\right.}{3b^2}$$

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*(3*(A*b - a*B)*AppellF1[1/2, -n, -
p, 3/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + B*(a
+ b*x)*AppellF1[3/2, -n, -p, 5/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x
))/(-(b*e) + a*f)]))/(3*b^2*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b
*e - a*f))^p)
```

Maple [F]

$$\int \frac{(Bx + A)\,(dx + c)^n\,(fx + e)^p}{\sqrt{bx + a}}dx$$

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A + Bx)(c + dx)^n(e + fx)^p}{\sqrt{a + bx}}\,dx = \int \frac{(Bx + A)(dx + c)^n(fx + e)^p}{\sqrt{bx + a}}\,dx$$

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(A + Bx)(c + dx)^n(e + fx)^p}{\sqrt{a + bx}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a)**(1/2),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(A + Bx)(c + dx)^n(e + fx)^p}{\sqrt{a + bx}}\,dx = \int \frac{(Bx + A)(dx + c)^n(fx + e)^p}{\sqrt{bx + a}}\,dx$$

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \int \frac{(Bx+A)(dx+c)^n(fx+e)^p}{\sqrt{bx+a}}\,dx$$

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \int \frac{(e+f\,x)^p\,(A+B\,x)\,(c+d\,x)^n}{\sqrt{a+b\,x}}\,dx$$

```
[In] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x)^(1/2),x)
[Out] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x)^(1/2), x)
```

3.139 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 530

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$$
$$= \frac{(bg-ah)^3(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(1+m,-n,-p,2+m,-\frac{d(}{b}\right.}{b^4(1+m)}$$
$$+\frac{3h(bg-ah)^2(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(2+m,-n,-p,3+m\right.}{b^4(2+m)}$$
$$+\frac{3h^2(bg-ah)(a+bx)^{3+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(3+m,-n,-p,4+m\right.}{b^4(3+m)}$$
$$+\frac{h^3(a+bx)^{4+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(4+m,-n,-p,5+m,-\frac{d(a+bx)}{bc-ad},\right.}{b^4(4+m)}$$

```
[Out] (-a*h+b*g)^3*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b
*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)/((b*(f*x+e)/(-a*f+b*e))^p)+3*h*(-a*h+b*g)^2*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+
e)^p*AppellF1(2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^
4/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)+3*h^2*(-a*h+b
*g)*(b*x+a)^(3+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(3+m,-n,-p,4+m,-d*(b*x+a)/(-a
*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(3+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*
x+e)/(-a*f+b*e))^p)+h^3*(b*x+a)^(4+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(4+m,-n,-
p,5+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(4+m)/((b*(d*x+c)/(-
a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {187, 165, 145, 144, 143}

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$$
$$=\frac{3h^2(bg-ah)(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+3,-n,-p,m+4,-\frac{d}{b}\right.}{b^4(m+3)}$$
$$+\frac{(bg-ah)^3(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+1,-n,-p,m+2,-\frac{d(}{b}\right.}{b^4(m+1)}$$
$$+\frac{3h(bg-ah)^2(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+2,-n,-p,m+3,\right.}{b^4(m+2)}$$
$$+\frac{h^3(a+bx)^{m+4}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+4,-n,-p,m+5,-\frac{d(a+bx)}{bc-ad},-\right.}{b^4(m+4)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
```

```
[Out] ((b*g - a*h)^3*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^4*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(3*h*(b*g - a*h)^2*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m
, -n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))
])/(b^4*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p
) + (3*h^2*(b*g - a*h)*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3
 + m, -n, -p, 4 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a
*f))])/(b^4*(3 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f
))^p) + (h^3*(a + b*x)^(4 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[4 + m, -n,
-p, 5 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
4*(4 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 187

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\text{integral} = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)^2\,dx}{b} + \frac{(bg-ah)\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx}{b}$$

$$= \frac{h^2 \int (a+bx)^{2+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b^2} + 2\frac{(h(bg-ah)) \int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b^2} + \frac{(bg-ah)^2 \int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx}{b^2}$$

$$= \frac{h^3 \int (a+bx)^{3+m}(c+dx)^n(e+fx)^p\,dx}{b^3} + \frac{(h^2(bg-ah)) \int (a+bx)^{2+m}(c+dx)^n(e+fx)^p\,dx}{b^3} + \frac{(h(bg-ah)^2) \int (a+bx)^{1+m}(c+dx)^n(e+fx)^p\,dx}{b^3} + 2\left(\frac{(h^2(bg-ah)) \int (a+bx)^{2+m}(c+dx)^n(e+fx)^p\,dx}{b^3} + \frac{(h(bg-ah)^2) \int (a+bx)^{1+m}(c+dx)^n(e+fx)^p\,dx}{b^3}\right) + \frac{(bg-ah)^3 \int (a+bx)^m(c+dx)^n(e+fx)^p\,dx}{b^3}$$

$$= \frac{\left(h^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^{3+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^3} + \frac{\left(h^2(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^3} + \frac{\left(h(bg-ah)^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^3} + 2\left(\frac{\left(h^2(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^3} + \frac{\left(h(bg-ah)^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^3}\right) + \frac{\left((bg-ah)^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^3}$$

$$= \frac{\left(h^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{3+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)}{b^3}$$

$$+\frac{\left(h^2(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-}\right.}{b^3}$$

$$+\frac{\left(h(bg-ah)^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-}\right.}{b^3}$$

$$+2\left(\frac{\left(h^2(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\right.}{b^3}\right.$$

$$+\frac{\left(h(bg-ah)^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{b}{be-}\right.}{b^3}$$

$$+\frac{\left((bg-ah)^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}\right.}{b^3}$$

$$= \frac{(bg-ah)^3(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d(}{b}\right.}{b^4(1+m)}$$

$$+\frac{h(bg-ah)^2(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(2+m;-n,-p;3+m;\right.}{b^4(2+m)}$$

$$+\frac{h^2(bg-ah)(a+bx)^{3+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(3+m;-n,-p;4+m;\right.}{b^4(3+m)}$$

$$+2\left(\frac{h(bg-ah)^2(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(2+m;-n,-p;3+\right.}{b^4(2+m)}\right.$$

$$+\frac{h^2(bg-ah)(a+bx)^{3+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(3+m;-n,-p;4+m;\right.}{b^4(3+m)}$$

$$+\frac{h^3(a+bx)^{4+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(4+m;-n,-p;5+m;-\frac{d(a+bx)}{bc-ad},\right.}{b^4(4+m)}$$

Mathematica [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3, x]
```

Maple [F]

$$\int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,(hx+g)^3\,dx$$

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \int (hx+g)^3(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="fricas")
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*(b*x + a)^m*(d*x + c)^n*
(f*x + e)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \int (hx+g)^3(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="maxima")
[Out] integrate((h*x + g)^3*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Giac [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \int (e+f\,x)^p\,(g+h\,x)^3\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

```
[In] int((e + f*x)^p*(g + h*x)^3*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(g + h*x)^3*(a + b*x)^m*(c + d*x)^n, x)
```

3.140 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 393

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$

$$= \frac{(bg-ah)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(1+m,-n,-p,2+m,-\frac{d(a+}{bc-}\right.}{b^3(1+m)}$$

$$+\frac{2h(bg-ah)(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(2+m,-n,-p,3+m,-\right.}{b^3(2+m)}$$

$$+\frac{h^2(a+bx)^{3+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(3+m,-n,-p,4+m,-\frac{d(a+bx)}{bc-ad},-\right.}{b^3(3+m)}$$

```
[Out] (-a*h+b*g)^2*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b
*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)/((b*(f*x+e)/(-a*f+b*e))^p)+2*h*(-a*h+b*g)*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)
^p*AppellF1(2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/
(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)+h^2*(b*x+a)^(3+
m)*(d*x+c)^n*(f*x+e)^p*AppellF1(3+m,-n,-p,4+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x
+a)/(-a*f+b*e))/b^3/(3+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e)
)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {187, 165, 145, 144, 143}

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$
$$= \frac{(bg-ah)^2(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+1,-n,-p,m+2,-\frac{d(a}{bc}\right.}{b^3(m+1)}$$
$$+\frac{2h(bg-ah)(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+2,-n,-p,m+3,\right.}{b^3(m+2)}$$
$$+\frac{h^2(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+3,-n,-p,m+4,-\frac{d(a+bx)}{bc-ad},\right.}{b^3(m+3)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
```

```
[Out] ((b*g - a*h)^2*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^3*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(2*h*(b*g - a*h)*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m,
-n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])
/(b^3*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
+ (h^2*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3 + m, -n, -p, 4
+ m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^3*(3 +
 m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rule 187

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b} \\ &\quad + \frac{(bg-ah)\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx}{b} \\ &= \frac{h^2\int (a+bx)^{2+m}(c+dx)^n(e+fx)^p\,dx}{b^2} \\ &\quad + 2\frac{(h(bg-ah))\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p\,dx}{b^2} \\ &\quad + \frac{(bg-ah)^2\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx}{b^2}\end{aligned}$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int(a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^2}$$
$$+2\frac{\left(h(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int(a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^2}$$
$$+\frac{\left((bg-ah)^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int(a+bx)^{m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b^2}$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)}{b^2}$$
$$+2\frac{\left(h(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{b}{be-}\right.}{b^2}$$
$$+\frac{\left((bg-ah)^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}\right.}{b^2}$$

$$= \frac{(bg-ah)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d(}{b}\right.}{b^3(1+m)}$$
$$+\frac{2h(bg-ah)(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(2+m;-n,-p;3+m;\right.}{b^3(2+m)}$$
$$+\frac{h^2(a+bx)^{3+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(3+m;-n,-p;4+m;-\frac{d(a+bx)}{bc-ad},\right.}{b^3(3+m)}$$

Mathematica [F]

$$\int(a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx=\int(a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2, x]
```

Maple [F]

$$\int (bx+a)^m (dx+c)^n (fx+e)^p (hx+g)^2 dx$$

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (hx+g)^2(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="fricas")
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (hx+g)^2(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="maxima")
[Out] integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (hx+g)^2(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="giac")
[Out] integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (e+f\,x)^p\,(g+h\,x)^2\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

```
[In] int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n, x)
```

3.141 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(1+m,-n,-p,2+m,-\frac{d(a+b}{bc-a}\right.}{b^2(1+m)} + \frac{h(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(2+m,-n,-p,3+m,-\frac{d(a+bx)}{bc-ad},-\frac{f}{}\right.}{b^2(2+m)}$$

```
[Out] (-a*h+b*g)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*x
+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)/
((b*(f*x+e)/(-a*f+b*e))^p)+h*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(2+m
,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(d*x+
c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {165, 145, 144, 143}

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+1,-n,-p,m+2,-\frac{d(a+b}{bc-a}\right.}{b^2(m+1)} + \frac{h(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+2,-n,-p,m+3,-\frac{d(a+bx)}{bc-ad},-\frac{f}{}\right.}{b^2(m+2)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
```

```
[Out] ((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n,
-p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (h
*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m, -n, -p, 3 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((
b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{h\int(a+bx)^{1+m}(c+dx)^n(e+fx)^p\,dx}{b}+\frac{(bg-ah)\int(a+bx)^m(c+dx)^n(e+fx)^p\,dx}{b}\\
&= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int(a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b}\\
&\quad+\frac{\left((bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b}\\
&= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^p\,dx}{b}\\
&\quad+\frac{\left((bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^p}{b}\\
&= \frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d(a+bx)}{bc-ad},-\right.}{b^2(1+m)}\\
&\quad+\frac{h(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(2+m;-n,-p;3+m;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(2+m)}
\end{aligned}
$$

Mathematica [F]

$$
\int(a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx=\int(a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx
$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x), x]
```

Maple [F]

$$\int (bx+a)^m (dx+c)^n (fx+e)^p (hx+g)\, dx$$

[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)

[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\, dx = \int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\, dx$$

[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="fricas")

[Out] integral((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\, dx = \text{Timed out}$$

[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g),x)

[Out] Timed out

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\, dx = \int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\, dx$$

[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="maxima")

[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="giac")
[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (e+f\,x)^p\,(g+h\,x)\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

```
[In] int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

3.142 $\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^p \left(\frac{b(e+fx)}{be-af}\right)^{-p} \operatorname{AppellF1}\left(1+m,-n,-p,2+m,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(1+m)}$$

```
[Out] (b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*x+a)/(-a*d+b
*c),-f*(b*x+a)/(-a*f+b*e))/b/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(
-a*f+b*e))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {145, 144, 143}

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx$$
$$= \frac{(a+bx)^{m+1}(c+dx)^n (e+fx)^p \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(\frac{b(e+fx)}{be-af}\right)^{-p} \operatorname{AppellF1}\left(m+1,-n,-p,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(m+1)}$$

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rubi steps

$$\text{integral} = \left((c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right) \int (a+bx)^m \left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n (e+fx)^p\,dx$$

$$= \left((c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^p \left(\frac{b(e+fx)}{be-af}\right)^{-p}\right) \int (a+bx)^m \left(\frac{bc}{bc-ad} + \frac{bdx}{bc-ad}\right)^n \left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^p dx$$

$$= \frac{(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^p \left(\frac{b(e+fx)}{be-af}\right)^{-p} F_1\Big(1+m;-n,-p;2+m;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+}{be-}}{b(1+m)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx = \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(1+m,-n,-p,2+m,\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right)}{b(1+m)}$$

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, (
d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Maple [F]

$$\int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,dx$$

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx = \int (bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="fricas")
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx = \int (bx+a)^m (dx+c)^n (fx+e)^p \, dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="maxima")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Giac [F]

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx = \int (bx+a)^m (dx+c)^n (fx+e)^p \, dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="giac")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx = \int (e+f\,x)^p\,(a+b\,x)^m\,(c+d\,x)^n \, dx$$

```
[In] int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n, x)
```

3.143 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \text{Int}\left(\frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx},x\right)$$

```
[Out] CannotIntegrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

```
[In] Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
[Out] Defer[Int][((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx=\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

```
[In] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
[Out] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(bx+a)^m\,(dx+c)^n\,(fx+e)^p}{hx+g}dx$$

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx=\int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="fricas")
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx=\text{Timed out}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p/(h*x+g),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="maxima")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Mupad [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(e+f\,x)^p\,(a+b\,x)^m\,(c+d\,x)^n}{g+h\,x}\,dx$$

```
[In] int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x),x)
[Out] int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x), x)
```

3.144 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 268

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \frac{(Ab-aB)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(1+m,-n,m+n,2+\right.}{b^2(1+m)} + \frac{B(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(2+m,-n,m+n,3+m,-\right.}{b^2(2+m)}$$

```
[Out] (A*b-B*a)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+
n)*AppellF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2
/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)+B*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^(-m-n)*
(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(2+m,-n,m+n,3+m,-d*(b*x+a)/(-a*d+b*c),
-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {165, 145, 144, 143}

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+1,-n,m+n,m\right.}{b^2(m+1)} + \frac{B(a+bx)^{m+2}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+2,-n,m+n,m+3,-\right.}{b^2(m+2)}$$

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
```

```
[Out] ((A*b - a*B)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x)
)/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b
*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(1 + m)*((b*(c + d*x))/(b*c
 - a*d))^n) + (B*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[2 + m, -n, m + n, 3 + m, -((d*(a + b*x)
)/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((b*(c + d*x))/
(b*c - a*d))^n)
```

<u>Rule 143</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

<u>Rule 144</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

<u>Rule 145</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

<u>Rule 165</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\text{integral} = \frac{B\int (a+bx)^{1+m}(c+dx)^n(e+fx)^{-m-n}\,dx}{b} + \frac{(Ab-aB)\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}\,dx}{b} + \frac{\left((Ab-aB)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{b}{be-}\right.}{b} + \frac{\left((Ab-aB)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{}{be}\right.}{b}$$

$$= \frac{(Ab-aB)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m;-n,m+n;2+\right.}{b^2(1+m)} + \frac{B(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(2+m;-n,m+n;3+m;-\right.}{b^2(2+m)}$$

Mathematica [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
[Out] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n), x]
```

Maple [F]

$$\int (bx+a)^{m} (Bx+A) (dx+c)^{n} (fx+e)^{-n-m} \, dx$$

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-n-m),x)
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-n-m),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n} \, dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n} \, dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="fricas")
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n} \, dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-m-n),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n} \, dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n} \, dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="maxima")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$
$$=\int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx=\int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n), x)
```

3.145 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 283

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$$
$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\operatorname{AppellF1}\left(1+m,-n,m+n,2+m,-\frac{d}{b}\right.}{bf(1+m)}$$
$$-\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\operatorname{AppellF1}\left(1+m,-n,1+m\right.}{f(be-af)(1+m)}$$

```
[Out] B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appel
lF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b/f/(1+m)/(
(b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)
*(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*d+b*
c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {165, 145, 144, 143}

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$$
$$= \frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\operatorname{AppellF1}\left(m+1,-n,m+n,m+2,-\frac{d(}{b}\right.}{bf(m+1)}$$
$$-\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\operatorname{AppellF1}\left(m+1,-n,m+n\right.}{f(m+1)(be-af)}$$

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b*c - a*d))
, -((f*(a + b*x))/(b*e - a*f))])/(b*f*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n
) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a +
b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*(
(b*(c + d*x))/(b*c - a*d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\text{integral} = \frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n}\,dx}{f} + \frac{(-Be+Af)\int (a+bx)^m(c+dx)^n(e+fx)^{-1-m-n}\,dx}{f}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}\,dx}{f} + \frac{\left((-Be+Af)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-1-m-n}\,dx}{f}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bf}{be-}\right.}{f} + \frac{\left(b(-Be+Af)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)}{f(be-af)}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m;-n,m+n;2+m;-\right.}{bf(1+m)} - \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m;-n,1+m\right.}{f(be-af)(1+m)}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.73

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{1-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{-1+m+n}\left(B(be-af)\operatorname{AppellF1}\left(1+m,-n,m\right.\right.}{f(be-af}$$

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(1 - m - n)*((b*(e + f*x))/(b*e -
a*f))^(-1 + m + n)*(B*(b*e - a*f)*AppellF1[1 + m, -n, m + n, 2 + m, (d*(a +
 b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + b*(-(B*e) + A*f)*App
ellF1[1 + m, -n, 1 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*
x))/(-(b*e) + a*f)]))/(f*(b*e - a*f)^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^
n)
```

Maple [F]

$$\int (bx+a)^m (Bx+A) (dx+c)^n (fx+e)^{-1-m-n} dx$$

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n} dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-1} dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="fricas
")
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n} dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-1-m-n),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n} dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-1} dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="maxima
")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$$
$$=\int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-1}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx=\int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+1}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1), x)
```

3.146 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 277

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(1+m,-n,1+m+n,2+m,-\right.}{f(be-af)(1+m)}$$
$$-\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1-m-n}\,\text{Hypergeometric2F1}\left(1+m,-n,2+\right.}{f(be-af)(1+m)}$$

```
[Out] B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appel
lF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b
*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*(f*
x+e)^(-1-m-n)*hypergeom([-n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x
+e))/f/(-a*f+b*e)/(1+m)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {165, 145, 144, 143, 134}

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$= \frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+1,-n,m+n+1,m+2,-\right.}{f(m+1)(be-af)}$$
$$-\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+\right.}{f(m+1)(be-af)}$$

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a + b*x))/(b*c - a
*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1
- m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*
c - a*d)*(e + f*x)))])/(f*(b*e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*
c - a*d)*(e + f*x)))^n)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
```

```
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\text{integral} = \frac{B\int(a+bx)^m(c+dx)^n(e+fx)^{-1-m-n}\,dx}{f} + \frac{(-Be+Af)\int(a+bx)^m(c+dx)^n(e+fx)^{-2-m-n}\,dx}{f}$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1-m-n}\,{}_2F_1\left(1+m,-n;2+m;-\frac{(de-}{(bc-}\right.}{f(be-af)(1+m)} + \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-1-m-n}\,dx}{f}$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1-m-n}\,{}_2F_1\left(1+m,-n;2+m;-\frac{(de-}{(bc-}\right.}{f(be-af)(1+m)} + \frac{\left(bB(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{}{be}\right.}{f(be-af)}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m;-n,1+m+n;2+m;\right.}{f(be-af)(1+m)} - \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1-m-n}\,{}_2F_1\left(1+m,-n;2+m;-\frac{(de-}{(bc-}\right.}{f(be-af)(1+m)}$$

Mathematica [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.78

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx =$$

$$-\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-1-m-n}\left(\frac{b(e+fx)}{be-af}\right)^n\left(B(e+fx)\left(\frac{b(e+fx)}{be-af}\right)^m\text{AppellF1}\left(1+m\right.\right.}{f(-b}$$

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*((b*(e + f*x))/(b*e
 - a*f))^n*(B*(e + f*x)*((b*(e + f*x))/(b*e - a*f))^m*AppellF1[1 + m, -n, 1
 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)
] + (-(B*e) + A*f)*Hypergeometric2F1[1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a +
 b*x))/((b*c - a*d)*(e + f*x))]))/(f*(-(b*e) + a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n))
```

Maple [F]

$$\int (bx+a)^m (Bx+A)(dx+c)^n (fx+e)^{-2-m-n}\,dx$$

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$=\int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-2}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-2-m-n),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-2}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="maxima
")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-2}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+2}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2), x)
```

3.147 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 263

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$$
$$=\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)}$$
$$-\frac{(b(Bce(1+m)+A(cf(1+n)-de(2+m+n)))+a(Adf(1+m)+B(de(1+n)-cf(2+m+n))))}{(be-af)^2(de-}$$

```
[Out] (-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-2-m-n)/(-a*f+b*e)/(-c*f+d*
e)/(2+m+n)-(b*(B*c*e*(1+m)+A*(c*f*(1+n)-d*e*(2+m+n)))+a*(A*d*f*(1+m)+B*(d*e
*(1+n)-c*f*(2+m+n))))*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom([-
n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^2/(-c*f+d*
e)/(1+m)/(2+m+n)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {160, 12, 134}

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$$
$$=\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)}$$
$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(m+1)-Bcf(m+n+2)+Bde(n+}{(m+1)(m+n}$$

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/((
b*e - a*f)*(d*e - c*f)*(2 + m + n)) - ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) -
A*d*e*(2 + m + n)) + a*(A*d*f*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))
*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 +
 m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e -
 a*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*
d)*(e + f*x)))^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rubi steps

$$\text{integral} = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \frac{\int (b(Bce(1+m)+Acf(1+n)-Ade(2+m+n))+a(Adf(1+m)+Bde(1+n)-Bcf(2+m+n)))}{(be-af)(de-cf)(2+m+n)}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \frac{(b(Bce(1+m)+Acf(1+n)-Ade(2+m+n))+a(Adf(1+m)+Bde(1+n)-Bcf(2+m}{(be-af)(de-cf)(2+m+n)}$$

$$= \frac{(Be - Af)(a + bx)^{1+m}(c + dx)^{1+n}(e + fx)^{-2-m-n}}{(be - af)(de - cf)(2 + m + n)}$$

$$- \frac{(b(Bce(1 + m) + Acf(1 + n) - Ade(2 + m + n)) + a(Adf(1 + m) + Bde(1 + n) - Bcf(2 + m}{(be - af)^2(de -}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.85

$$\int (a + bx)^m(A + Bx)(c + dx)^n(e + fx)^{-3-m-n}\,dx =$$

$$- \frac{(a + bx)^{1+m}(c + dx)^n(e + fx)^{-2-m-n}\left((-Be + Af)(c + dx) + \frac{(b(Bce(1+m)+Acf(1+n)-Ade(2+m+n))+a(Adf(1}{}\right.}{(be - af)(de - cf)(2 +}$$

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-2 - m - n)*((-(B*e) + A*f)*(c
+ d*x) + ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) - A*d*e*(2 + m + n)) + a*(A*d*f
*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))*(e + f*x)*Hypergeometric2F1[
1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*
e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n)))/((b
*e - a*f)*(d*e - c*f)*(2 + m + n)))
```

Maple [F]

$$\int (bx + a)^m (Bx + A) (dx + c)^n (fx + e)^{-3-m-n}\,dx$$

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

Fricas [F]

$$\int (a + bx)^m(A + Bx)(c + dx)^n(e + fx)^{-3-m-n}\,dx$$
$$= \int (Bx + A)(bx + a)^m(dx + c)^n(fx + e)^{-m-n-3}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-3-m-n),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-3}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="maxima
")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-3}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+3}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3), x)
```

3.148 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 558

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx$$
$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)}$$
$$+ \frac{(af(Adf(2+m)+B(de(1+n)-cf(3+m+n)))+b(Be(de+cf(1+m))+Af(cf(2+n)-de(4+}{(be-af)^2(de-cf)^2(2+m+n)(3+m+n)}$$
$$+ \frac{((2+m+n)(abcdf(Be-Af)+bde((aBcf+A(bde-bcf-adf))(3+m+n)-(Be-Af)(bc(1+}{}$$

```
[Out] (-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-3-m-n)/(-a*f+b*e)/(-c*f+d*
e)/(3+m+n)+(a*f*(A*d*f*(2+m)+B*(d*e*(1+n)-c*f*(3+m+n)))+b*(B*e*(d*e+c*f*(1+
m))+A*f*(c*f*(2+n)-d*e*(4+m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-2-m
-n)/(-a*f+b*e)^2/(-c*f+d*e)^2/(2+m+n)/(3+m+n)+((2+m+n)*(a*b*c*d*f*(-A*f+B*e
)+b*d*e*((a*B*c*f+A*(-a*d*f-b*c*f+b*d*e))*(3+m+n)-(-A*f+B*e)*(b*c*(1+m)+a*d
*(1+n)))-(a*d+b*c)*f*((a*B*c*f+A*(-a*d*f-b*c*f+b*d*e))*(3+m+n)-(-A*f+B*e)*(
b*c*(1+m)+a*d*(1+n))))-(b*c*(1+m)+a*d*(1+n))*(a*f*(A*d*f*(2+m)+B*(d*e*(1+n)
-c*f*(3+m+n)))+b*(B*e*(d*e+c*f*(1+m))+A*f*(c*f*(2+n)-d*e*(4+m+n)))))*(b*x+a
)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom([-n, 1+m],[2+m],-(-c*f+d*e)*(b
*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^3/(-c*f+d*e)^2/(1+m)/(2+m+n)/(3+m+n)/(
((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {160, 12, 134}

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx$$
$$= \frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(-bde(a(Adf(m+2)-Bcf(m+n+}{}$$
$$+\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-3}}{(m+n+3)(be-af)(de-cf)}$$
$$+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(e+fx)^{-m-n-2}(af(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Af(cf(}{(m+n+2)(m+n+3)(be-af)^2(de-cf)^2}$$

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-3 - m - n))/((
b*e - a*f)*(d*e - c*f)*(3 + m + n)) + ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n)
- B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*
(4 + m + n))))*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/
((b*e - a*f)^2*(d*e - c*f)^2*(2 + m + n)*(3 + m + n)) + (((2 + m + n)*(a*b*
c*d*f*(B*e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m
+ n)) + a*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d
)*f*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 +
m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*
f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(
1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(a + b*x)^(1 + m)*(c + d*x
)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*
f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)^3*(d*e - c*f)^2*(1 +
m)*(2 + m + n)*(3 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x))
)^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] &&  !IntegerQ[n]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rubi steps

$$\text{integral} = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} - \frac{\int (a+bx)^m(c+dx)^n(e+fx)^{-3-m-n}(b(Bce(1+m)+Acf(2+n)-Ade(3+m+n))+a(Adf(2+m}{(be-af)(de-cf)(3+m+n)}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(Be(de+cf(1+m))+Af(cf(2+n)-}{(be-af)^2(de-cf)^2(2+m+n)(3+m} + \frac{\int((2+m+n)(abcdf(Be-Af)-bde(b(Bce(1+m)+Acf(2+n)-Ade(3+m+n))+a(A}{}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(Be(de+cf(1+m))+Af(cf(2+n)-}{(be-af)^2(de-cf)^2(2+m+n)(3+m} + \frac{((2+m+n)(abcdf(Be-Af)-bde(b(Bce(1+m)+Acf(2+n)-Ade(3+m+n))+a(Adf}{}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(Be(de+cf(1+m))+Af(cf(2+n)-}{(be-af)^2(de-cf)^2(2+m+n)(3+m} + \frac{((2+m+n)(abcdf(Be-Af)-bde(b(Bce(1+m)+Acf(2+n)-Ade(3+m+n))+a(Adf}{}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 508, normalized size of antiderivative = 0.91

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx =$$

$$-\frac{(a+bx)^{1+m}(c+dx)^n(e+fx)^{-3-m-n}\left(-((Be-Af)(c+dx))-\frac{(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(Be(}{(be-af)(}\right.}{}$$

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-3 - m - n)*(-((B*e - A*f)*(c +
 d*x)) - ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e
*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n))))*(c + d*x)*(e +
 f*x))/((b*e - a*f)*(d*e - c*f)*(2 + m + n)) - (((2 + m + n)*(a*b*c*d*f*(B*
e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a
*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d)*f*(b*(B
*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 + m) + B*d*
e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*f*(A*d*f*
(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) +
 A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(e + f*x)^2*Hypergeometric2F1[1 + m
, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*e - a
*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)
*(e + f*x)))^n)))/((b*e - a*f)*(d*e - c*f)*(3 + m + n)))
```

Maple [F]

$$\int (bx+a)^m\,(Bx+A)\,(dx+c)^n\,(fx+e)^{-4-m-n}\,dx$$

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-4}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-4-m-n),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-4}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="maxima
")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-4}\,dx$$

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+4}}\,dx$$

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4), x)
```

3.149 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4}+\frac{b\arcsin(dx)}{2d^3}$$

```
[Out] 1/2*b*arcsin(d*x)/d^3-1/3*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/6*(3*b*d^2*x+6*a*d
^2+4*c)*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1623, 1823, 794, 222}

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{\sqrt{1-d^2x^2}(2(3ad^2+2c)+3bd^2x)}{6d^4}+\frac{b\arcsin(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*Sqrt[1 - d^2*x^2])/d^2 - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x(-2c-3ad^2-3bd^2x)}{\sqrt{1-d^2x^2}}\,dx}{3d^2} \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-4c-6ad^2-3bd^2x-2cd^2x^2)}{6d^4} + \frac{b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (Sqrt[1 - d^2*x^2]*(-4*c - 6*a*d^2 - 3*b*d^2*x - 2*c*d^2*x^2))/(6*d^4) + (b
*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.76

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(2\,\mathrm{csgn}(d)c\,d^2x^2\sqrt{-d^2x^2+1}+3\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)b\,d^2x+6\,\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\,a\,d^2+4\,\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\,c-3\,\mathrm{a}\right.}{6d^4\sqrt{-d^2x^2+1}}$
risch	$\frac{(2c\,d^2x^2+3b\,d^2x+6a\,d^2+4c)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{6d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{b\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(-d^2*x^2+1)^(1/2)+3
*(-d^2*x^2+1)^(1/2)*csgn(d)*b*d^2*x+6*csgn(d)*(-d^2*x^2+1)^(1/2)*a*d^2+4*cs
gn(d)*(-d^2*x^2+1)^(1/2)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)+(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}cx^2}{3\,d^2} - \frac{\sqrt{-d^2x^2+1}bx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}a}{d^2} + \frac{b\arcsin(dx)}{2\,d^3} - \frac{2\sqrt{-d^2x^2+1}c}{3\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{6\,bd\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right) - (6\,ad^2 + (2\,(dx+1)c + 3\,bd - 4\,c)(dx+1) - 3\,bd + 6\,c)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*d*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^2 + (2*(d*x + 1)*c +
3*b*d - 4*c)*(d*x + 1) - 3*b*d + 6*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Mupad [B] (verification not implemented)

Time = 8.05 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.09

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,b\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3} - \frac{14\,b\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5} + \frac{2\,b\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7} - \frac{2\,b\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4} - \frac{\sqrt{1-d\,x}\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

```
[In] int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x
 + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d
) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

3.150 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arcsin(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 1829, 655, 222}

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(2ad^2+c)\arcsin(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] ((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*d^2)*ArcTan[(d*x)/(-1
+ Sqrt[1 - d^2*x^2])])/d^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.61 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)dcx-2\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)a\,d^2+2\,\operatorname{csgn}(d)d\sqrt{-d^2x^2+1}\,b-\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)c\right)\operatorname{csgn}(}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(cx+2b)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(2a\,d^2+c)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{a\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin(dx)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\sqrt{-dx+1} - 2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
[Out] -1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2 +
 c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3} - \frac{14\,c\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5} + \frac{2\,c\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7} - \frac{2\,c\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.151 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\arcsin(dx)}{d}-a\text{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] b*arcsin(d*x)/d-a*arctanh((-d^2*x^2+1)^(1/2))-c*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1623, 1823, 858, 222, 272, 65, 214}

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -a\text{arctanh}\left(\sqrt{1-d^2x^2}\right)+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}-\frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2}\end{aligned}$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{2b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - a\log(x) + a\log\left(-1+\sqrt{1-d^2x^2}\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (2*b*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])
/d - a*Log[x] + a*Log[-1 + Sqrt[1 - d^2*x^2]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.61 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(-\operatorname{csgn}(d)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2-\operatorname{csgn}(d)\sqrt{-d^2x^2+1}\,c+\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-(dx+1)(dx-1)}}\right)bd\right)\operatorname{csgn}(d)}{d^2\sqrt{-d^2x^2+1}}$	96

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^2*(-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*
d^2-csgn(d)*(-d^2*x^2+1)^(1/2)*c+arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2
))*b*d)*csgn(d)/(-d^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\sqrt{-dx+1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.85 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.10

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*c/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(44) = 88.

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.08

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right|\right)-\left(\pi+2\arcta\right.}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(
sqrt(2) - sqrt(-d*x + 1)) + 2)) - a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1))
/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) - 2)) - (pi + 2*a
rctan(1/2*sqrt(d*x + 1)*((sqrt(2) - sqrt(-d*x + 1))^2/(d*x + 1) - 1)/(sqrt(
2) - sqrt(-d*x + 1))))*b*d + sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{\sqrt{1-d\,x}\,\left(\frac{c}{d^2}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}$$

```
[In] int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d))/
(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(d^2)^(1/2))))/(d^2)^(1/2)
```

3.152 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\,\text{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] c*arcsin(d*x)/d-b*arctanh((-d^2*x^2+1)^(1/2))-a*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1623, 1821, 858, 222, 272, 65, 214}

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\,\text{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx
\end{aligned}$$

$$= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)$$

$$= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}$$

$$= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{2c\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - b\log(x) + b\log\left(-1+\sqrt{1-d^2x^2}\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (2*c*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d
 - b*Log[x] + b*Log[-1 + Sqrt[1 - d^2*x^2]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.02

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\operatorname{csgn}(d)dbx-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)da+\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)cx\right)\sqrt{-dx+1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{\sqrt{-d^2x^2+1}\,xd}$	97
risch	$\frac{a\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{x\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{c\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}-b\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{(-dx+1)(dx+1)}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$	129

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-arctanh(1/(-d^2*x^2+1)^(1/2))*csgn(d)*d*b*x-(-d^2*x^2+1)^(1/2)*csgn(d)*d*
a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c*x)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*
csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\sqrt{-dx+1}ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.60

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iadG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{adG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}a}{x}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(44) = 88$.

Time = 0.40 (sec) , antiderivative size = 282, normalized size of antiderivative = 5.88

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\frac{4ad^2\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)^2-4}+bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx-}}{\sqrt{2-\sqrt{-}}}\right.\right.}{d}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] -(4*a*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2
) - sqrt(-d*x + 1)))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x
```

```
+ 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4) + b*d*log(abs(-(sqrt(2) - sqrt(-d*x
 + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - b*d
*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2)
 - sqrt(-d*x + 1)) - 2)) - (pi + 2*arctan(1/2*sqrt(d*x + 1)*((sqrt(2) - sqr
t(-d*x + 1))^2/(d*x + 1) - 1)/(sqrt(2) - sqrt(-d*x + 1))))*c)/d
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

```
[In] int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1))/
(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(d*x
 + 1)^(1/2))/x
```

3.153 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 71

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] -1/2*(a*d^2+2*c)*arctanh((-d^2*x^2+1)^(1/2))-1/2*a*(-d^2*x^2+1)^(1/2)/x^2-b
*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1623, 1821, 821, 272, 65, 214}

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\left(ad^2+2c\right)\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*Sqrt[1 - d^2*x^2])/x^2 - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{1}{2}\int \frac{-2b-(2c+ad^2)\,x}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{4}\left(-2c-ad^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)
\end{aligned}
$$

$$= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}\left(a+\frac{2c}{d^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)$$

$$= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}(2c+ad^2)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{1}{2}\left(-\frac{(a+2bx)\sqrt{1-d^2x^2}}{x^2} - (2c+ad^2)\log(x) + (2c+ad^2)\log\left(-1+\sqrt{1-d^2x^2}\right)\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-(((a + 2*b*x)*Sqrt[1 - d^2*x^2])/x^2) - (2*c + a*d^2)*Log[x] + (2*c + a*d
^2)*Log[-1 + Sqrt[1 - d^2*x^2]])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)^2\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2x^2+2\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)c\,x^2+2\sqrt{-d^2x^2+1}\,bx+\sqrt{-d^2x^2+1}\,a\right)}{2\sqrt{-d^2x^2+1}\,x^2}$	108
risch	$\frac{\sqrt{dx+1}\,(dx-1)(2bx+a)\sqrt{(-dx+1)(dx+1)}}{2x^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}} - \frac{\left(c+\frac{a\,d^2}{2}\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx+1)}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$	113

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
a*d^2*x^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*c*x^2+2*(-d^2*x^2+1)^(1/2)*b*x+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(ad^2+2\,c)x^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\sqrt{-dx+1}}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{\sqrt{-d^2x^2+1}b}{x} - \frac{\sqrt{-d^2x^2+1}a}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(61) = 122$.

Time = 0.42 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right|\right.}{}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/2*((a*d^3 + 2*c*d)*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + s
qrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - (a*d^3 + 2*c*d)*log(abs(-(s
qrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x
 + 1)) - 2)) - 4*(a*d^3*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*
x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 - 2*b*d^2*((sqrt(2) - sqrt(-d*x + 1))/
sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 + 4*a*d^3*((sqr
t(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x +
 1))) + 8*b*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(
sqrt(2) - sqrt(-d*x + 1))))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sq
rt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.39

$$\begin{aligned}\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx &= c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)\\ &\quad-\frac{\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^4}{2\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\,\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\,\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}\\ &\quad+\frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2}-\frac{a\,d^2\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2}\\ &\quad-\frac{b\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{32\,\left(\sqrt{d\,x+1}-1\right)^2}\end{aligned}$$

```
[In] int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((
d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^
(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2) - 1)^2
/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x +
 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2*((1 - d
*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.154 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 87

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{cx^2\sqrt{-1+dx}\sqrt{1+dx}}{3d^2} + \frac{\sqrt{-1+dx}\sqrt{1+dx}(2(2c+3ad^2)+3bd^2x)}{6d^4} + \frac{b\operatorname{arccosh}(dx)}{2d^3}$$

```
[Out] 1/2*b*arccosh(d*x)/d^3+1/3*c*x^2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/6*(3*b*d
^2*x+6*a*d^2+4*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1624, 1823, 794, 223, 212}

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{(1-d^2x^2)\,(2(3ad^2+2c)+3bd^2x)}{6d^4\sqrt{dx-1}\sqrt{dx+1}} + \frac{b\sqrt{d^2x^2-1}\operatorname{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}} - \frac{cx^2(1-d^2x^2)}{3d^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c +
3*a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) +
 (b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 +
d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-1+d^2x^2}\int\frac{x(a+bx+cx^2)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int\frac{x(2c+3ad^2+3bd^2x)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$
\begin{aligned}
&= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,dx, x, \frac{x}{\sqrt{-1+d^2x^2}}\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$
\begin{aligned}
&\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx \\
&= \frac{\sqrt{-1+dx}\sqrt{1+dx}(3d^2(2a+bx)+2c(2+d^2x^2))+6bd\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{6d^4}
\end{aligned}
$$

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2)) + 6*b
*d*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*d^4)
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

method	result
risch	$\frac{\left(2c\,d^2x^2+3b\,d^2x+6a\,d^2+4c\right)\sqrt{dx+1}\,\sqrt{dx-1}}{6d^4}+\frac{b\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(dx+1)(dx-1)}}{2d^2\sqrt{d^2}\,\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\,\operatorname{csgn}(d)c\,d^2x^2\sqrt{d^2x^2-1}+3\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)b\,d^2x+6\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)a\,d^2+4\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)c+3\ln\left(\left(\sqrt{d^2x^2-1}\right.\right.\right.}{6d^4\sqrt{d^2x^2-1}}$

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*c*d^2*x^2+3*b*d^2*x+6*a*d^2+4*c)*(d*x+1)^(1/2)*(d*x-1)^(1/2)/d^4+1/2
*b/d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-1)
)^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{dx-1}}{6\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d^2x^2-1}cx^2}{3\,d^2} + \frac{\sqrt{d^2x^2-1}bx}{2\,d^2} + \frac{\sqrt{d^2x^2-1}a}{d^2} + \frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3} + \frac{2\,\sqrt{d^2x^2-1}c}{3\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqrt(
d^2*x^2 - 1)*c/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\,(2\,ad^{11}-bd^{10}+2\,cd^9)}{d^{12}}\right)-\frac{6\,b\log(\sqrt{dx+1}-\sqrt{dx-1})}{d^2}}{6\,d}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
- 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d*x
 + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 12.70 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.66

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d\,x-1}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}+\frac{d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{d\,x+1}-1\right)}^8}} + \frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}$$

```
[In] int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3*d^2) +
(2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.155 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 52

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{(2b+cx)\sqrt{-1+dx}\sqrt{1+dx}}{2d^2}+\frac{(c+2ad^2)\,\text{arccosh}(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arccosh(d*x)/d^3+1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/
d^2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(52) = 104.

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.60, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {915, 1829, 655, 223, 212}

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{\sqrt{d^2x^2-1}(2ad^2+c)\,\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left((c+2ad^2)\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad +\frac{\left((c+2ad^2)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,dx,x,\frac{x}{\sqrt{-1+d^2x^2}}\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}
$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{(c+2ad^2)\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{d(2b+cx)\sqrt{-1+dx}\sqrt{1+dx}+2(c+2ad^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{2d^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x] + 2*(c + 2*a*d^2)*ArcTanh[Sqrt[
(-1 + d*x)/(1 + d*x)]])/(2*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 5.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.85

method	result
risch	$\frac{(cx+2b)\sqrt{dx-1}\,\sqrt{dx+1}}{2d^2}+\frac{(2a\,d^2+c)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(dx+1)(dx-1)}}{2d^2\sqrt{d^2}\,\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)dcx+2\ln\left(\left(\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^2+2\,\operatorname{csgn}(d)d\sqrt{d^2x^2-1}\,b+\ln\left(\left(\sqrt{d^2x^2-1}\,\operatorname{csgn}(d\right.\right.}{2d^3\sqrt{d^2x^2-1}}$

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/2*(2*a*d^2+c)/d^2*ln(x*d^2/
(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^
(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{dx-1}-(2\,ad^2+c)\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(44) = 88.

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}cx}{2\,d^2} + \frac{\sqrt{d^2x^2-1}b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.54

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\,(2\,ad^2+c)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
- 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 12.84 (sec) , antiderivative size = 312, normalized size of antiderivative = 6.00

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{b\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - \frac{\frac{14\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}+\frac{d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{d\,x+1}-1\right)^8}}$$

```
[In] int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(
1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.156 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+\frac{b\operatorname{arccosh}(dx)}{d}+a\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] b*arccosh(d*x)/d+a*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+c*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/d^2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(55) = 110.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1624, 1823, 858, 223, 212, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{a\sqrt{d^2x^2-1}\arctan\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\operatorname{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}-\frac{c(1-d^2x^2)}{d^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,dx, x, \frac{x}{\sqrt{-1+d^2x^2}}\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx, x, \sqrt{-1+d^2x^2}\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+2a\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2b\text{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
[Out] (c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + 2*a*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)
]] + (2*b*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result
default	$\frac{\left(-\operatorname{csgn}(d)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)c+\ln\left(\left(\sqrt{(dx+1)(dx-1)}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)bd\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d}{d^2\sqrt{d^2x^2-1}}$

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+(d^2*x^2-1)^(1/2)*csgn(d)*c+ln(
(((d*x+1)*(d*x-1))^(1/2)*csgn(d)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.59 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -a\arcsin\left(\frac{1}{d|x|}\right) + \frac{b\log\left(2\,d^2x + 2\sqrt{d^2x^2-1}d\right)}{d} + \frac{\sqrt{d^2x^2-1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*c/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) - \frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d} + \frac{\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2} - \frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - a\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right) - \ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

```
[In] int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) - 1
i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((
d*x + 1)^(1/2) - 1)))*1i
```

3.157 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x} + \frac{c\operatorname{arccosh}(dx)}{d} + b\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] c*arccosh(d*x)/d+b*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+a*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/x
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1624, 1821, 858, 223, 212, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{a(1-d^2x^2)}{x\sqrt{dx-1}\sqrt{dx+1}} + \frac{b\sqrt{d^2x^2-1}\arctan\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}} + \frac{c\sqrt{d^2x^2-1}\operatorname{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(c\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx,x,x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad+\frac{\left(c\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,dx,x,\frac{x}{\sqrt{-1+d^2x^2}}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad+\frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx,x,\sqrt{-1+d^2x^2}\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{b\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad+\frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x} + 2b\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2c\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + 2*b*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
 + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result	siz
risch	$\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{x}+\frac{\left(\frac{c\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-1}\right)}{\sqrt{d^{2}}}-b\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$	95
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\operatorname{csgn}(d)dbx+\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(d)da+\ln\left(\left(\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)cx\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{\sqrt{d^{2}x^{2}-1}\,xd}$	96

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x+(c*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/
(d^2)^(1/2)-b*arctan(1/(d^2*x^2-1)^(1/2)))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^
(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{ad^2x + 2\,bdx\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right) + \sqrt{dx+1}\sqrt{dx-1}ad - cx\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right.}{dx}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.24 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.93

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
```

```
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -b\arcsin\left(\frac{1}{d|x|}\right)+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}a}{x}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*a/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{8\,ad^2}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}-\frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

```
[In] int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i)
)/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)^(
1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((d*
x + 1)^(1/2) - 1)))*1i
```

3.158 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{x}+\frac{1}{2}(2c+ad^2)\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] 1/2*(a*d^2+2*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/2*a*(d*x-1)^(1/2)*(d*
x+1)^(1/2)/x^2+b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1624, 1821, 821, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d^2x^2-1}(ad^2+2c)\arctan\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{x\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*(1 - d^2*x^2))/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*Arc
Tan[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+(2c+ad^2)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{4\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx, x, \sqrt{-1+d^2x^2}\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{(2c+ad^2)\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(a+2bx)\sqrt{-1+dx}\sqrt{1+dx}}{2x^2} + (2c+ad^2)\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
[Out] ((a + 2*b*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (2*c + a*d^2)*ArcTan[S
qrt[(-1 + d*x)/(1 + d*x)]]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(2bx+a)}{2x^{2}}-\frac{\left(c+\frac{a\,d^{2}}{2}\right)\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$	76
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)^{2}\left(\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)a\,d^{2}x^{2}+2\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)c\,x^{2}-2\sqrt{d^{2}x^{2}-1}\,bx-\sqrt{d^{2}x^{2}-1}\,a\right)}{2\sqrt{d^{2}x^{2}-1}\,x^{2}}$	103

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(2*b*x+a)/x^2-(c+1/2*a*d^2)*arctan(1/(d^2*x
^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2\,bdx^2+2\,(ad^2+2\,c)x^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\sqrt{dx-1}}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right) - c\arcsin\left(\frac{1}{d|x|}\right) + \frac{\sqrt{d^2x^2-1}b}{x} + \frac{\sqrt{d^2x^2-1}a}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 - 1
)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(67) = 134$.

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{(ad^3+2\,cd)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) + \frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\right)\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.45 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.81

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}}-c\left(\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}-\frac{a\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{a\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

```
[In] int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2
))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.159 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 116

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{3x^3}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+\frac{(3c+2ad^2)\sqrt{-1+dx}\sqrt{1+dx}}{3x}+\frac{1}{2}bd^2\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] 1/2*b*d^2*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/3*a*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/x^3+1/2*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+1/3*(2*a*d^2+3*c)*(d*x-1)^(1
/2)*(d*x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1624, 1821, 849, 821, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{(1-d^2x^2)(2ad^2+3c)}{3x\sqrt{dx-1}\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{3x^3\sqrt{dx-1}\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\arctan\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(a*(1 - d^2*x^2))/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(
```

```
3*x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-
1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int\frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int\frac{3b+(3c+2ad^2)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int\frac{2(3c+2ad^2)+3bd^2x}{x^2\sqrt{-1+d^2x^2}}\,dx}{6\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad-\frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(bd^2\sqrt{-1+d^2x^2}\right)\int\frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad-\frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(bd^2\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+d^2x}}\,dx,x,x^2\right)}{4\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad+\frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx,x,\sqrt{-1+d^2x^2}\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad-\frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}}+\frac{bd^2\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{\sqrt{-1+dx}\sqrt{1+dx}(3x(b+2cx)+a(2+4d^2x^2))}{6x^3}+bd^2\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)))/(6*x^3
) + b*d^2*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
```

Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(4a\,d^{2}x^{2}+6c\,x^{2}+3bx+2a)}{6x^{3}}-\frac{b\,d^{2}\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\sqrt{(dx+1)(dx-1)}}{2\sqrt{dx-1}\,\sqrt{dx+1}}$	89
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)^{2}\left(3\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)b\,d^{2}x^{3}-4\sqrt{d^{2}x^{2}-1}\,a\,d^{2}x^{2}-6\sqrt{d^{2}x^{2}-1}\,c\,x^{2}-3\sqrt{d^{2}x^{2}-1}\,bx-2\sqrt{d^{2}x^{2}-1}\,a\right)}{6\sqrt{d^{2}x^{2}-1}\,x^{3}}$	123

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(4*a*d^2*x^2+6*c*x^2+3*b*x+2*a)/x^3-1/2*b*d
^2*arctan(1/(d^2*x^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x+1
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+2\,(2\,ad^3+3\,cd)x^3+(2\,(2\,ad^2+3\,c)x^2+3\,bx+2\,a)\sqrt{dx+1}\sqrt{c}}{6\,x^3}$$

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\,\sqrt{d^2x^2-1}ad^2}{3\,x}+\frac{\sqrt{d^2x^2-1}c}{x}+\frac{\sqrt{d^2x^2-1}b}{2\,x^2}+\frac{\sqrt{d^2x^2-1}a}{3\,x^3}$$

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2*
x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(92) = 184.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^8-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-\right.}}{\left(\left(\sqrt{dx+1}-\sqrt{}\right.\right.}}{3\,d}$$

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.62

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}} - \frac{b\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2} + \frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} + \frac{\sqrt{d\,x-1}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}} + \frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

```
[In] int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/((((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*
1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (c
*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d^2*x^2)
/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((d*x - 1
)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```